211en15

15

CIRCLES

You are already familiar with geometrical figures such as a line segment, an angle, a triangle, a quadrilateral and a circle. Common examples of a circle are a wheel, a bangle, alphabet O, etc. In this lesson we shall study in some detail about the circle and related concepts.

OBJECTIVES

After studying this lesson, you will be able to

- *define a circle*
- *give examples of various terms related to a circle*
- *illustrate congruent circles and concentric circles*
- *identify and illustrate terms connected with circles like chord, arc, sector, segment, etc.*
- *verify experimentally results based on arcs and chords of a circle*
- *use the results in solving problems*

EXPECTED BACKGROUND KNOWLEDGE

- Line segment and its length
- Angle and its measure
- Parallel and perpendicular lines
- Closed figures such as triangles, quadrilaterals, polygons, etc.
- Perimeter of a closed figure
- Region bounded by a closed figure
- Congruence of closed figures

15.1 CIRCLE AND RELATED TERMS

15.1.1 Circle

A circle is a collection of all points in a plane which are at a constant distance from a fixed point in the same plane.

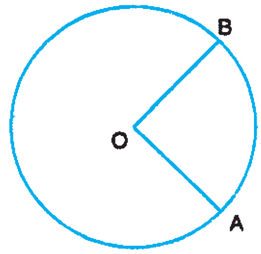


Fig. 15.1

Radius : A line segment joining the centre of the circle to a point on the circle is called its radius.

In Fig. 15.1, there is a circle with centre O and one of its radius is OA. OB is another radius of the same circle.

Activity for you : Measure the length OA and OB and observe that they are equal. Thus

All radii (plural of radius) of a circle are equal

The length of the radius of a circle is generally denoted by the letter 'r'. It is customry to write radius instead of the length of the radius.

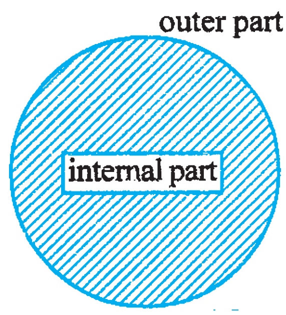


Fig. 15.2

A closed geometric figure in the plane divides the plane into three parts namely, the inner part of the figure, the figure and the outer part. In Fig. 15.2, the shaded portion is the inner part of the circle, the boundary is the circle and the unshaded portion is the outer part of the circle.

Activity for you

(a) Take a point Q in the inner part of the circle (See Fig. 15.3). Measure OQ and find that OQ < r. The inner part of the circle is called **the interior of the circle**.

(b) Now take a point P in the outer part of the circle (Fig. 15.3). Measure OP and find that OP > r. The outer part of the circle is called **the exterior of the circle.**

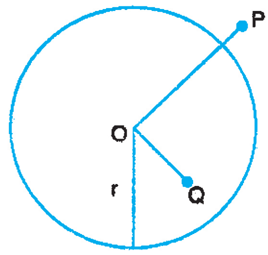


Fig. 15.3

15.1.2 Chord

A line segment joining any two points of a circle is called a chord. In Fig. 15.4, AB, PQ and CD are three chords of a circle with centre O and radius r. The chord PQ passes through the centre O of the circle. Such a chord is called a diameter of the circle. Diameter is usually denoted by 'd'.

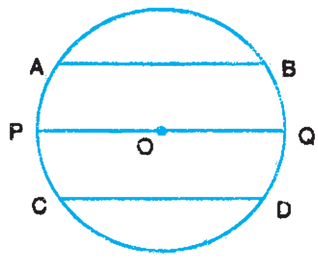


Fig. 15.4

A chord passing though the centre of a circle is called its diameter.

Activity for you :

Measure the length d of PQ, the radius r and find that d is the same as 2r. Thus we have

$d = 2r$

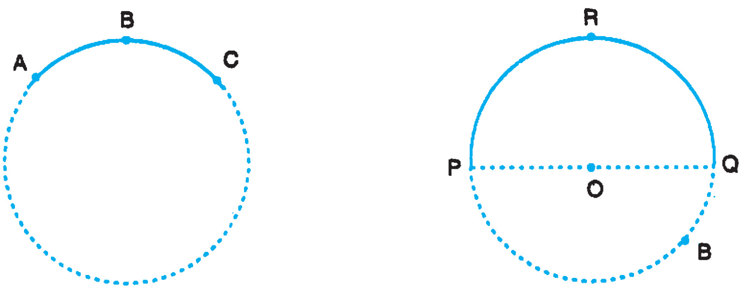
i.e. the diameter of a circle = twice the radius of the circle.

Measure the length PQ, AB and CD and find that PQ > AB and PQ > CD, we may conclude

Diameter is the longest chord of a circle.

15.1.3 Arc

A part of a circle is called an arc. In Fig. 15.5(a) ABC is an arc and is denoted by arc ABC



(a) (b)

Fig. 15.5

15.1.4 Semicircle

A diameter of a circle divides a circle into two equal arcs, each of which is known as a semicircle.

In Fig. 15.5(b), PQ is a diameter and $\widehat{PRQ}$ is semicircle and so is $\widehat{PBQ}$.

15.1.5 Sector

The region bounded by an arc of a circle and two radii at its end points is called a sector.

In Fig. 15.6, the shaded portion is a sector formed by the arc PRQ and the unshaded portion is a sector formed by the arc PTQ.

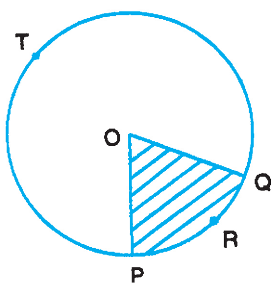


Fig. 15.6

15.1.6 Segment

A chord divides the interior of a circle into two parts,

each of which is called a segment. In Fig. 15.7, the shaded region PAQP and the unshaded region PBQP are both segments of the circle. PAQP is called a minor segment and PBQP is called a major segment.

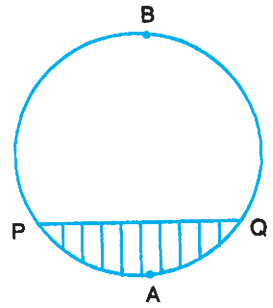


Fig. 15.7

15.1.7 Circumference

Choose a point P on a circle. If this point moves along the circle once and comes back to its original position then the distance covered by P is called the circumference of the circle

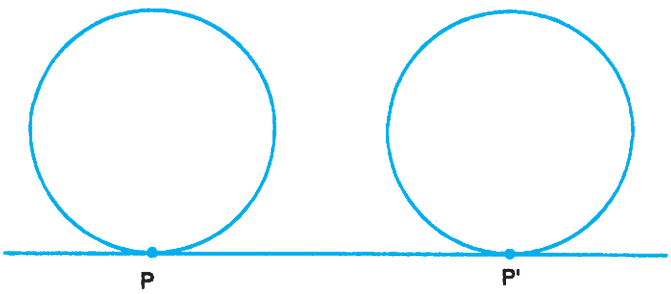


Fig. 15.8

Activity for you :

Take a wheel and mark a point P on the wheel where it touches the ground. Rotate the wheel along a line till the point P comes back on the ground. Measure the distance between the Ist and last position of P along the line. This distance is equal to the circumference of the circle. Thus,

The length of the boundary of a circle is the circumference of the circle.

Activity for you

Consider different circles and measures their circumference(s) and diameters. Observe that in each case the ratio of the circumference to the diameter turns out to be the same.

The ratio of the circumference of a circle to its diameter is always a constant. This constant is universally denoted by Greek letter π.

Therefore, $\frac{c}{d} = \frac{c}{2r} = \pi$, where c is the circumference of the circle, d its diameter and r is its radius.

An approximate value of π is $\frac{22}{7}$. Aryabhata -I (476 A.D.), a famous Indian Mathematician gave a more accurate value of π which is 3.1416. In fact this number is an irrational number.

15.2 MEASUREMENT OF AN ARC OF A CIRCLE

Consider an arc PAQ of a circle (Fig. 15.9). To measure its length we put a thread along PAQ and then measure the length of the thread with the help of a scale.

Similarly, you may measure the length of the arc PBQ.

15.2.1 Minor arc

An arc of circle whose length is less than that of a semi-circle of the same circle is called a minor arc. PAQ is a minor arc (See Fig. 15.9)

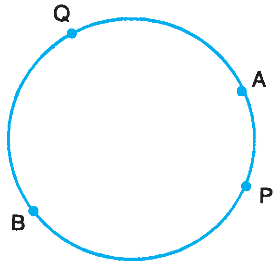


Fig. 15.9

15.2.2 Major arc

An arc of a circle whose length is greater than that of a semicircle of the same circle is called a major arc. In Fig. 15.9, arc PBQ is a major arc.

15.3 CONCENTRIC CIRCLES

Circles having the same centre but different radii are called concentric circles (See Fig. 15.10).

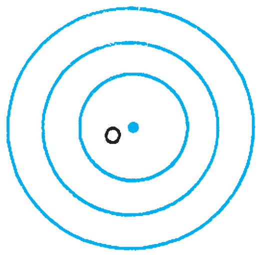


Fig. 15.10

15.4 CONGRUENT CIRCLES OR ARCS

Two cirlces (or arcs) are said to be congruent if we can superimpose (place) one over the other such that they cover each other completely.

15.5 SOME IMPORTANT RULES

Activity for you :

(i) Draw two circles with cenre O_1 and O_2 and radius r and s respectively (See Fig. 15.11)

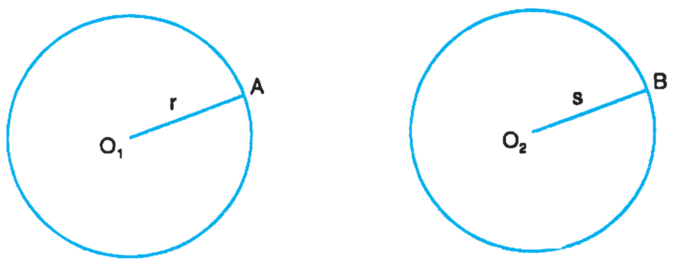


Fig. 15.11

(ii) Superimpose the circle (i) on the circle (ii) so that O_1 coincides with O_2.

(iii) We observe that circle (i) will cover circle (ii) if and only if r = s

Two circles are congruent if and only if they have equal radii.

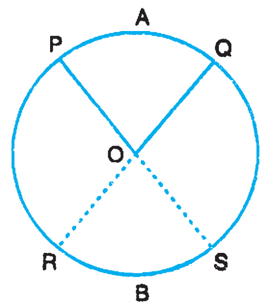


Fig. 15.12

In Fig. 15.12 if arc PAQ = arc RBS then $\angle$ POQ = $\angle$ ROS and conversely if $\angle$ POQ = $\angle$ ROS

then arc PAQ = arc RBS.

Two arcs of a circle are congruent if and only if the angles subtended by them at the centre are equal.

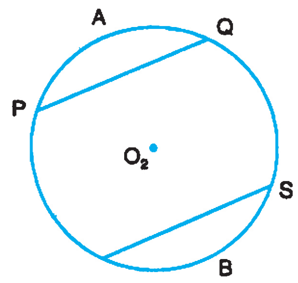


Fig. 15.13

In Fig. 15.13, if arc PAQ = arc RBS

then PQ = RS

and conversely if PQ = RS then

arc PAQ = arc RBS.

Two arcs of a circle are congruent if and only if their corresponding chords are equal.

Activity for you :

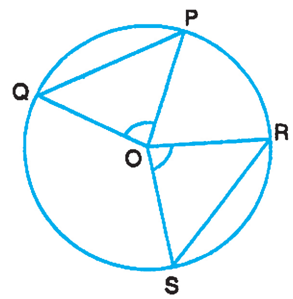


Fig. 15.14

(i) Draw a circle with centre O

(ii) Draw equal chords PQ and RS (See Fig. 15.14)

(iii) Join OP, OQ, OR and OS

(iv) Measure $\angle$ POQ and $\angle$ ROS

We observe that $\angle$ POQ = $\angle$ ROS

Conversely if $\angle$ POQ = $\angle$ ROS

then PQ = RS

Equal chords of a circle subtend equal angles at the centre and conversely if the angles subtended by the chords at the centre of a circle are equal, then the chords are equal.

Note : *The above results also hold good in case of congruent circles.*

We take some examples using the above properties :

Example 15.1 : In Fig. 15.15, chord PQ = chord RS. Show that chord PR = chord QS.

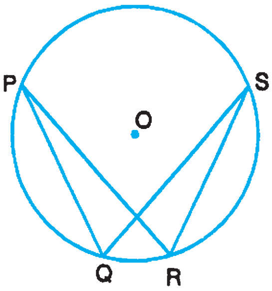


Fig. 15.15

Solution : The arcs corresponding to equal chords PQ and RS are equal.

Add to each arc, the arc QR,

yielding arc PQR = arc QRS

$\therefore$ chord PR = chord QS

Example 15.2 : In Fig. 15.16, arc AB = arc BC, $\angle$AOB = 30° and $\angle$AOD = 70°. Find $\angle$COD.

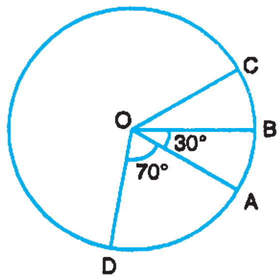


Fig. 15.16

Solution : Since arc AB = arc BC

$\therefore$ $\angle AOB = \angle BOC$

(Equals arcs subtend equal angles at the centre)

$\therefore$ $\angle BOC = 30°$

Now $\angle COD = \angle COB + \angle BOA + \angle AOD$

$= 30° + 30° + 70°$

$= 130°$.

Activity for you :

(i) Draw a circle with centre O (See Fig. 15.17).

(ii) Draw a chord PQ.

(iii) From O draw ON $\perp$ PQ

(iv) Measure PN and NQ

You will observe that

PN = NQ.

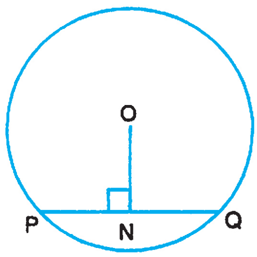


Fig. 15.17

The perpendicular drawn from the centre of a circle to a chord bisects the chord.

Activity for you :

(i) Draw a circle with centre O (See Fig. 15.18).

(ii) Draw a chord PQ.

(iii) Find the mid point M of PQ.

(iv) Join O and M.

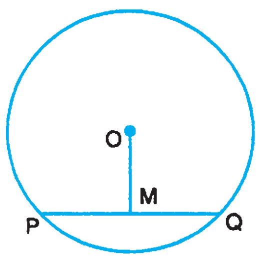


Fig. 15.18

(v) Measure $\angle$ OMP or $\angle$ OMQ with set square or protractor.

We observe that $\angle$ OMP = $\angle$ OMQ = 90^0.

The line joining the centre of a circle to the mid point of a chord is perpendicular to the chord.

Activity for you :

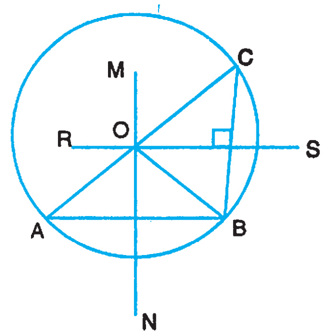


Fig. 15.20

Take three non collinear points A, B and C. Join AB and BC. Draw perpendicular bisectors MN and RS of AB and BC respectively.

Since A, B, C are not collinear, MN is not parallel to RS. They will intersect only at one point O. Join OA, OB and OC and measure them.

We observe that OA = OB = OC

Now taking O as the centre and OA as radius draw a circle which passes through A, B and C.

Repeat the above procedure with another three non-collinear points and observe that there is only one circle passing through three given non-collinear points.

There is one and only one circle passing through three non-collinear points.

Note. It is important to note that a circle can not be drawn to pass through three collinear points.

Activity for you :

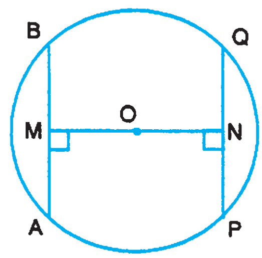


Fig. 15.20a

(i) Draw a circle with centre O [Fig. 15.20a]

(ii) Draw two equal chords AB and PQ of the circle.

(iii) Draw OM $\perp$ PQ and ON $\perp$ PQ

(iv) Measure OM and ON and observe that they are equal.

Equal chords of a circle are equidistant from the centre.

In Fig. 15.20 b, OM = ON

Measure and observe that AB = PQ. Thus,

Chords, that are equidistant from the centre of a circle, are equal.

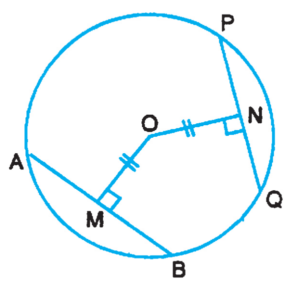


Fig. 15.20b

The above results hold good in case of congruent circles also.

We now take a few examples using these properties of circle.

Examples 15.3 : In Fig. 15.21, O is the centre of the circle and ON ⊥ PQ. If PQ = 8 cm and ON = 3 cm, find OP.

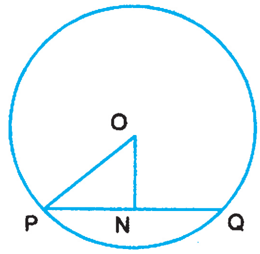


Fig. 15.21

Solution: ON ⊥ PQ (given) and since perpendicular drawn from the centre of a circle to a chord bisects the chord.

∴ PN = NQ = 4 cm

In a right triangle OPN,

∴ $OP^2 = PN^2 + ON^2$

or $OP^2 = 4^2 + 3^2 = 25$

∴ OP = 5 cm.

Examples 15.4 : In Fig. 15.22, OD is perpendicular to the chord AB of a circle whose centre is O and BC is a diameter. Prove that CA = 2OD.

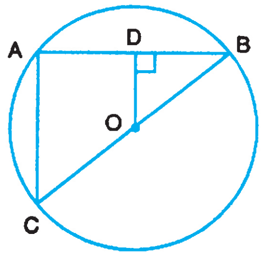


Fig. 15.22

Solution : Since OD ⊥ AB (Given)

∴ D is the mid point of AB (Perpendicular through the centre bisects the chord)

Also O is the mid point of CB (Since CB is a diameter)

Now in ΔABC, O and D are mid points of the two sides BC and BA of the triangle ABC. Since the line segment joining the mid points of any two sides of a triangle is parallel and half of the third side.

∴ $OD = \frac{1}{2} CA$

i.e. CA = 2OD.

Example 15.5 : A regular hexagon is inscribed in a circle. What angle does each side of the hexagon subtend at the centre?

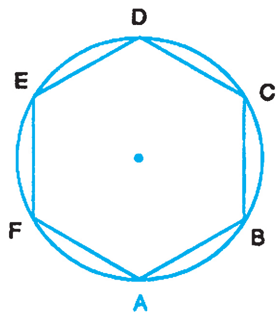


Fig. 15.23

Solution : A regular hexagon has six sides which are equal. Therefore each side subtends the same angle at the centre.

Let us suppose that a side of the hexagon subtends an angle x° at the centre.

Then, we have

$$6x^\circ = 360^\circ \Rightarrow x = 60^\circ$$

Hence, each side of the hexagon subtends an angle of 60° at the centre.

Example 15.6 : In Fig. 15.24, two parallel chords PQ and AB of a circle are of lengths 7 cm and 13 cm respectively. If the distance between PQ and AB is 3 cm, find the radius of the circle.

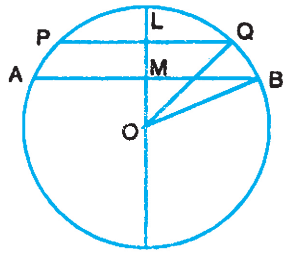


Fig. 15.24

Solution : Let O be the centre of the circle. Draw perpendicular bisector OL of PQ which also bisects AB at M. Join OQ and OB (Fig. 15.24)

Let OM = x cm and radius of the circle be r cm

Then $OB^2 = OM^2 + MB^2$ and $OQ^2 = OL^2 + LQ^2$

$$\therefore \quad r^2 = x^2 + \left(\frac{13}{2}\right)^2 \qquad \text{...(i)}$$

$$\text{and} \quad r^2 = (x+3)^2 + \left(\frac{7}{2}\right)^2 \qquad \text{...(ii)}$$

Therefore from (i) and (ii),

$$x^2 + \left(\frac{13}{2}\right)^2 = (x+3)^2 + \left(\frac{7}{2}\right)^2$$

$$\therefore \quad 6x = \frac{169}{4} - 9 - \frac{49}{4}$$

or 6x = 21

$$\therefore \quad x = \frac{7}{2}$$

$$\therefore \quad r^2 = \left(\frac{7}{2}\right)^2 + \left(\frac{13}{2}\right)^2 = \frac{49}{4} + \frac{169}{4} = \frac{218}{4}$$

$$\therefore \quad r = \frac{\sqrt{218}}{2}$$

Hence the radius of the circle is $r = \dfrac{\sqrt{218}}{2}$ cm.

CHECK YOUR PROGRESS 15.1

In questions 1 to 5, fill in the blanks to make each of the statements true.

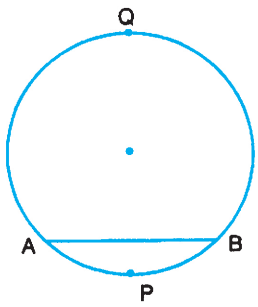


Fig. 15.25

1. In Fig. 15.25,

   (i) AB is a ... of the circle.

   (ii) Minor arc corresponding to AB is... .

2. A ... is the longest chord of a circle.

3. The ratio of the circumference to the diameter of a circle is always ... .

4. The value of π as 3.1416 was given by great Indian Mathematician... .

5. Circles having the same centre are called ... circles.

6. Diameter of a circle is 30 cm. If the length of a chord is 20 cm, find the distance of the chord from the centre.

7. Find the circumference of a circle whose radius is

   (i) 7 cm (ii) 11 cm. $\left(\text{Take } \pi = \dfrac{22}{7}\right)$

8. In the Fig. 15.26, RS is a diameter which bisects the chords PQ and AB at the points M and N respectively. Is PQ || AB ? Given reasons.

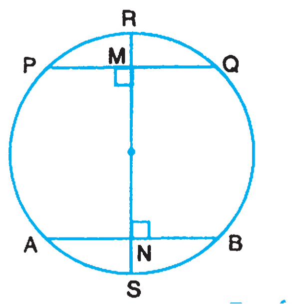


Fig. 15.26

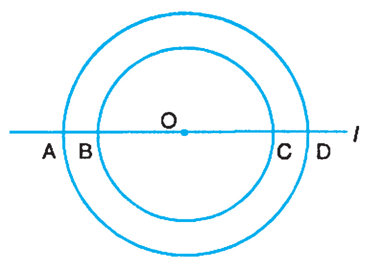


Fig. 15.27

9. In Fig. 15.27, a line *l* intersects the two concentric circles with centre O at points A, B, C and D. Is AB = CD ? Give reasons.

LET US SUM UP

- The circumference of a circle of radius r is equal to 2π r.

- Two arcs of a circle are congurent if and only if either the angles subtended by them at the centre are equal or their corresponding chords are equal.
- Equal chords of a circle subtend equal angles at the centre and vice versa.
- Perpendicular drawn from the centre of a circle to a chord bisects the chord.
- The line joining the centre of a circle to the mid point of a chord is perpendicular to the chord.
- There is one and only one circle passng through three non-collinear points.
- Equal chords of a circle are equidistant from the centre and the converse.

TERMINAL EXERCISE

1. If the length of a chord of a circle is 16 cm and the distance of the chord from the centre is 6 cm, find the radius of the circle.
2. Two circles with centres O and O′ (See Fig. 15.28) are congruent. Find the length of the arc CD.

Fig. 15.28

3. A regular pentagon is inscribed in a circle. Find the angle which each side of the pentagon subtends at the centre.
4. In Fig. 15.29, AB = 8 cm and CD = 6 cm are two parallel chords of a circle with centre O. Find the distance between the chords.

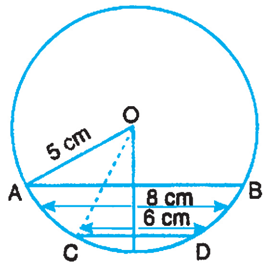


Fig. 15.29

5. In Fig.15.30 arc PQ = arc QR, $\angle POQ = 15^0$ and $\angle SOR = 110^o$. Find $\angle SOP$.

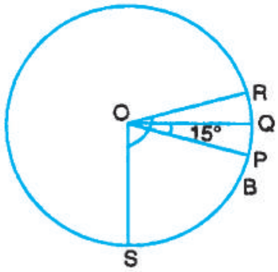


Fig. 15.30

6. In Fig. 15.31, AB and CD are two equal chords of a circle with centre O. Is chord BD = chord CA? Give reasons.

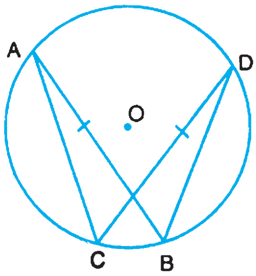


Fig. 15.31

7. If AB and CD are two equal chords of a circle with centre O (Fig. 15.32) and $OM \perp AB$, $ON \perp CD$. Is OM = ON ? Give reasons.

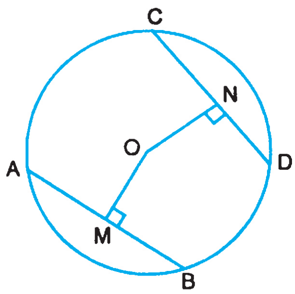


Fig. 15.32

8. In Fig. 15.33, AB = 14 cm and CD = 6 cm are two parallel chords of a circle with centre O. Find the distance between the chords AB and CD.

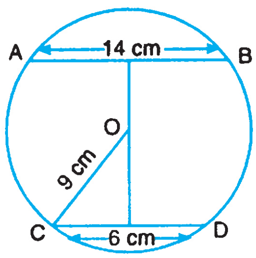


Fig. 15.33

9. In Fig. 15.34, AB and CD are two chords of a circle with centre O, intersecting at a point P inside the circle.

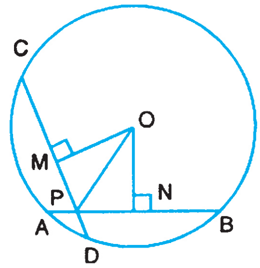


Fig. 15.34

OM $\perp$ CD, ON $\perp$ AB and $\angle$OPM = $\angle$OPN. Now answer:

Is (i) OM = ON, (ii) AB = CD ? Give reasons.

10. C_1 and C_2 are concentric circles with centre O (See Fig. 15.35), l is a line intersecting C_1 at points P and Q and C_2 at points A and B respectively, ON $\perp l$, is PA = BQ? Give reasons.

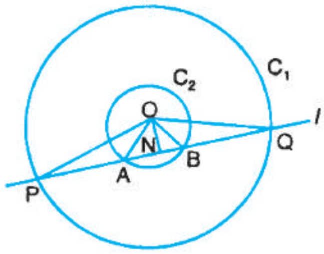


Fig. 15.35

ANSWERS TO CHECK YOUR PROGRESS

15.1

1. (i) Chord (ii) APB 2. Diameter 3. Constant
4. Aryabhata-I 5. Concentric 6. $5\sqrt{5}$ cm.
7. (i) 44 cm (ii) 69.14 cm 8. Yes 9. Yes

ANSWERS TO TERMINAL EXERCISE

1. 10 cm 2. 2a cm 3. 72°
4. 1 cm 5. 80°
6. Yes (Equal arcs have corresponding equal chrods of acircle)
7. Yes (equal chords are equidistant from the centre of the circle)
8. $10\sqrt{2}$ cm 9. (i) Yes (ii) Yes ($\Delta OMP \cong \Delta ONP$)
10. Yes (N is the middle point of chords PQ and AB).