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QUADRATIC EQUATIONS

- **Quadratic polynomial:** A polynomial of degree2
- **Quadratic equation:** An equation having degree 2.
- **General form of a quadratic equation:** $ax^2 + bx + c = 0$, $a \neq 0$ where a, b, c are real numbers and x is a variable.
- **Roots of a quadratic equation:** Values of variable which satisfy a quadratic equation. α is a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

  A quadric equation has two roots.

  Zeros of a quadratic polynomial and the roots of the corresponding quadratic equation are the same.
- **Methods for solution of quadratic equation:**
  (i) Factor method

  (ii) Using the quadratic formula
- **Factor method of solving $ax^2 + bx + c = 0$, $a \neq 0$:** Factorise $ax^2 + bx + c$, $a \neq 0$ into a product of two linear factors. Equate each factor to zero and get the values of the variable. These values are the required roots of the given quadratic equation.
- **Quadratic formula :** The roots of the equation $ax^2 + bx + c = 0$ are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

- **Discriminant :** The expression $b^2 - 4ac$ is called discriminant of the equation $ax^2 + bx + c = 0$ and denoted by D.
- **Nature of Roots :** A quadratic equation

  $ax^2 + bx + c = 0$ $(a \neq 0)$ has

  (i) two distinct real roots if $D = b^2 - 4ac > 0$

  (ii) two equal (or coincident) and real roots if $D = b^2 - 4ac = 0$

  (iii) no real root if $D = b^2 - 4ac < 0$.
- **Word Problems or daily life problems:** To solve a word problem using quadratic equations convert the given problem in the form of a quadratic equation and then solve the equation by using factor method or quadratic formula.

CHECK YOUR PROGRESS:

1. Which of the following is not a quadratic equation?

   (A) $(x - 1)(x + 3) = 6$ (B) $x + \frac{1}{x} = 7$

   (C) $3x^2 - 5x + 2 = 0$ (D) $x^2 + 2\sqrt{x} + 3 = 0$

2. If the quadratic equation $3x^2 + mx + 2 = 0$ has real and equal roots, then the value of m is :

   (A) $-\sqrt{6}$ (B) $\sqrt{6}$ (C) $\frac{\sqrt{6}}{2}$ (D) $\pm 2\sqrt{6}$

3. The discriminant of the quadratic equation $5x^2 - 6x - 2 = 0$ is :

   (A) 56 (B) 66 (C) 76 (D) 86

4. If one root of the quadratic equation $x^2 - \alpha x - 5 = 0$ is 5 then the other root is :
   (A) –1 (B) 1 (C) $-\alpha$ (D) α
5. Roots of the quadratic equation $x^2 - 14x + 45 = 0$ are:
   (A) real and equal (B) real and distinct
   (C) not real (D) none of these
6. Solve the following equations by factor method:
   (i) $x^2 + 3x = 18$ (B) $2x^2 + 5x - 3 = 0$
7. Solve the follwoing quadratic equations using quadratic formula:
   (i) $3x^2 - 4x - 7 = 0$ (ii) $6x^2 - 19x + 15 = 0$
8. The sum of the ages (in years) of a father and his son is 60 and the product of their ages is 576. Find their ages.
9. Find two consecutive odd positive integers if the sum of their squares is 290.
10. The product of the digits of a two digit number is 12. When 9 is added to the number, the digits interchange their places. Find the number.

STRETCH YOUSELF

1. If –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equaion $P(x^2 + x) + k = 0$ has equal roots, find the value of K.
2. Find the value of K for which the quadratic equation $x^2 - 4x + K = 0$ has two real and distinct roots.
3. Solve the equation:

$$\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}, \; x \neq 0, -1.$$

4. If x = 2 and x = 3 are the roots of the equation $3x^2 - 2kx + 2m = 0$, find the values of k and m.
5. Find the value of k for which the quadratic equation $x^2 - 2x(1+3k) + 7(3+2k) = 0$ has real and eqaul roots.

ANSWERS

CHECK YOUR PROGRESS :

1. D 2. D 3. C 4. A
5. B 6. (i) 3, –6 (ii) $\frac{1}{2}, -3$
7. (i) -1, $\frac{7}{3}$ (ii) $\frac{3}{2}, \frac{5}{3}$
8. Father's age = 48 years, son's is age = 12 years.
9. 11, 13 10. 34

STRETCH YOURSELF:

1. $\frac{7}{4}$ 2. K < 4 3. $\left(\frac{-5}{2}, \frac{3}{2}\right)$
4. $k = \frac{15}{2}$, m = 9 5. k = 2 or $k = \frac{-10}{9}$