1

SETS

311en01

Let us consider the following situation : One day Mrs. and Mr. Mehta went to the market. Mr. Mehta purchased the following objects/items. "a toy, one kg sweets and a magazine". Where as Mrs. Mehta purchased the following objects/items. "Lady fingers, Potatoes and Tomatoes".

In both the examples, objects in each collection are well defined. What can you say about the collection of students who speak the truth ? Is it well defined? Perhaps not. A set is a collection of well defined objects. For a collection to be a set it is necessary that it should be well defined.

The word well defined was used by the German Mathematician George Cantor (1845- 1918 A.D) to define a set. He is known as father of set theory. Now-a-days set theory has become basic to most of the concepts in Mathematics. In this lesson we shall discuss some basic definitions and operations involving sets.

OBJECTIVES

After studying this lesson, you will be able to :

- define a set and represent the same in different forms;
- define different types of sets such as, finite and infinite sets, empty set, singleton set, equivalent sets, equal sets, sub sets and cite examples thereof;
- define and cite examples of universal set, complement of a set and difference between two sets;
- define union and intersection of two sets;
- represent union and intersection of two sets, universal set, complement of a set, difference between two sets by Venn Diagram;

EXPECTED BACKGROUND KNOWLEDGE

- Number systems,

1.1 SOME STANDARD NOTATIONS

Before defining different terms of this lesson let us consider the following examples:

(i) collection of tall students in your school.	(i) collection of those students of your school whose height is more than 180 cm.
(ii) collection of honest persons in your colony.	(ii) collection of those people in your colony who have never been found involved in any theft case.
(iii) collection of interesting books in your school library.	(iii) collection of Mathematics books in your school library.
(iv) collection of intelligent students in your school.	(iv) collection of those students in your school who have secured more than 80% marks in annual examination.

In all collections written on left hand side of the vertical line the term tallness, interesting, honesty, intelligence are not well defined. In fact these notions vary from individual to individual. Hence these collections can not be considered as sets.

While in all collections written on right hand side of the vertical line, 'height' 'more than 180 cm.' 'mathematics books' 'never been found involved in theft case,' ' marks more than 80%' are well defined properties. Therefore, these collections can be considered as sets.

If a collection is a set then each object of this collection is said to be an element of this set. A set is usually denoted by capital letters of English alphabet and its elements are denoted by small letters.

For example, A = {toy elephant, packet of sweets, magazines.}

Some standard notations to represent sets :

N : the set of natural numbers

W : the set of whole numbers

Z : the set of integers

Z^+: the set of positve integers

Z^-: the set of negative integers

Q : the set of rational numbers

I : the set of irrational numbers

R : the set of real numbers

C : the set of complex numbers

Other frequently used symbols are :

$\in$: 'belongs to'

$\notin$: 'does not belong to'

$\exists$: There exists, $\nexists$: There does not exist.

For example N is the set of natural numbers and we know that 2 is a natural number but -2 is not a natural number. It can be written in the symbolic form as $2 \in N$ and $-2 \notin N$.

1. 2 REPRESENTATION OF A SET

There are two methods to represent a set.

1.2.1 (i) Roster method (Tabular form)

In this method a set is represented by listing all its elements, separating them by commas and enclosing them in curly bracket.

If V be the set of vowels of English alphabet, it can be written in Roster form as :

$V = \{ a, e, i, o, u\}$

(ii) If A be the set of natural numbers less than 7. then

$A=\{1, 2, 3, 4, 5, 6\}$, is in the Roster form.

Note : To write a set in Roster form elements are not to be repeated i.e. all elements are taken as distinct. For example if A be the set of letters used in the word mathematics, then

$A = \{m, a, t, h, e, i, c, s\}$

1.2.2 Set-builder form

In this form elements of the set are not listed but these are represented by some common property.

Let V be the set of vowels of English alphabet then V can be written in the set builder form as:

$V = \{x : x \text{ is a vowel of English alphabet}\}$

(ii) Let A be the set of natural numbers less than 7. then $A = \{x : x \in N \text{ and } x < 7\}$

Note : Symbol ':' read as 'such that'

Example: 1.1 Write the following in set -builder form :

(a) $A=\{ -3,-2,-1,0,1,2,3 \}$ (b) $B = \{3,6,9,12\}$

Solution : (a) $A = \{ x : x \in Z \text{ and } -3 \le x \le 3 \}$

(b) $B = \{ x : x = 3n \text{ and } n \in N, n \le 4 \}$

Example: 1.2 Write the following in Roster form.

(a) $C = \{ x : x \in N \text{ and } 50 \le x \le 60 \}$

(b) $D = \{x : x \in R \text{ and } x^2 - 5x + 6 = 0\}$

Solution : (a) $C =\{50, 51, 52,53,54,55,56,57,58,59,60\}$

(b) $x^2 - 5x + 6 = 0$

$$\Rightarrow \quad (x-3)(x-2)=0 \quad \Rightarrow \quad x=3,2.$$

$$\therefore \quad D=\{2,3\}$$

1.3 CLASSIFICATION OF SETS

1.3.1 Finite and infinite sets

Let A and B be two sets where

A = {x : x is a natural number}

B = {x : x is a student of your school}

As it is clear that the number of elements in set A is not finite while number of elements in set B is finite. A is said to be an infinite set and B is said to be a finite set.

A set is said to be finite if its elements can be counted and it is said to be infinite if it is not possible to count upto its last element.

1.3.2 Empty (Null) Set : Consider the following sets.

$$A=\{x : x \in R \text{ and } x^2+1=0\}$$

B = {x : x is number which is greater than 7 and less than 5}

Set A consists of real numbers but there is no real number whose square is -1. Therefore this set consists of no element. Similiarly there is no such number which is less than 5 and greater than 7. Such a set is said to be a null (empty) set. It is denoted by the symbol ϕ or { }

A set which has no element is said to be a null/empty/void set, and is denoted by ϕ. or {}

1.3.3 Singleton Set : Consider the following set :

A = {x : x is an even prime number}

As there is only one even prime number namely 2, so set A will have only one element. Such a set is said to be singleton. Here A = $\{2\}$.

A set which has only one element is known as singleton.

1.3.4 Equal and equivalent sets : Consider the following examples.

(i) $A=\{1,2,3\}$, $B=\{2,1,3\}$ (ii) $D=\{1,2,3\}$, $E=\{a,b,c\}$.

In example (i) Sets A and B have the same elements. Such sets are said to be equal sets and it is written as A = B. In example (ii) sets D and E have the same number of elements but elements are different. Such sets are said to be equivalent sets and are written as A $\approx$ B.

Two sets A and B are said to be equivalent sets if they have same number of elements but they are said to be equal if they have not only the same number of elements but elements are also the same.

1.3.5 Disjoint Sets : Two sets are said to be disjoint if they do not have any common element. For example, sets A= { 1,3,5} and B = { 2,4,6 } are disjoint sets.

Example 1.3 Given that $A = \{2, 4\}$ and $B = \{x : x \text{ is a solution of } x^2 + 6x + 8 = 0\}$

Are A and B disjoint sets ?

Solution : If we solve $x^2 + 6x + 8 = 0$, we get

$x = -4, -2$. $\therefore$ $B = \{-4, -2\}$ and $A = \{2, 4\}$

Clearly ,A and B are disjoint sets as they do not have any common element.

Example 1.4 If $A = \{x : x \text{ is a vowel of English alphabet}\}$

and $B = \{y : y \in N \text{ and } y \leq 5\}$ Is (i) $A = B$ (ii) $A \approx B$?

Solution : $A = \{a, e, i, o, u\}$, $b = \{1, 2, 3, 4, 5\}$.

Each set is having five elements but elements are different

$\therefore$ $A \neq B$ but $A \approx B$.

Example 1.5 Which of the following sets

$A = \{x : x \text{ is a point on a line}\}$, $B = \{y : y \in N \text{ and } y \leq 50\}$ are finite or infinite ?

Solution : As the number of points on a line is uncountable (cannot be counted) so A is an infinite set while the number of natural numbers upto fifty can be counted so B is a finite set.

Example 1.6 Which of the following sets

$A = \{x : x \text{ is irrational and } x^2 - 1 = 0\}$.

$B = \{x : x \in z \text{ and } -2 \leq x \leq 2\}$ are empty?

Solution : Set A consists of those irrational numbers which satisfy $x^2 - 1 = 0$. If we solve $x^2 - 1 = 0$ we get $x = \pm 1$. Clearly ± 1 are not irrational numbers. Therefore A is an empty set.

But $B = \{-2, -1, 0, 1, 2\}$. B is not an empty set as it has five elements.

Example 1.7 Which of the following sets are singleton ?

$A = \{x : x \in Z \text{ and } x - 2 = 0\}$ $\qquad B = \{y : y \in R \text{ and } y^2 - 2 = 0\}$.

Solution : Set A contains those integers which are the solution of $x - 2 = 0$ or $x = 2$. $\therefore$ $A = \{2\}$.

$\Rightarrow$ A is a singleton set.

B is a set of those real numbers which are solutions of $y^2 - 2 = 0$ or $y = \pm\sqrt{2}$

$\therefore$ $B = \{-\sqrt{2}, \sqrt{2}\}$ Thus, B is not a singleton set.

CHECK YOUR PROGRESS 1.1

1. Which of the following collections are sets ?
   (i) The collection of days in a week starting with S.
   (ii) The collection of natural numbers upto fifty.
   (iii) The collection of poems written by Tulsidas.
   (iv) The collection of fat students of your school.

2. Insert the appropriate symbol in blank spaces. If A = $\{1, 2, 3\}$.
   (i) 1..........A (ii) 4........A.

3. Write each of the following sets in the Roster form :
   (i) $A = \{x : x \in z \text{ and } -5 \le x \le 0\}$.
   (ii) $B = \{x : x \in R \text{ and } x^2 - 1 = 0\}$.
   (iii) C = {x : x is a letter of the word banana}.
   (iv) D = {x : x is a prime number and exact divisor of 60}.

4. Write each of the following sets in the set builder form ?
   (i) A = {2, 4, 6, 8, 10} (ii) B = {3, 6, 9,...... ∞ }
   (iii) C = {2, 3, 5, 7} (iv) $D = \{-\sqrt{2}, \sqrt{2}\}$

   Are A and B disjoints sets ?

5. Which of the following sets are finite and which are infinite ?
   (i) Set of lines which are parallel to a given line.
   (ii) Set of animals on the earth.
   (iii) Set of Natural numbers less than or equal to fifty.
   (iv) Set of points on a circle.

6. Which of the following are null set or singleton ?
   (i) $A = \{x : x \in R \text{ and } x \text{ is a solution of } x^2 + 2 = 0\}$.
   (ii) $B = \{x : x \in Z \text{ and } x \text{ is a solution of } x - 3 = 0\}$.
   (iii) $C = \{x : x \in Z \text{ and } x \text{ is a solution of } x^2 - 2 = 0\}$.
   (iv) D = {x : x is a student of your school studying in both the classes XI and XII }

7. In the following check whether A = B or $A \approx B$.
   (i) A = {a}, B = {x : x is an even prime number}.
   (ii) A = {1, 2, 3, 4}, B = {x : x is a letter of the word guava}.
   (iii) $A = \{x : x \text{ is a solution of } x^2 - 5x + 6 = 0\}, B = \{2, 3\}$.

1.4 SUB- SET

Let set A be a set containing all students of your school and B be a set containing all students of class XII of the school. In this example each element of set B is also an element of set A. Such a set B is said to be subset of the set A. It is written as $B \subseteq A$

Consider $D = \{1, 2, 3, 4, ...\}$, $E = \{..... -3 -2, -1, 0, 1, 2, 3, ...\}$

Clearly each element of set D is an element of set E also $\therefore\ D \subseteq E$

If A and B are any two sets such that each element of the set A is an element of the set B also, then A is said to be a subset of B.

Remarks

(i) Each set is a subset of itself i.e. $A \subseteq A$.

(ii) Null set has no element so the condition of becoming a subset is automatically satisfied. Therefore null set is a subset of every set.

(iii) If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

(iv) If $A \subseteq B$ and $A \neq B$ then A is said to be a proper subset of B and B is said to be a super set of A. i.e. $A \subset B$ or $B \supset A$.

Example 1.8 If $A = \{x : x$ is a prime number less than $5\}$ and

$B = \{y : y$ is an even prime number$\}$, then is B a proper subset of A ?

Solution : It is given that

$A = \{2, 3\}$, $B = \{2\}$.

Clearly $B \subseteq A$ and $B \neq A$

We write $B \subset A$ and say that B is a proper subset of A.

Example 1.9 If $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5\}$. is $A \subseteq B$ or $B \subseteq A$?

Solution : Here $1 \in A$ but $1 \notin B \Rightarrow A \not\subseteq B$. Also $5 \in B$ but $5 \notin A \Rightarrow B \not\subseteq A$.

Hence neither A is a subset of B nor B is a subset of A.

Example 1.10 If $A = \{a, e, i, o, u\}$, $B = \{e, i, o, u, a\}$

Is $A \subseteq B$ or $B \subseteq A$ or both ?

Solution : Here in the given sets each element of set A is an element of set B also

$\therefore \quad A \subseteq B \quad$ .... (i)

and each element of set B is an element of set A also. $\therefore\ B \subseteq A$ ......(ii)

From (i) and (ii) $A = B$

1.4.1 Number of Subsets of a Set :

Let A = $\{x\}$, then the subsets of A are ϕ, A.

Note that $n(A) = 1$, number of subsets of A = 2 = 2^1

Let A = {2, 4}, then the subsets of A are ϕ, {4}, {2}, {2, 4}.

Note that $n(A) = 2$, number of subsets of A = 4 = 2^2

Let A = {1, 3, 5}, then subsets of A are ϕ, {1}, {3}, {5}, {1, 3}, {1, 5}, {3, 5}, {1, 3, 5}.

Note that $n(A) = 3$, number of subsets of A = 8 = 2^3

> If A is a set with $n(A) = p$, then the number of subsets of A = 2^p and number of proper subsets of A = $2^p - 1$.

Subsets of real Numbers :

We know some standard sets of numbers as-

The set of natural numbers	N = {1, 2, 3, 4, ...}
The set of whole numbers	W = {0, 1, 2, 3, 4, ...}
The set of Integers	Z = {........., –4, –3, –2, –1, 0, 1, 2, 3, 4, ...}
The set of Rational numbers	$Q = \left\{x : x = \frac{p}{q}, p, q \in Z \text{ and } q \neq 0\right\}$

The set of irrational numbers denoted by I.

I = $\{x : x \in R \text{ and } x \notin Q\}$ i.e. all real numbers that are not rational

These sets are subsets of the set of real numbers. Some of the obvious relations among these subsets are

$$N \subset W \subset Z \subset Q,\ Q \subset R,\ I \subset R,\ N \not\subset I$$

1.4.2 INTERVALS AS SUBSETS OF REAL NUMBERS

An interval I is a subset of R such that if $x, y \in$ I and z is any real numbers between x and y then $z \in$ I.

> Any real number lying between two different elements of an interval must be contained in the interval.

If $a, b \in$R and $a < b$, then we have the following types of intervals :

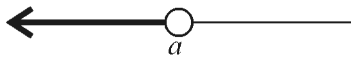
(i) The set $\{x \in R : a < x < b\}$ is called an open interval and is denoted by (a, b). On the number line it is shown as :

(ii) The set $\{x \in R : a \leq x \leq b\}$ is called a closed interval and is denoted by $[a, b]$. On the number line it is shown as :

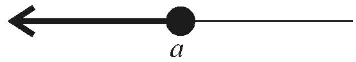
(iii) The set $\{x \in R : a < x \le b\}$ is an interval, open on left and closed on right. It is denoted by $(a, b]$. On the number line it is shown as :

(iv) The set $\{x \in R : a \le x < b\}$ is an interval, closed on left and open on right. It is denoted by $[a, b)$. On the number line it is shown as :

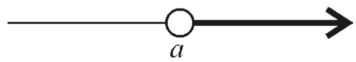
(v) The set $\{x \in R : x < a\}$ is an interval, which is dentoed by $(-\infty, a)$. It is open on both sides. On the number line it is shown as :



(vi) The set $\{x \in R : x \le a\}$ is an interval which is denoted by $(-\infty, a]$. It is closed on the right. On the number line it is shown as :



(vii) The set $\{x \in R : x > a\}$ is an interval which is denoted by (a, ∞). It is open on the both sides. On the number line it is shown as :



(viii) The set $\{x \in R : x \ge a\}$ is an interval which is denoted by $[a, \infty)$. It is closed on left. On the number line it is shown as :

First four intervals are called finite intervals and the number $b - a$ (which is always positive) is called the length of each of these four intervals (a, b), $[a, b]$, $(a, b]$ and $[a, b)$.

The last four intervals are called infinite intervals and length of these intervals is not defined.

1.5 POWER SET

Let $\quad A = \{a, b\}$ then, Subset of A are ϕ, $\{a\}$, $\{b\}$ and $\{a, b\}$.

If we consider these subsets as elements of a new set B (say) then, $B = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

B is said to be the power set of A.

Notation : Power set of a set A is denoted by P(A).
and it is the set of all subsets of the given set.

Example 1.11 Write the power set of each of the following sets :

(i) $\quad A = \{x : x \in R \text{ and } x^2 + 7 = 0\}$.

(ii) $\quad B = \{y : y \in N \text{ and } 1 \le y \le 3\}$.

Solution :

(i) Clearly A = ϕ (Null set), $\therefore$ ϕ is the only subset of given set, $\therefore$ P (A)={ ϕ }

(ii) The set B can be written as {1, 2, 3}

Subsets of B are ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}.

$\therefore$ P (B) = { ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3} }.

Example 1.12 Write each of the following sets as intervals :

(i) $\{x \in R : -1 < x \le 2\}$ (ii) $\{x \in R : 1 \ge 2x - 3 \ge 0\}$

Solution : (i) The given set = $\{x \in R : -1 < x \le 2\}$

Hence, Interval of the given set = (–1, 2]

(ii) The given set = $\{x \in R : 1 \ge 2x - 3 \ge 0\}$

$\Rightarrow \{x \in R : 4 \ge 2x \ge 3\}$, $\Rightarrow \left\{x \in R : 2 \ge x \ge \frac{3}{2}\right\}$

$\Rightarrow \left\{x \in R : \frac{3}{2} \le x \le 2\right\}$, Hence, Interval of the given set = $\left[\frac{3}{2}, 2\right]$

1.6 UNIVERSAL SET

Consider the following sets.

A = {x : x is a student of your school}

B = {y : y is a male student of your school}

C = {z : z is a female student of your school}

D = {a : a is a student of class XII in your school}

Clearly the set B, C, D are all subsets of A. A can be considered as the universal set for this particular example. Universal set is generally denoted by U. In a particular problem a set U is said to be a universal set if all the sets in that problem are subsets of U.

Remarks

(i) Universal set does not mean a set containing all objects of the universe.

(ii) A set which is a universal set for one problem may not be a universal set for another problem.

Example 1.13 Which of the following sets can be considered as a universal set ?

X = {x : x is a real number}

Y = {y : y is a negative integer}

Z = {z : z is a natural number}

Solution : As it is clear that both sets **Y** and **Z** are subset of **X**.

$\therefore$ **X** is the universal set for this problem.

1.7 VENN DIAGRAM

British mathematician John Venn (1834 – 1883 AD) introduced the concept of diagrams to represent sets. According to him universal set is represented by the interior of a rectangle and other sets are represented by interior of circles.

For example if U= {1, 2, 3, 4, 5}, A = {2, 4} and B = {1,3}, then these sets can be represented as

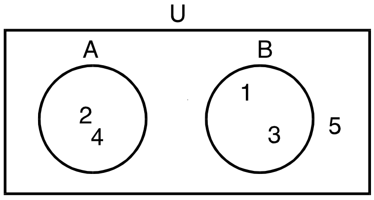


Fig. 1.1

Diagramatical representation of sets is known as a Venn diagram.

1.8 DIFFERENCE OF SETS

Consider the sets

$$A = \{1, 2, 3, 4, 5\} \text{ and } B = \{2, 4, 6\}.$$

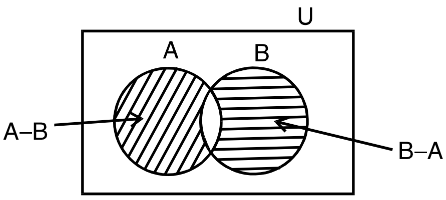
A new set having those elements which are in A but not in B is said to be the difference of sets A and B and it is denoted by A – B. $\therefore$ A – B= {1, 3, 5}

Similiarly a set of those elements which are in B but not in A is said to be the difference of B and A and it is devoted by B – A. $\therefore$ B – A = {6}

In general, if A and B are two sets then

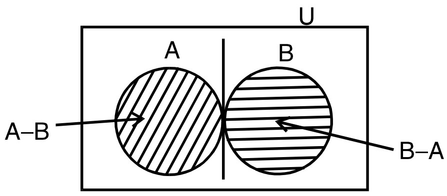
$$A - B = \{\, x : x \in A \text{ and } x \notin B \,\} \text{ and } B - A = \{\, x : x \in B \text{ and } x \notin A \,\}$$

Difference of two sets can be represented using Venn diagram as :



When A and B are not **disjoint** sets

or



When A and B are **disjoint** sets

Fig. 1.2 **Fig. 1.3**

1.9. COMPLEMENT OF A SET

Let **X** denote the universal set and **Y, Z** its sub sets where

X = {x : x is any member of a family}

Y = {x : x is a male member of the family}

Z = {x : x is a female member of the family}

$X - Y$ is a set having female members of the family.

$X - Z$ is a set having male members of the family.

$X - Y$ is said to be the complement of Y and is usally denoted by Y' or Y^c.

$X - Z$ is said to be complement of Z and denoted by Z' or Z^c.

If U is the universal set and A is its subset then the complement of A is a set of those elements which are in U but not in A. It is denoted by A' or A^c.

$A' = U - A = \{x : x \in U \text{ and } x \notin A\}$

The complement of a set can be represented using Venn diagram as :

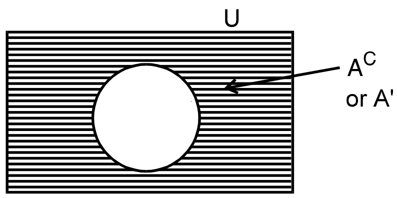


Fig. 1.4

Remarks

(i) Difference of two sets can be found even if none is a subset of the other but complement of a set can be found only when the set is a subset of some universal set.

(ii) $\phi^c = U$. (iii) $U^c = \phi$.

Example 1.14 Given that

$A = \{x : x$ is a even natural number less than or equal to $10\}$

and $B = \{x : x$ is an odd natural number less than or equal to $10\}$

Find (i) $A - B$ (ii) $B - A$ (iii) is $A - B = B - A$?

Solution : It is given that

$A = \{2, 4, 6, 8, 10\}$, $B = \{1, 3, 5, 7, 9\}$

Therefore, (i) $A - B = \{2, 4, 6, 8, 10\}$, (ii) $B - A = \{1, 3, 5, 7, 9\}$

(iii) Clearly from (i) and (ii) $A - B \neq B - A$.

Example 1.15 Let U be the universal set and A its subset where

$U = \{x : x \in N$ and $x \le 10\}$

$A = \{y : y$ is a prime number less than $10\}$

Find (i) A^c (ii) Represent A^c in Venn diagram.

Solution : It is given

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. and $A = \{2, 3, 5, 7\}$

(i) $A^c = U - A = \{1, 4, 6, 8, 9, 10\}$

(ii)

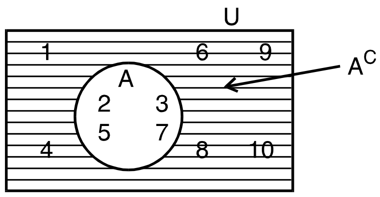


Fig. 1.5

1.9.1 Properties of complement of sets

1. Complement Law's

   (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$

2. De Morgan's Law

   (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

3. Law of double complementation $(A')' = A$

4. Law of empty set and universal set i.e $\phi' = U$ and $U' = \phi$

1. Verification of Complement Law

Let $U = \{1, 2, 3, \ldots\ldots\ldots 10\}$ and $A = \{2, 4, 6, 8, 10\}$

Then $A' = \{1, 3, 5, 7, 9\}$

Now, $A \cup A' = \{1, 2, 3, 4, \ldots\ldots\ldots\ldots, 10\} = U$ and $A \cap A' = \phi$

Hence, $A \cup A' = U$ and $A \cap A' = \phi$

2. Verification of De Morgan's Law

Let $U = \{1, 2, 3, \ldots\ldots., 9\}$ and $A = \{2, 4, 6, 8\}$, $B = \{2, 3, 5, 7\}$

Hence, $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

and $(A \cup B)' = U - (A \cup B) = \{1, 9\}$ ...(1)

Now $A' = U - A = \{1, 3, 5, 7, 9\}$ and $B' = U - B = \{1, 4, 6, 8, 9\}$

$\therefore$ $A' \cap B' = \{1, 9\}$ ...(2)

From (1) and (2), $(A \cup B)' = A' \cap B'$

Also $A \cap B = \{2\}$

$\therefore$ $(A \cap B)' = U - (A \cap B) = \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(3)

and $A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(4)

From (3) and (4), we get $(A \cap B)' = A' \cup B'$

Verification of $(A')' = A$

Let $U = \{1, 2, 3, \ldots\ldots\ldots\ldots, 10\}$ and $A = \{1, 2, 3, 5, 7, 9\}$

Then $A' = U - A = \{4, 6, 8, 10\}$ $(A')' = U - A' = \{1, 2, 3, 5, 7, 9\} = A$

Thus from the definition of the complement of a subset A of the universal set U it follows that $(A')' = A$

CHECK YOUR PROGRESS 1.2

1. Insert the appropriate symbol in the blank spaces, given that A={1, 3, 5, 7, 9}

   (i) ϕ ....A (ii) {2, 3, 9}..........A

   (iii) 3...........A (iv) 10..................A

2. Given that A = {a, b}, how many elements P(A) has ?

3. Let $A = \{ \phi, \{1\}, \{2\}, \{1,2\}\}$. Which of the following is true or false ?

   (i) $\{1,2\} \subset A$ (ii) $\phi \in A$.

4. Which of the following statements are true or false ?

   (i) Set of all boys, is contained in the set of all students of your school.

   (ii) Set of all boy students of your school, is contained in the set of all students of your school.

   (iii) Set of all rectangles, is contained in the set of all quadrilaterals.

   (iv) Set of all circles having centre at origin is contained in the set of all ellipses having centre at origin.

5. If A = { 1, 2, 3, 4, 5 } , B= {5, 6, 7} find (i) A – B (ii) B – A.

6. Let N be the universal set and A, B, C, D be its subsets given by

   $A = \{x : x \text{ is a even natural number}\}$, $B = \{x : x \in N \text{ and } x \text{ is a multiple of } 3\}$

   $C = \{x : x \in N \text{ and } x \geq 5\}$, $D = \{x : x \in N \text{ and } x \leq 10\}$

   Find complements of A, B, C and D respectively.

7. Write the following sets in the interval form.

   (a) $\{x \in R : -8 < x < 3\}$ (b) $\{x \in R : 3 \leq 2x \angle 7\}$

8. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4} and B = {2, 4, 6, 8}, then verify the following

   (i) $(A')' = A$ (ii) $(B')' = B$

   (iii) $A \cap A' = \phi$ (iv) $(A \cup B)' = A' \cap B'$

1.10. INTERSECTION OF SETS

Consider the sets

$A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6\}$

It is clear, that there are some elements which are common to both the sets A and B. Set of these common elements is said to be interesection of A and B and is denoted by $A \cap B$.

Here $A \cap B = \{2, 4\}$

If A and B are two sets then the set of those elements which belong to both the sets is said to be the intersection of A and B. It is denoted by $A \cap B$. $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A \cap B$ can be represented using Venn diagram as :

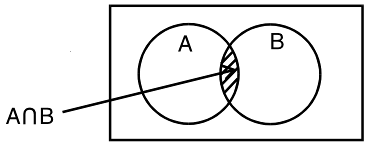


Fig. 1.6

Remarks

If $A \cap B = \phi$ then A and B are said to be **disjoint sets**. In Venn diagram disjoint sets can be represented as

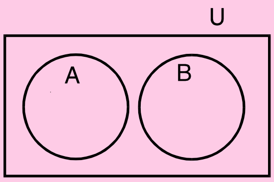


Fig.1.7

Example 1.16 Given that

$A = \{x : x$ is a king out of 52 playing cards$\}$

and $B = \{ y : y$ is a spade out of 52 playing cards$\}$

Find (i) $A \cap B$ (ii) Represent $A \cap B$ using Venn diagram .

Solution : (i) As there are only four kings out of 52 playing cards, therefore the set A has only four elements. The set B has 13 elements as there are 13 spade cards but out of these 13 spade cards there is one king also. Therefore there is one common element in A and B.

$\therefore A \cap B = \{$ King of spade$\}$.

(ii)

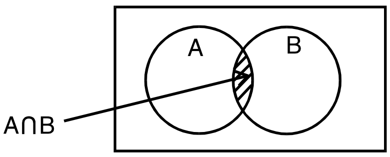
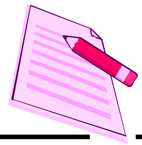


Fig. 1.8



1.11 UNION OF SETS

Consider the following examples :

(i) A is a set having all players of Indian men cricket team and B is a set having all players of Indian women cricket team. Clearly A and B are disjoint sets. Union of these two sets is a set having all players of both teams and it is denoted by $A \cup B$.

(ii) D is a set having all players of cricket team and E is the set having all players of Hockey team, of your school. Suppose three players are common to both the teams then union of D and E is a set of all players of both the teams but three common players to be written once only.

If A and B are any two sets then union of A and B is the set of those elements which belong to A or B.

In set builder form : $A \cup B = \{x : x \in A \text{ or } x \in B\}$

OR

$$A \cup B = \{x : x \in A - B \text{ or } x \in B - A \text{ or } x \in A \cap B\}$$

$A \cup B$ can be represented using Venn diagram as :

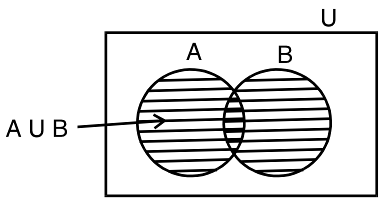


Fig. 1.9

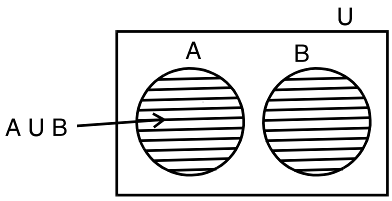


Fig. 1.10

$$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B).$$

or $$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

where n $A \cup B$ stands for number of elements in $A \cup B$.

Example 1.17 $A = \{x : x \in Z^+ \text{ and } x \le 5\}$, B = {y : y is a prime number less than 10}

Find (1) $A \cup B$ (ii) represent $A \cup B$ using Venn diagram.

Solution : We have,

A={1, 2, 3, 4, 5} B = {2, 3, 5, 7}. $\therefore$ $A \cup B$ = {1, 2, 3, 4, 5, 7}.

(ii)

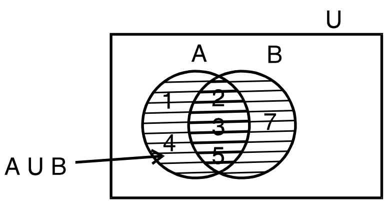


Fig.1.11

CHECK YOUR PROGRESS 1.3

1. Which of the following pairs of sets are disjoint and which are not ?

   (i) $\{x : x$ is an even natural number$\}$, $\{y : y$ is an odd natural number$\}$

   (ii) $\{x : x$ is a prime number and divisor of 12$\}$, $\{ y : y \in N$ and $3 \le y \le 5 \}$

   (iii) $\{x : x$ is a king of 52 playing cards$\}$, $\{ y : y$ is a diamond of 52 playing cards$\}$

   (iv) $\{1, 2, 3, 4, 5\}$, $\{a, e, i, o, u\}$

2. Find the intersection of A and B in each of the following :

   (i) $A = \{ x : x \in Z \}$, $B = \{ x : x \in N \}$ (ii) $A = \{$Ram, Rahim, Govind, Gautam$\}$

   $B = \{$Sita, Meera, Fatima, Manprit$\}$

3. Given that $A = \{1, 2, 3, 4, 5\}$, $B = \{5, 6, 7, 8, 9, 10\}$

   find (i) $A \cup B$ (ii) $A \cap B$.

4. If $A = \{ x : x \in N \}$, $B = \{ y : y \in z$ and $-10 \le y \le 0 \}$, find $A \cup B$ and write your answer in the Roster form as well as in set-builder form.

5. If $A = \{2, 4, 6, 8, 10\}$, $B \{8, 10, 12, 14\}$, $C = \{14, 16, 18, 20\}$.

   Find (i) $A \cup (B \cup C)$ (ii) $A \cap (B \cap C)$.

6. Let $U = \{1, 2, 3, ...10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{ 1, 3, 5, 7, 9, 10\}$

   Find (i) $(A \cup B)'$ (ii) $(A \cap B)'$ (iii) $(B')'$ (iv) $(B - A)'$.

7. Draw Venn diagram for each of the following :

   (i) $A \cap B$ when $B \subset A$ (ii) $A \cap B$ when A and B are disjoint sets.

   (iii) $A \cap B$ when A and B are neither subsets of each other nor disjoint sets.

8. Draw Venn diagram for each of the following :

   (i) $A \cup B$ when $A \subset B$. (ii) $A \cup B$ when A and B are disjoint sets.

   (iii) $A \cup B$ when A and B are neither subsets of each other nor disjoint sets.

9. Draw Venn diagram for each the following :

   (i) $A - B$ and $B - A$ when $A \subset B$.

   (ii) $A - B$ and $B - A$ when A and B are disjoint sets.

   (iii) $A - B$ and $B - A$ when A and B are neither subsets of each other nor disjoint sets.

LET US SUM UP

- Set is a well defined collection of objects.
- To represent a set in Roster form all elements are to be written but in set builder form a set is represented by the common property of its elements.

- If the elements of a set can be counted then it is called a finite set and if the elements cannot be counted, it is infinite.
- If each element of set A is an element of set B, then A is called sub set of B.
- For two sets A and B, $A - B$ is a set of those elements which are in A but not in B.
- Complement of a set A is a set of those elements which are in the universal set but not in A. i.e. $A^c = U - A$
- Intersection of two sets is a set of those elements which belong to both the sets.
- Union of two sets is a set of those elements which belong to either of the two sets.
- Any set 'A' is said to be a subset of a set 'B' if every element of A is contained in B.
- Empty set is a subset of every set.
- Every set is a subset of itself.
- The set 'A' is a proper subset of set 'B' iff $A \subseteq B$ and $A \neq B$
- The set of all subsets of a given set 'A' is called power set of A.
- Two sets A and B are equal iff $A \subseteq B$ and $B \subseteq A$
- If $n(A) = p$ then number of subsets of $A = (2)^p$
- (a, b), $[a, b]$, $(a, b]$ and $[a, b)$ are finite intervals as their length $b - a$ is real and finite.
- Complement of a set A with respect to U is denoted by A' and defined as $A' = \{x : x \in U \text{ and } x \notin A\}$
- $A' = U - A$
- If $A \subset U$, then $A' \subset U$
- Properties of complement of set A with respect to U
  - $A \cup A' = U$ and $A \cap A' = \phi$
  - $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$
  - $(A')' = A$
  - $\phi' = U$ and $U' = \phi$

SUPPORTIVE WEB SITES

http://www.mathresource.iitb.ac.in/project/indexproject.html

http://mathworld.wolfram.com/SetTheory.html

http://www-history.mcs.st-andrews.ac.uk/HistTopics/Beginnings_of_set_theory.html

TERMINAL EXERCISE

1. Which of the following statements are true or false :

   (i) $\{1, 2, 3\} = \{1, \{2\}, 3\}$. (ii) $\{1, 2, 3\} = \{3, 1, 2\}$.

   (iii) $\{a, e, o\} = \{a, b, c\}$. (iv) $\{\phi\} = \{\ \}$

2. Write the set in Roster form represented by the shaded portion in the following.

   (i) $A = \{1, 2, 3, 4, 5\}$

   $B = \{5, 6, 7, 8, 9\}$

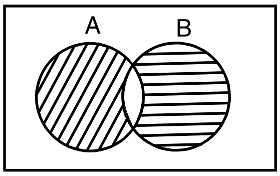


Fig. 1.12

   (ii) $A = \{1, 2, 3, 4, 5, 6\}$

   $B = \{2, 6, 8, 10, 12\}$

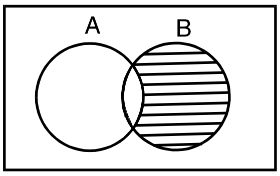


Fig. 1.13

3. Represent the follwoing using Venn diagram.

   (i) $(A \cup B)'$ provided A and B are not disjoint sets.

   (ii) $(A \cap B)'$ provided A and B are disjoint sets.

   (iii) $(A - B)'$ provided A and B are not disjoint sets.

4. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A\{2, 4, 6, 8\}$, $B = \{1, 3, 5, 7\}$

   Verify that

   (i) $(A \cup B)' = A' \cap B'$

   (ii) $(A \cap B)' = A' \cup B'$

   (iii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

5. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,

   $A = \{1, 3, 5, 7, 9\}$.

   $B = \{2, 4, 6, 8, 10\}$, $C = \{1, 2, 3\}$.

   Find (i) $A' \cap (B - C)$. (ii) $A \cup (B \cup C)$

   (iii) $A' \cap (B \cup C)'$ (iv) $(A \cap B)' \cup C'$

6. What does the shaded portion represent in each of the following Venn diagrams :

(i)

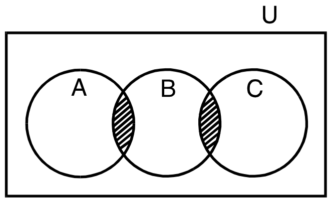


Fig. 1.14

(ii)

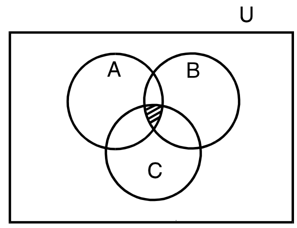


Fig. 1.15

(iii)

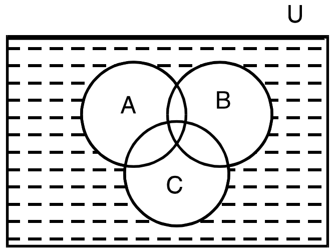


Fig. 1.16

(iv)

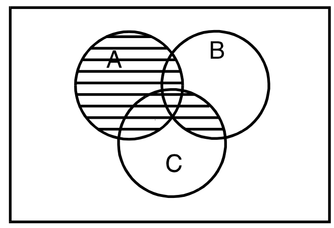


Fig. 1.17

7. Draw Venn diagram for the following :

(i) $A' \cap (B \cup C)$ (ii) $A' \cap (C - B)$

Where A,B,C are not disjoint sets and are subsets of the universal set U.

8. Verify De Morgan's Law if $U = \{x : x \in N \text{ and } x \leq 10\}$

$A = \{x : x \in U \text{ and } x \text{ is a prime number}\}$and

$B = \{x : x \in U \text{ and } x \text{ is a factor of } 24\}$

9. Examine whether the following statements are true or false :

(*a*) $\{a, e\} \subset \{x : x$ is a vowel in the English alphabet]

(*b*) $\{1, 2, 3\} \subset \{1, 3, 5\}$ (*c*)$\{a, b, c\} \subset \{a, b, c\}$ (*d*) $\phi \subset \{1, 3, 5\}$

10. Write down all the subsets of the following sets :

(*a*) $\{a\}$ (*b*) $\{1, 2, 3\}$ (*c*) ϕ

11. Write down the following as intervals :

(*a*) $\{x : x \in R, -4 < x \leq 6\}$ (*b*) $\{x : x \in R, -12 < x < -10\}$

(*c*) $\{x : x \in R, 0 \leq x < 7\}$ (*d*) $\{x : x \in R, 3 \leq x \leq 4\}$

ANSWERS

CHECK YOUR PROGRESS 1.1

1. (i), (ii), (iii) are sets.
2. (i) $\in$ (ii) $\notin$
3. (i) $A = \{-5, -4, -3, -2, -1, 0\}$ (ii) $B = \{-1, 1\}$,
   (iii) $C = \{a, b, n\}$ (iv) $D = \{2, 3, 5\}$.
4. (i) $A = \{x : x \text{ is a even natural number less than or equal to ten}\}$.
   (ii) $B = \{x : x \in N \text{ and } x \text{ is a multible of } 3\}$.
   (iii) $C = \{x : x \text{ is a prime number less than } 10\}$.
   (iv) $D = \{x : x \in R \text{ and } x \text{ is a solution of } x^2 - 2 = 0\}$.
5. (i) Infinite (ii) Finite (iii) Finite (iv) Infinite
6. (i) Null (ii) Singleton (iii) Null (iv) Null
7. (i) $A \approx B$ (ii) $A \approx B$ (iii) $A = B$.

CHECK YOUR PROGRESS 1.2

1. (i) $\subset$ (ii) $\not\subset$ (iii) $\in$ (iv) $\notin$
2. 4
3. (i) False (ii) True
4. (i) False (ii) True (iii) True (iv) False
5. (i) $\{1, 2, 3, 4\}$ (ii) $\{6, 7\}$.
6. $A^c = \{x : x \text{ is an odd natural number}\}$

   $B^c = \{x : x \in N \text{ and } x \text{ is not a multiple of } 3\}$

   $C^c = \{1, 2, 3, 4\}$. $D^c = \{11, 12, 13, ...\}$
7. (a) (-8,3) (b) $\left[\frac{3}{2}, \frac{7}{2}\right)$

CHECK YOUR PROGRESS 1.3

1. (i) Disjoint (ii) Not disjoint (iii) Not disjoint (iv) Disjoint
2. (i) $\{x : x \in N\}$ (ii) ϕ
3. (i) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ (ii) $\{5\}$

4. Roster from $\{-10, -9, -8, ...0, 1, 2, 3, ...\}$

   Set builder from $\{x : x \in Z \text{ and } -10 \le x \le \infty\}$

5. (i) $\{x : x \text{ is a even natural number less than equal to } 20\}$. (ii) ϕ

6. (i) ϕ (ii) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

   (iii) $\{1, 3, 5, 7, 9, 10\}$ (iv) $\{2, 4, 6, 8, 10\}$

7. (i)

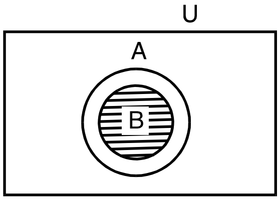


Fig. 1.18

(ii)

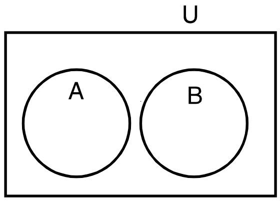


Fig. 1.19

(iii)

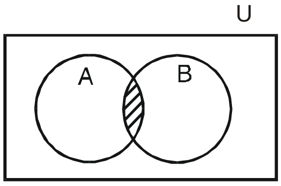


Fig. 1.20

8. (i)

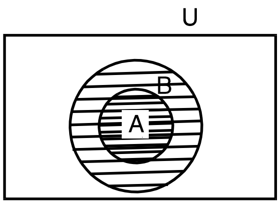


Fig. 1.21

(ii)

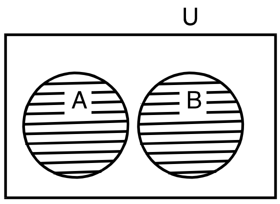


Fig. 1.22

(iii)

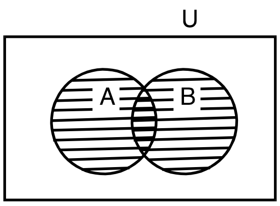
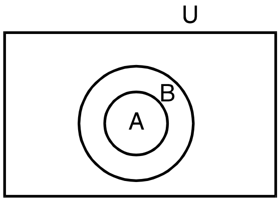
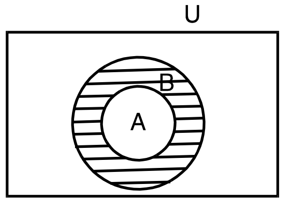


Fig. 1.23

9. (i)



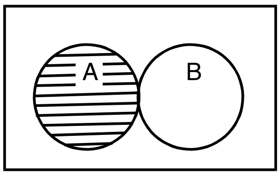
$A - B = \phi$



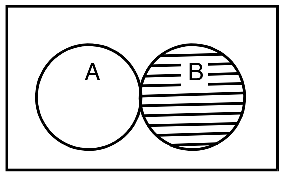
$B - A =$ Shaded portion.

Fig. 1.24

(ii)



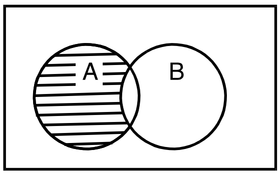
$A - B = A$



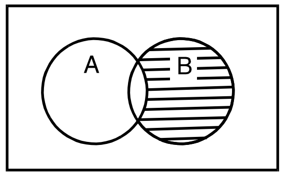
$B - A = B$

Fig. 1.25

(iii)



$A - B =$ Shaded Portion



$B - A =$ Shaded Portion

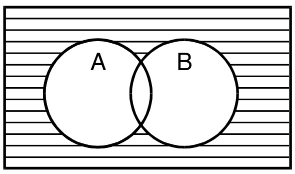
Fig. 1.26

TERMINAL EXERCISE

1. (i) False (ii) True (iii) False (iv) False

2. (i) $\{1, 2, 3, 4, 6, 7, 8, 9\}$ (ii) $\{8, 10, 12\}$

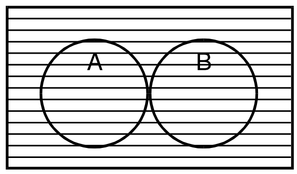
3. (i)



$(A \cup B)' =$ Shaded Portion

Fig. 1.27

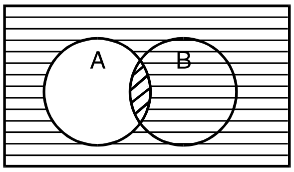
(ii)



$(A \cap B)' =$ Shaded Portion

Fig. 1.28

(iii)



$(A - B)' =$ Shaded Portion

Fig. 1.29

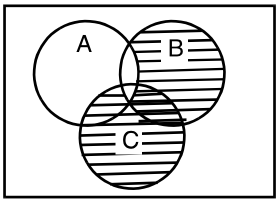
5. (i) $\{4, 6, 8, 10\}$ (ii) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

   (iii) ϕ (iv) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

6. (i) $(A \cap B) \cap (B \cap C)$ (ii) $(A \cap B) \cap C$

   (iii) $[(A \cup B) \cup C]'$ (iv) $A \cup (B \cap C)$.

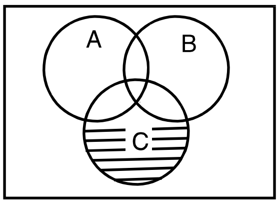
7. (i)



$A' \cap (B \cup C) =$ Shaded Portion

Fig.1.30

(ii)



$A' \cap (C - B) =$ Shaded Portion

Fig. 1.31

9. (*a*) True (*b*) False

   (*c*) True (*d*) True

10. (*a*) ϕ, $\{a\}$ (*b*) ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}

    (c) ϕ

11. (*a*) (– 4, 6] (*b*) (–12, – 10)

    (*c*) [0, 7) (*d*) [3, 4]