3

TRIGONOMETRIC FUNCTIONS-I

Circular Measure of Angle

- An angle is a union of two rays with the common end point. An angle is formed by the rotation of a ray as well.
- Negative and positive angles are formed according as the rotation is clockwise or anticlock-wise.

A Unit Circle

- when a line segment makes one complete rotation, its end point describes a circle.
- In case the length of the rotating line be one unit then the circle described will be a circle of unit radius. Such a circle is termed as **unit circle**.

A radian

- A radian is the measure of an angle subtended at the centre of a circle by an arc equal in length to the radius (r) of the circle

Relation between Degree and Radian

- An arc of unit length subtends an angle of 1 radian. The circumference 2π subtendan angle of 2π radian

Relation Between Length of an Arc and Radius of the Circle

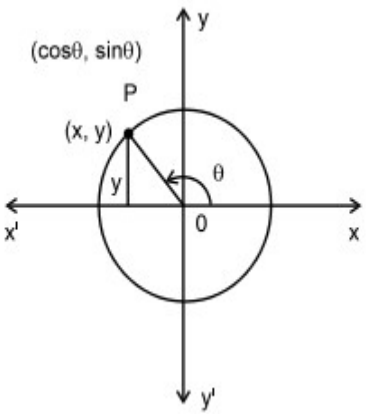
- The angle subtended by an arc of a circle at the center of the circle

  Is give by the ratio of the length of the arc and the radius of the circle.
- $\theta = \frac{l}{r}$

TRIGONOMETRIC FUNCTIONS

I Quadrant	II Quadrant	III Quadrant	IV Quadrant
All	Sin,cosec	tan,cot	cos ,sec
Positive	Positive	Positive	Positive

Relation Between Trigonometric Functions



$$sin^2\theta + cos^2\theta = 1$$

$$cosec^2\theta = 1 + cot^2\theta$$

$$sec^2\theta = 1 + tan^2\theta$$

$$x = \sin\theta\,,$$

$$y = \cos\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

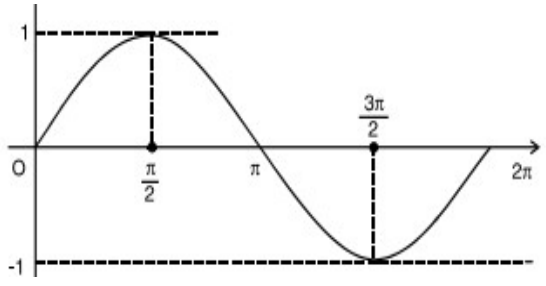
Trigonometric Functions Of Some Specific Real Numbers

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D.

Graphs of Trigonometric Functions

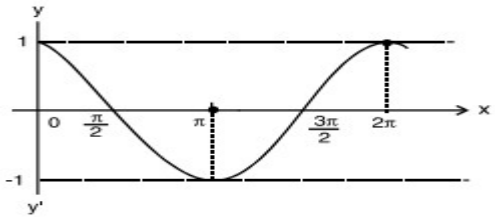
- The importance of the graph of functions stems from the fact that this is a convenient way of presenting many properties of the functions.
- By observing the graph we can examine several characteristic properties of the functions such as
  - A. periodicity,
  - B. intervals in which the function is increasing or decreasing
  - C. symmetry about axes,
  - D. maximum and minimum points of the graph in the given interval

Graphs of Trigonometric function

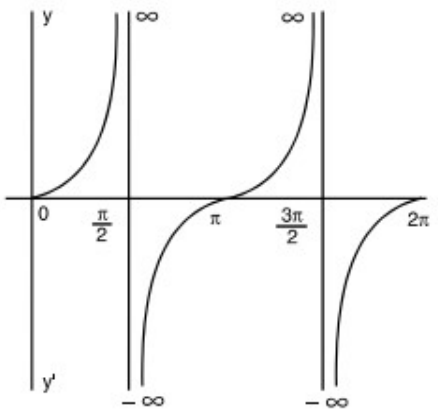


Variation of sinθ from 0 to 2π

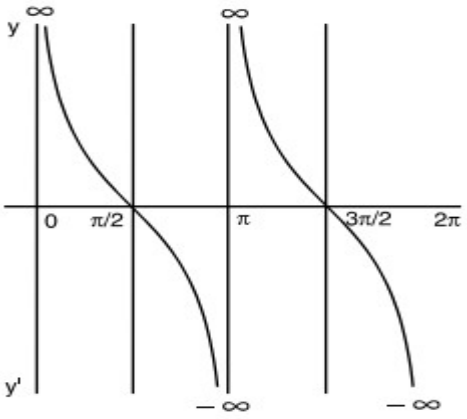
	sin	cos	tan	cosec	sec	cot
0	0	1	0	ND	1	ND
$\frac{\pi}{6}$	0.5	0.87	0.58	2	1.15	1.73
$\frac{\pi}{3}$	0.87	0.5	+N.D.	1.5	2	0.58
$\frac{\pi}{2}$	1	0	-1.73	1	-ND	0
$\frac{2\pi}{3}$	0.87	0.5	-0.58	1.15	-2	-.58
$\frac{5\pi}{6}$	0.5	-0.87	0	2	-1.15	-1.73
π	0	-1	0.58	-ND	-1	ND
$\frac{7\pi}{6}$	-0.5	-0.87	1.73	-2	-1.15	1.73
$\frac{4\pi}{3}$	-0.87	-0.5	+	-1.15	-2	.58
$\frac{3\pi}{2}$	-1	0	-1.73	-1	-ND	0
$\frac{5\pi}{3}$	-0.87	0.5	-0.58	-1.15	2	-.58
$\frac{11\pi}{6}$	-0.5	0.87	0	2	1.15	-1.73
2π	0	1	0	-ND		-ND



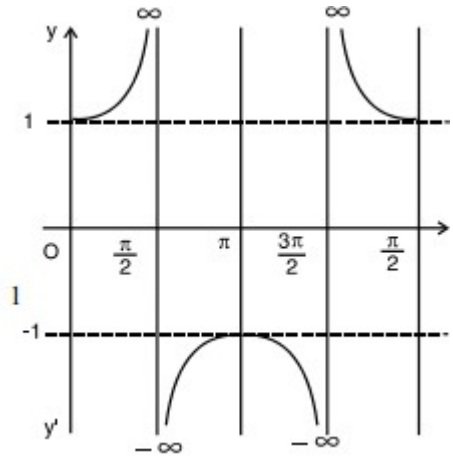
Variation of cos θ from 0 to 2π



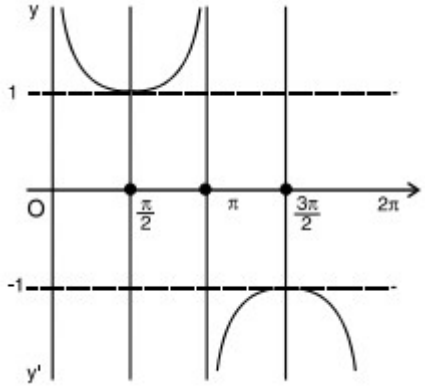
Variation of tan θ from 0 to 2π



Variation of cot θ from 0 to 2π



Variation of sec θ from 0 to 2π



Variation of cosec θ varies from 0 to 2π

PERIODICITY OF THE TRIGONOMETRIC FUNCTIONS

- A function f (x) is said to be periodic if its value is unchanged when the value of the variable is increased by a constant, that is
- if f (x + p) = f (x) for all x.
- If p is smallest positive constant of this type, then p is called the period of the function f (x).
- If f (x) is a periodic function with period p, then 1/ f(x)is also a periodic function with period p.

Check Your Progress

Q1 The value of $\frac{\pi}{5}$ radians is equal to:

(A) 18^o
(B) 36^o
(C) 45^o
(D) 90^o

Q2 In a triangle two angles are 50^o and

70^o. The measure of third angle of the triangle in radian is:

(A) $\frac{\pi}{2}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Q3 The angle in radians subtended by an arc of length 20cm at the center of a circle of radius 45cm is equal to:

(A) $\frac{9}{4}$ radians

(B) $\frac{4}{9}$ radians

(C) $\frac{2}{3}$ radians

(D) $\frac{3}{2}$ radians

Q4 The minimum value of $\sin\theta$ is equal to:

(A) 1
(B) 0
(C) 2
(D) – 1

Q5 In which point, the graph of $\tan\theta$ is discontinuous?

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{\pi}{4}$

(D) 2π

Stretch yourself

Q1 Draw the graph of $\cos\theta$ where θ varies from 0 to 2π, write any two major observations.

Q2 Prepare a table to write the values of trigonometric functions $\sin\theta$, $\cos\theta$ & $\tan\theta$ where θ takes values 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ & $\frac{\pi}{2}$

Q3 Draw the graph of $\tan\theta$ and write any two observations.

Q4 Find the period of

(i) $x = 3\sin 2y$ (ii) $x = \cos\frac{y}{2}$

Q5 Write the periods of trigonometric functions
(i) $\sin x$ and
(ii) $\cos x$

Answer to Check your Progress

Q1(B)

$\frac{\pi}{5}$ radians

$= \left(\frac{360}{2\pi} \times \frac{\pi}{5}\right)^o$

$= 36^o$

Q2 (B)

3^{rd} angle
$= 60^o$
In radian

$\Rightarrow \frac{2\pi}{360^o} \times 6^o$

$= \frac{\pi}{3}$ radians

Q3 (B)

$\theta = \frac{l}{r}$

$= \frac{20}{45}$ radians

$= \frac{4}{9}$ radians

Q4 (D)

Q5(A)

Answer to stretch yourself

Q 1 Draw the graph
(i) minimum value of $\cos\theta = -1$ and maximum = 1
(ii)It is continuous in every where

Q 2 Prepare the table and write the value table and write the values of
$\sin\theta,\ \cos\theta$ & $\tan\theta$

Q 3
(i) The value of $\tan\theta$ lies between
$-\infty + 0 + \infty$
(ii)Its period is π

Q 4
(i) π (ii) 4π

Q 5
(i) $\sin x = \sin(x + 2n\pi)$ where $n = 0, \pm 1,$ …….
(ii) $\cos x = \cos(x + 2n\pi)$ where
$n = 0, \pm 1 \pm 2,$ …….