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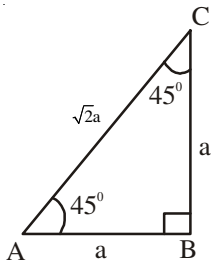
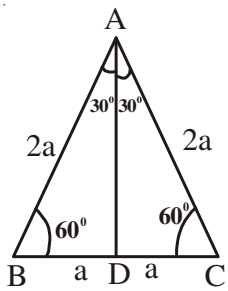
TRIGONOMETRIC RATIOS OF SOME SPECIAL ANGLES

- **Trigonometic ratios of angle 45^0:** In Δ ABC, $\angle B = 90^0$, $\angle A = 45^0$ then $\angle C = 45^0$ and AB = BC = a then AC = $\sqrt{2}a$



In Δ ABC, $\sin 45^0 = \frac{1}{\sqrt{2}}$, $\cos 45^0 = \frac{1}{\sqrt{2}}$, $\tan 45^0 = 1$, $\cot 45^0 = 1$, $\operatorname{cosec} 45^0 = \sqrt{2}$, $\sec 45^0 = \sqrt{2}$

- **Trigonometric ratios of 30^0 and 60^0:** In an equilateral triangle ABC with side 2a, AD = $\sqrt{3}a$



In Δ ADB,

$\sin 30^0 = \frac{1}{2}$,

$\cos 30^0 = \frac{\sqrt{3}}{2}$

In Δ ADB

$\sin 60^0 = \frac{\sqrt{3}}{2}$

$\cos 60^0 = \frac{1}{2}$

$\tan 30^0 = \frac{1}{\sqrt{3}}$

$\operatorname{cosec} 30^0 = 2$

$\sec 30^0 = \frac{2}{\sqrt{3}}$

$\cot 30^0 = \sqrt{3}$

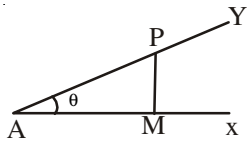
$\tan 60^0 = \sqrt{3}$

$\operatorname{cosec} 60^0 = \frac{2}{\sqrt{3}}$

$\sec 60^0 = 2$

$\cot 60^0 = \frac{1}{\sqrt{3}}$

- **Ttrigonometric Ratios of 0^0 and 90^0:** Let $\angle XAY = \theta$.



In Δ AMP, we have

$\sin\theta = \frac{PM}{AP}$, $\cos\theta = \frac{AM}{AP}$, $\tan\theta = \frac{PM}{AM}$

If θ becomes 0^0, then PM = 0, AM = AP

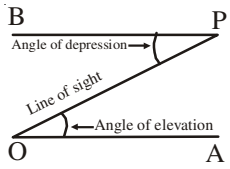
If θ becomes 90^0, then AM = 0, AP = PM

If $\theta = 0^0$, then	If $\theta = 90^0$, then
$\sin 0^0 = 0$	$\sin 90^0 = 1$
$\cos 0^0 = 1$	$\cos 90^0 = 0$
$\tan 0^0 = 0$	$\tan 90^0$ = Not defined
$\operatorname{cosec} 0^0 = \frac{1}{0} =$ not defined	$\operatorname{cosec} 90^0 = 1$
$\sec 0^0 = \frac{1}{1} = 1$	$\sec 90^0$ = not defined
$\cot 0^0 = \frac{1}{0} =$ not defined	$\cot 90^0 = 0$

Trigonometric Ratios of $0^0, 30^0, 45^0, 60^0$ and 90^0

$\theta \rightarrow$ / ratio	0^0	30^0	45^0	60^0	90^0
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cot	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
cosec	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined

- **Applications of trigonometry :**



- **Line of sight:** If an observer is at O and the point P is under consideration then the line OP is called line of sight of the point P.
- **Angle of elevation:** Angle between the line of sight and the horizontal line OA is known as angle of elevation of point P as seen from O.
- **Angle of depression:** If an observer is at P and the object under consideration is at O , then the $\angle BPO$ is known as angle of depression of O as seen from P.
- **Relation between angle of elevation and angle of depression**: Angle of elevation of a point P as seen form O is equal to the angle of depression of O as seen from P.

CHECK YOUR PROGRESS:

1. $\frac{2\tan 30^0}{1+\tan^2 30^0} + \frac{1-\tan 45^0}{1+\tan^2 45^0}$ is equal to :

   (A) $\sin 60^0$ (B) $\sin 30^0$ (C) $\cos 60^0$ (D) $\tan 60^0$

2. The ratio of the length of a rod to its shadow is $1 : \sqrt{3}$. The angle of elevation of sun is:

   (A) 30^0 (B) 45^0 (C) 60^0 (D) 90^0

3. Evaluate $\tan^2 45^0 - \sin^2 60^0 + 2\cos^2 30^0$.

4. In Δ ABC, right angled at C, AC = $2\sqrt{3}$ cm and BC = 2cm. Find $\angle$ A and $\angle$ B.

5. If $\sin(A + B) = 1$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$, $0^0 < A + B \leq 90^0$ and $A \geq B$ find A and B.

STRETCH YOURSELF

1. The string of a kite is 100m long and it makes an angle of 60^0 with the horizontal. Assuming that there is no slack in the string, calculate the height of the kite.
2. A 12m heigh tree is broken by the wind in such a way that its top touches the ground and makes an angle of 30^0 with the ground. Find the height at which tree is broken.
3. Find the value of A, if $\sin 2A = 2\sin A$ where $0 \leq A < 90^0$.

ANSWERS

CHECK YOUR PROGRESS :

1. A
2. A
3. $\frac{7}{4}$
4. $\angle A = 30^0$, $\angle B = 60^0$
5. $\angle A = 60^0$, $\angle B = 30^0$

STRETCH YOURSELF :

1. $50\sqrt{3}$ m
2. 4m
3. 0^0