2

RELATIONS AND FUNCTIONS-I

Cartesian product of Two Sets

Let A={1, 2}, B= {3, 4, 5}. Set of all ordered pairs of elements of A and B is {(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)} B ×A = {(3, 1),(3, 2),(4, 1),(4, 2),(5, 1),(5, 2)}	A × B and is called the Cartesian product of sets A and B. i.e. A×B ={(1, 3), (1, 4),(1, 5),(2, 3),(2, 4),(2, 5)} Cartesian product of sets B and A is denoted by B×A.

In the set builder form:

A$\times B = \{(a, b): a\epsilon A and b\epsilon B\} and$

B$\times A = \{(b, a): b\epsilon B and a\epsilon A\}$

Number of elements in the Cartesian product of two finite sets

Number of elements in Cartesian product of two finite sets A and B i.e. n(A × B) = n(A). n(B)	Example A = {1,2},B ={x,y} A ×B ={(1,x),(2,x),(1,y),(2,y)

Cartesian product of the set of real numbers R with itself up to R × R ×

Ordered triplet A × A × A = {(a, b, c) : a, b, c∈ A } Here (a, b, c) is called an ordered triplet.	A {1, 2} form the set A ×A ×A ={(1,1,1),(1,1,2),(1,2,1),(1,2,2),(2,1,1),(2,1,2),(2,2,1)(2,2,2}

Here (a, b, c) is called an ordered triplet. R}

Relations

If A and B are two sets then a relation R from A toB is a sub set of A×B. ,

If R =ϕ is called a void relation.

(ii) R=A×B, R is called a universal relation.

(iii) If R is a relation defined from A to A, it is called a relation defined on A.

(iv) R={(a,a) $\forall a \in A\}$,is called the identity relation

Domain and Range of a Relation

If R is a relation between two sets then the set of first elements (components) of all the ordered pairs of R is called Domain and

set of 2nd elements of all the ordered pairs of R is called range, of the given relation

Co-domain of a Relation

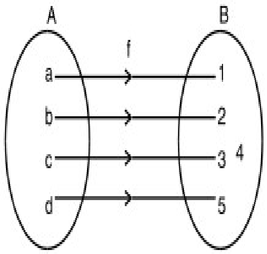
If R is a relation from A to B, then B is called co domain of R.

For example, let A = {1, 3, 4, 5, 7} and B = {2, 4, 6, 8} and R be the relation 'is one less than' from A to B, then R = {(1, 2), (3, 4), (5, 6), (7, 8)} so co domain of R = {2, 4, 6, 8}

Function

Function is a special type of relation.

F:A→B is a rule of correspondence from A to B such that to every element of A∃ a unique element in B



A={a,b,c,d}

B={1,2,3}

f :A→B
={(a,1)(b,2)(c,3)(d,4)}

(i) the set B will be termed as co-domain and

(ii) (ii) the set {1, 2, 3, 5} is called the range. From the above we can conclude that range is a subset of co-domain.

(iii) Symbolically

f

f : A→ $B or A \rightarrow B$

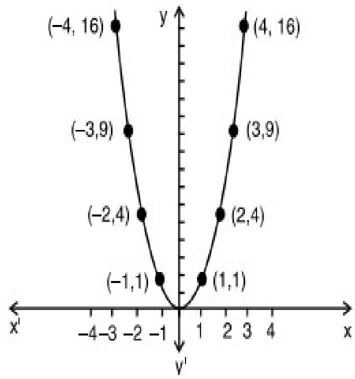
Real Valued Function of a real Variable

A function which has either R or one of its subsets as its range is called a real valued function. Further, if its domain is also either R or a subset of R, then it is called a real function.

GRAPHICAL REPRESENTATION OF FUNCTIONS

Y= X^2

x	y
0	0
1	1
-1	1
2	4
-2	4
3	9
-3	9
4	16
-4	16



SOME SPECIAL FUNCTIONS

Monotonic Function

F:A→ Bbe a function then F is said to be monotonic on an interval (a,b) if it is either→Let F : A B increasing or decreasing on that interval.

- For function to be increasing on an interval (a,b)

$x_1 < x_2 \Rightarrow F(x_1) < F(x_2) \forall x_1 x_2 \in (a,b)$

- for function to be decreasing on (a,b)

$x_1 > x_2 \Rightarrow F(x_1) > F(x_2) \forall x_1 x_2 \in (a,b)$

Even Function

A function is said to be an even function if for each x of domain F(- x) =F(x)

Odd Function

A function is said to be an odd function if for each x

f(-x) = - f(x)

Greatest Inter Function

F(x) = [x] which is the greatest integer less than or equal to x f(x) is called Greatest Integer Function

Polynomial Function

Any function defined in the form of a polynomial is called a polynomial function.

Rational Function

Function of the type f(x) $=\frac{g(x)}{h(x)}$, where h(x) ≠ 0 and g(x) and h(x)are polynomial functions are called rational functions.

Reciprocal Function:

Functions of the type $y=\frac{1}{x}$, x≠ 0 is called a reciprocal function.

Exponential Function

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots\cdots\cdots \frac{x^n}{n!} \cdots$$

This is called exponential theorem, infinite series is called the exponential series.

f(x) $=e^x$, where x is any real number is called exponential function

Logarithmic Functions

$$y = e^x \text{ or } x = \log_e y$$

$$y = \log_e x$$

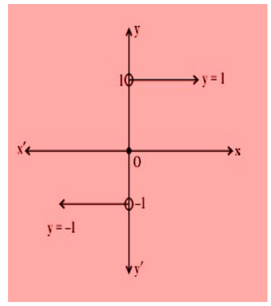
Identity Function

Let R be the set of real numbers. Define the real valued functionf:R→ R by y = f(x)=x . for each x∈ R.Such a function is called the identity function

Constant Function

The function f:R→R by y= f(x) =c, x∈R where c is constant and each x∈R

Signum Function



The function f: R→R defined by f(x) $= \begin{cases} 1 \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -1 \text{ if } x < 0 \end{cases}$ is called signum function. The domain of the signum function is R and the range is the set {–1, 0, 1}.

Sum, difference, product and quotient of functions

Addition of two real functions:

Let f:X →R and g: X → R be any two function, where X ⊂ R, Then (f + g) : X → R by (f + g) (x) = f(x) + g(x), for all x ∈ X	Example F(x) $=x^2$,g(x) =2x+1 (f +g)(x) = f(x)+ g (x) $=x^2 +2x + 1$

(ii) **Subtraction of a real function**

Let f:X →R and g: X → R be any two functions, where X ⊂ R, Then (f - g): X → R by (f - g) (x) = f(x) - g(x), for all x ∈ X	Example F(x) $=x^2$, g(x) =2x+1 (f -g) (x) = f(x)- g (x) $=x^2 -2x - 1$

(iv) **Multiplication of two real functions :**

Let f:X $\rightarrow$R and g: X $\rightarrow$ R be any two functions, where X $\subset$ R, Then (f g): X $\rightarrow$ R by (f g) (x) = f(x) g(x), for all x $\in$ X	Example F(x) =x^2, g(x) =2x+1 (f g)(x) = f(x) g (x) =$2x^3 + x^2$

Quotient of two real functions

Let f:X $\rightarrow$R and g: X $\rightarrow$ R be any two functions, where X $\subset$ R, Then (f /g) : X $\rightarrow$ R by (f g) (x) = f(x) / g(x), for all x $\in$ X	Example F(x) =x^2, g(x) =2x+1 (f /g)(x) = f(x)/ g (x) =x^2/2x + 1

Check your Progress

Q 1 If $n(A)=3$, and $n(B)=5$, then $n(A\times B)$ is equal to:

(A) 8
(B) 15
(C) 5
(D) 3

Q 2 In relation R = {(1,3), (2,6) (3,9), (4,12)} The domain of R is:

(A) {1,2,3,4}
(B) {3,6,9,12}

Q 3 If $(x-3, y+4)=(5-x, 4+y)$, then the value of x is equal to:

(A) 2
(B) 4
(C) 8
(D) 6

Q 4 The total number of relations from a set consisting of 'm' elements to a set consisting of 'n' elements is equal to

(A) $m+n$
(B) mn
(C) 2^{mn}
(D) $m-n$

Q5 If the function is in the form of $f(-x)=-f(x)$, then the function is:

(A) Negative function
(B) Odd function
(C) Even function
(D) Step Function

Stretch yourself

Q1 Let A = {1,2,3,4,6} and R be the relation on A defined by

$$R=\{(a,b): a,b\in A \text{ and } a \text{ divides } b\}$$

(i) Write R in roster form
(ii) Find Domain & Range of R

Q2 Let $A=\{7,9,11\}, B=\{13,15,17\}$ and $R=\{(x,y): x\in A \ \& \ y\in B, x-y \text{ is odd}\}$
Show that relation 'R' is an empty relation

Q3 If A = {1,2}, B = {a,b} find out total number of possible relations from A to B

Q4 Find the domain and range of relation R, where

$$R=\left\{(x,y): y=x+\frac{8}{x}, x,y\in N, x<9\right\}$$

Q5 Draw the graph of modulus function

and find out domain and range of modulus function.

Answer to check yourself

Q1 B
Q2 A
Q3 B
Q4 C
Q5 B

Answer to stretch yourself

Q1 (i) R = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)}

(ii) Domain = {1,2,3,4,6}
Range = {1,2,3,4,6}

Q2 Let $A \times B$ = {(7,13), (7,15), (7,17), (9,13), (9,17), (11,13), (11,15), (11,17)}
None of the order pair is showing that first minus second component is odd. Hence the relation is an empty relation.

Q3 Total number of possible relation are 2^4 i.e. 16.

Q4 Domain = {1,2,4,8}
Range = {9,6}

Q5 In modulus function
Domain is $(-\infty, \infty)$
Range is $(0, \infty)$