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Complex Numbers

• Complex Number

Any number which can be expressed in the form of $a + b\,i$ where $i = \sqrt{-1}$ and a, b are real number is called as complex number. It is denoted as $Z = a + b\,i$

For example $Z = 3 + 7\,i$ is a complex number.

• Positive Integral powers of i

$i = \sqrt{-1} = i$

$i^2 = (\sqrt{-1})^2 = -1$

$i^3 = 2^2.2 = (-1).i = -2$

$2^4 = (2^2)^2 = (-1)^2 = 1$

If n is a positive integer such that $n > 4$, then to find 2^n, we first divide n by 4

• Conjugate of a Complex Number

The conjugate of a complex number is obtained by changing the sign of the imaginary part.

Let $Z = a + b\,i$, be the complex number

$\overline{Z} = a - b\,i$, is the conjugate of Z

(i) The conjugate of a real number is the number 1+ self. $Z = \overline{Z}$

(ii) If Z is a purely imaginary number then $\overline{Z} = -Z$.

(iii) Conjugate of the conjugate of a complex number is the number itself i.e. $(\overline{Z}) = Z$

• Modules of a Complex number

$Z = a + b\,i$, be the complex number $|Z|$ is the modulus of Z is given by $|Z| = \sqrt{a^2 + b^2}$

(i) $|Z| = 0 <=> Z = 0$
(ii) $|Z| = |\overline{Z}|$
(iii) $|Z| = |-Z|$

• Equality of Two Complex Numbers

Two complex number are equal, if and only if their real parts and imaginary parts are respectively equal.

If $a + b\,i = c + d\,i$, then

$a = c$ and $b = d$

• Operations on Complex Numbers

(A) Addition of Complex Numbers

If $Z_1 = a + b\,i$ and $Z_2 = c + d\,i$ are two complex numbers, then their addition is defined as

$$Z_1 + Z_2 = (a + c) + i\,(b + d)$$

(B) Subtraction of Complex Numbers

To subtract a complex number from another, the corresponding real and imaginary parts are separately subtracted.

$Z_1 = a + b\,i\ ,\ \ Z_2 = c + d\,i$

$Z_1 - Z_2 = (a - c) + i\,(b - d)$

(C) Multiplication of Two Complex Numbers

If $(a + b\,i)$ and $(c + d\,i)$ are two complex numbers, their product is defined as the complex number $(ac - bd) + (ad + bc)\,i$

(D) Division of Two Complex Numbers

Division of complex numbers involves multiplying both numerator and denominator with the conjugate of the denominator

Let $Z_1 = a + b\,i\ ,\ \ Z_2 = c + d\,i$

$$\frac{Z_1}{Z_2} = \frac{(ac-bd) + (bc-ad)\,i}{c^2+d^2}$$

- **Square root of a Complex Number**

  (i) $a + i\,b$ has two square roots in each case and two square roots just differ in their sign.

  (ii) Square root of a complex number is also a complex number.

Check Your Progress

Q1 The number which can be expressed in the form of $a+bi$ where $i=\sqrt{-1}$ is termed as:

(A) Natural number
(B) Rational number
(C) Real number
(D) Complex number

Q2 What is the value of $1+i^{10}+2$?

(A) 1
(B) 0
(C) 2
(D) 3

Q3 The conjugate of the complex number $5+3i$ is expressed as:

(A) $3+5i$
(B) $5-3i$
(C) $5+3i$
(D) $3-5i$

Q4 What is the conjugate of $(2+i)^2$?

(A) $3-4i$
(B) $3+4i$
(C) $4+3i$
(D) $4-3i$

Q5 If the complex number $z=1+2i$, then the modules of $-z$ is equal to:

(A) 5
(B) $\sqrt{5}$
(C) 3
(D) $\sqrt{3}$

Q6 What should be added with $3+2i$ in order to get the result $7-i$?

(A) $4-3i$
(B) $3+4i$
(C) $2+7i$
(D) $7-2i$

Q7 The product of two complex number

$(1+2i)$ and $(1-3i)$ is:

(A) $2+6i$
(B) $1+6i$
(C) $7-i$
(D) $1-6i$

Q8 The modulus of the complex number $\frac{3+4i}{2+i}$ is equal to:

(A) $2\sqrt{5}$
(B) $\sqrt{5}$
(C) $\frac{1}{\sqrt{5}}$
(D) $5\sqrt{2}$

Q9 The multiplicative inverse of the complex number $3+4i$ is equal to:

(A) $\frac{3}{25}+\frac{4}{25}i$
(B) $\frac{4}{9}+\frac{1}{12}i$
(C) $\frac{4}{9}-\frac{1}{12}i$
(D) $\frac{3}{25}-\frac{4}{25}i$

Q10 If $z_1=(2+i), z_2=(1-i),$ and $z_3=(2+3i),$ then the value of $z_1+(z_2+z_3)$ is equal to:

(A) $3+5i$
(B) $5+3i$
(C) $6+5i$
(D) $5+6i$

Q11 What will be the additive inverse of $-10-5i$?

(A) $10+5i$
(B) $-10+5i$
(C) $5+10i$
(D) $-5+10i$

Q12 When $3+i$ is divided by $4-2i$ the resulting complex number is:

(A) $5+7i$
(B) $3-2i$
(C) $2+3i$
(D) $\frac{1}{2}+\frac{1}{2}i$

Q13 The modulus of the complex number $(1+i)(4-3i)$ is equal to:

(A) $2\sqrt{5}$
(B) $3\sqrt{7}$
(C) $5\sqrt{2}$
(D) $7\sqrt{3}$

Q14 What is the modulus of the complex number $z=a+bi$ in the polar form?

(A) $r=a+b$
(B) $r=\sqrt{a^2+b^2}$
(C) $r=\frac{ab}{a+b}$
(D) $r=\frac{a+b}{\sqrt{a^2+b^2}}$

Q15 What will be the value of x and y if $2x+3yi=4-9i$?

(A) $x=4, y=-9$
(B) $x=2, y=9i$
(C) $x=2$ and $y=-3$
(D) $x=2$ and $y=3i$

Answer to check your Progress

1D	2 C	3 B	4 A	5B
6 A	7 C	8 B	9 D	10 B

11A 12 D 13 C 14 B 15 C

Stretch Yourself

1. Find the conjugate of $\frac{3+2i}{5-3i}$
2. If $z_1 = 2 + i$, $z_2 = 3 - 2i$, then find value of $\left|\frac{2z_2 + z_1 - 5 - i}{2z_1 - z_2 + 3 - i}\right|^2$
3. Find $\frac{1+c+is}{1+c-is}$, If $c^2 + s^2 = 1$
4. If $(\sqrt{3} + i)^{100} = 2^{99}$ (a + ib), then find $a^2 + b^2$
5. Find the polar form of $\frac{1+7i}{(2-i)^2}$