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5

GRAVITATION

Have you ever thought why a ball thrown upward always comes back to the ground? Or a coin tossed in air falls back on the ground. Since times immemorial, human beings have wondered about this phenomenon. The answer was provided in the 17th century by Sir Isaac Newton. He proposed that the gravitational force is responsible for bodies being attracted to the earth. He also said that it is the same force which keeps the moon in its orbit around the earth and planets bound to the Sun. It is a universal force, that is, it is present everywhere in the universe. In fact, it is this force that keeps the whole universe together.

In this lesson you will learn Newton's law of gravitation. We shall also study the acceleration caused in objects due to the pull of the earth. This acceleration, called acceleration due to gravity, is not constant on the earth. You will learn the factors due to which it varies. You will also learn about gravitational potential and potential energy. You will also study Kepler's laws of planetary motion and orbits of artificial satellites of various kinds in this lesson. Finally, we shall recount some of the important programmes and achievements of India in the field of space research.

OBJECTIVES

After studying this lesson, you should be able to:

- *state the law of gravitation;*
- *calculate the value of acceleration due to gravity of a heavenly body;*
- *analyse the variation in the value of the acceleration due to gravity with height, depth and latitude;*
- *distinguish between gravitaitonal potential and gravitational potential energy;*
- *identify the force responsible for planetary motion and state Kepler's laws of planetary motion;*

- *calculate the orbital velocity and the escape velocity;*
- *explain how an artificial satellite is launched;*
- *distinguish between polar and equatorial satellites;*
- *state conditions for a satellite to be a geostationary satellite;*
- *calculate the height of a geostationary satellite and list their applications; and*
- *state the achievements of India in the field of satellite technology.*

5.1 LAW OF GRAVITATION

It is said that Newton was sitting under a tree when an apple fell on the ground. This set him thinking: since all apples and other objects fall to the ground, there must be some force from the earth acting on them. He asked himself: Could it be the same force which keeps the moon in orbit around the earth? Newton argued that at every point in its orbit, the moon would have flown along a tangent, but is held back to the orbit by some force (Fig. 5.1). Could this continuous 'fall' be due to the same force which forces apples to fall to the ground? He had deduced from Kepler's laws that the force between the Sun and planets varies as $1/r^2$. Using this result he was able to show that it is the same force that keeps the moon in its orbit around the earth. Then he generalised the idea to formulate the universal law of gravitation as.

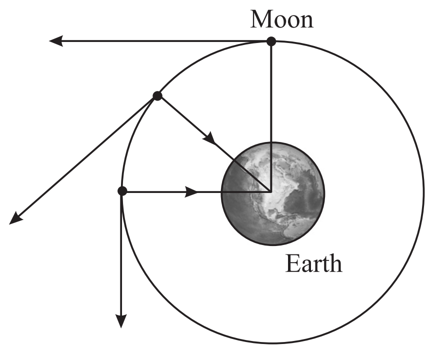


Fig. 5.1 : At each point on its orbit, the moon would have flown off along a tangent but the attraction of the earth keeps it in its orbit.

Every particle attracts every other particle in the universe with a force which varies as the product of their masses and inversely as the square of the distance between them. Thus, if m_1 and m_2 are the masses of the two particles, and r is the distance between them, the magnitude of the force F is given by.

$$F \propto \frac{m_1 m_2}{r^2}$$

or

$$F = G\frac{m_1 m_2}{r_2} \qquad (5.1)$$

The constant of proportionality, G, is called **the universal constant of gravitation. Its value remains the same between any two objects everywhere**

in the universe. This means that if the force between two particles is F on the earth, the force between these particles kept at the same distance anywhere in the universe would be the same.

One of the extremely important characteristics of the gravitational force is that it is always attractive. It is also one of the fundamental forces of nature.

Remember that the attraction is mutual, that is, particle of mass m_1 attracts the particle of mass m_2 and m_2 attracts m_1. Also, the force is along the line joining the two particles.

Knowing that the force is a vector quantity, does Eqn. (5.1) need modification? The answer to this question is that the equation should reflect both magnitude and the direction of the force. As stated, the gravitational force acts along the line joining the two particles. That is, m_2 attracts m_1 with a force which **is along the line joining the two particles** (Fig. 5.2). If the force of attraction exerted by m_1 on m_2 is denoted by $\mathbf{F}_{12}$ and the distance between them is denoted by $\mathbf{r}_{12}$, then the vector form of the law of gravitation is

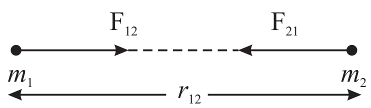


Fig. 5.2 : The masses m_1 and m_2 are placed at a distance r_{12} from eact other. The mass m_1 attracts m_2 with a force F_{12}.

$$\mathbf{F}_{12} = G\frac{m_1 m_2}{\mathbf{r}_{12}}\,\hat{\mathbf{r}}_{12} \qquad (5.2)$$

Here $\hat{r}_{12}$ is a unit vector from m_1 to m_2

In a similar way, we may write the force exerted by m_2 on m_1 as

$$\mathbf{F}_{21} = -G\frac{m_1 m_2}{\mathbf{r}_{21}^2}\,\hat{\mathbf{r}}_{21} \qquad (5.3)$$

As $\hat{\mathbf{r}}_{12} = -\hat{\mathbf{r}}_{21}$, from Eqns. (5.2) and (5.3) we find that

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \qquad (5.4)$$

The forces $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ are equal and opposite and form a pair of forces of action and reaction in accordance with Newton's third law of motion. Remember that $\hat{\mathbf{r}}_{12}$ and $\hat{\mathbf{r}}_{21}$ have unit magnitude. However, the directions of these vectors are opposite to each other.

Unless specified, in this lesson we would use only the magnitude of the gravitational force.

The value of the constant G is so small that it could not be determined by Newton or his contemporary experimentalists. It was determined by Cavendish for the first time about 100 years later. Its accepted value today is 6.67×10^{-11} Nm^{2}kg^{-2}. It is because of the smallness of G that the gravitational force due to ordinary objects is not felt by us.

Example 5.1 : Kepler's third law states (we shall discuss this in greater details later) that if r is the mean distance of a planet from the Sun, and T is its orbital period, then r^3/T^2 = const. Show that the force acting on a planet is inversely proportional to the square of the distance.

Solution : Assume for simplicity that the orbit of a planet is circular. (In reality, the orbits are nearly circular.) Then the centripetal force acting on the planet is

$$F = \frac{m\upsilon^2}{r}$$

where υ is the orbital velocity. Since $\upsilon = r\omega = \frac{2\pi r}{\mathrm{T}}$, where T is the period, we can rewrite above expression as

$$F = m\left(\frac{2\pi r}{\mathrm{T}}\right)^2 \Big/ r$$

or
$$F = \frac{4\pi^2 mr}{\mathrm{T}^2}$$

But $\mathrm{T}^2 \propto r^3$ or $\mathrm{T}^2 = Kr^3$ (Kelpler's 3rd law)

where K is a constant of proportionality. Hence

$$\therefore \qquad F = \frac{4\pi^2 mr}{Kr^3} = \frac{4\pi^2}{K} \times \frac{m}{r^2} = \frac{4\pi^2 m}{K} \cdot \frac{1}{r^2}$$

or
$$F \propto \frac{1}{r^2} \qquad \left(\because \frac{4\pi^2 m}{K} \text{ is constant for a planet}\right)$$

Before proceedins further, it is better that you check your progress.

INTEXT QUESTIONS 5.1

1. The period of revolution of the moon around the earth is 27.3 days. Remember that this is the period with respect to the fixed stars (the period of revolution with respect to the moving earth is about 29.5 days; it is this period that is used to fix the duration of a month in some calendars). The radius of moon's orbit is 3.84×10^8 m (60 times the earth's radius). Calculate the centripetal acceleration of the moon and show that it is very close to the value given by 9.8 ms^{-2} divided by 3600, to take account of the variation of the gravity as $1/r^2$.

2. From Eqn. (5.1), deduce dimensions of G.

3. Using Eqn. (5.1), show that G may be defined as the magnitude of force between two masses of 1 kg each separated by a distance of 1 m.

4. The magnitude of force between two masses placed at a certain distance is F. What happens to F if (i) the distance is doubled without any change in masses, (ii) the distance remains the same but each mass is doubled, (iii) the distance is doubled and each mass is also doubled?

5. Two bodies having masses 50 kg and 60 kg are seperated by a distance of 1m. Calculate the gravitational force between them.

5.2 ACCELERATION DUE TO GRAVITY

From Newton's second law of motion you know that a force **F** exerted on an object produces an acceleration **a** in the object according to the relation

$$\mathbf{F} = m\mathbf{a} \tag{5.5}$$

The force of gravity, i.e., the force exerted by the earth on a body lying on or near its surface, also produces an acceleration in the body. The acceleration produced by the force of gravity is called the **acceleration due to gravity**. It is denoted by the symbol g. According to Eq. (5.1), the magnitude of the force of gravity on a particle of mass m on the earth's surface is given by

$$F = \mathrm{G}\frac{m\,M}{R^2} \tag{5.6}$$

where M is the mass of the earth and R is its radius. From Eqns. (5.5) and (5.6), we get

$$mg = \mathrm{G}\,\frac{m\,M}{R^2}$$

or

$$g = \mathrm{G}\frac{M}{R^2} \tag{5.7}$$

Remember that the force due to gravity on an object is directed towards the center of the earth. It is this direction that we call vertical. Fig. 5.3 shows vertical directions at different places on the earth. The direction perpendicular to the vertical is called the **horizontal** direction.

Once we know the mass and the radius of the earth, or of any other celestial body such as a planet, the value of g at its surface can be calculated using Eqn. (5.7). On the surface of the earth, the value of g is taken as 9.8 $\mathrm{ms^{-2}}$.

Given the mass and the radius of a satellite or a planet, we can use Eqn. (5.7) to find the acceleration due to the gravitational attraction of that satellite or planet.

Before proceeding further, let us look at Eqn. (5.7) again. The acceleration due to gravity produced in a body is independent of its mass. This means that a heavy ball and a light ball will fall with the same velocity. **If we drop these balls from a certain height at the same time, both would reach the ground simultaneously.**

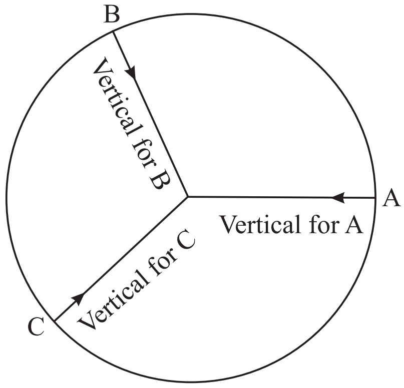


Fig. 5.3 : The vertical direction at any place is the direction towards the centre of earth at that point

ACTIVITY 5.1

Take a piece of paper and a small pebble. Drop them simultaneously from a certain height. Observe the path followed by the two bodies and note the times at which they touch the ground. Then take two pebbles, one heavier than the other. Release them simultaneously from a height and observe the time at which they touch the ground.

Fall Under Gravity

The fact that a heavy pebble falls at the same rate as a light pebble, might appear a bit strange. Till sixteenth century it was a common belief that a heavy body falls faster than a light body. However, the great scientist of the time, Galileo, showed that the two bodies do indeed fall at the same rate. It is said that he went up to the top of the Tower of Pisa and released simultaneously two iron balls of considerably different masses. The balls touched the ground at the same time. But when feather and a stone were made to fall simultaneously, they reached the ground at different times. Galileo argued that the feather fell slower because it experienced greater force of buoyancy due to air. He said that if there were no air, the two bodies would fall together. In recent times, astronauts have performed the feather and stone experiment on the moon and verified that the two fall together. Remember that the moon has no atmosphere and so no air.

Under the influence of gravity, a body falls vertically downwards towards the earth. For small heights above the surface of the earth, the acceleration due to gravity does not change much. Therefore, the equations of motion for initial and final velocities and the distance covered in time t are given by

$$v = u + gt$$

$$s = ut + (\frac{1}{2})gt^2$$

and $$v^2 = u^2 + 2gs. \quad (5.8)$$

It is important to remember that g **is always directed vertically downwards, no matter what the direction of motion of the body is.** A body falling with an acceleration equal to g is said to be in **free-fall**.

From Eqn. (5.8) it is clear that if a body begins to fall from rest, it would fall a distance $h = (1/2)gt^2$ in time t. So, a simple experiment like dropping a heavy coin from a height and measuring its time of fall with the help of an accurate stop watch could give us the value of g. If you measure the time taken by a five-rupee coin to fall through a distance of 1 m, you will find that the average time of fall for several trials is 0.45 s. From this data, the value of g can be calculated. However, in the laboratory you would determine g by an indirect method, using a simple pendulum.

You must be wondering as to why we take radius of the earth as the distance between the earth and a particle on its surface while calculating the force of gravity on that particle. When we consider two discreet particles or mass points, the separation between them is just the distance between them. But when we calculate gravitational force between extended bodies, what distance do we take into account? To resolve this problem, the concept of **centre of gravity of a body is introduced**. This is a point such that, as far as the gravitational effect is concerned, we may replace the whole body by just this point and the effect would be the same. For geometrically regular bodies of uniform density, such as spheres, cylinders, rectangles, the geometrical center is also the centre of gravity. That is why we choose the center of the earth to measure distances to other bodies. For irregular bodies, there is no easy way to locate their centres of gravity.

Where is the center of gravity of metallic ring located? It should lie at the center the ring. But this point is outside the mass of the body. It means that the centre of gravity of a body may lie outside it. Where is your own centre of gravity located? Assuming that we have a regular shape, it would be at the centre of our body, somewhere beneath the navel.

Later on in this course, you would also learn about the **centre of mass** of a body. This is a point at which the whole mass of the body can be assumed to be concentrated. In a **uniform gravitational field**, the kind we have near the earth, the centre of gravity coincides with the centre of mass.

The use of centre of gravity, or the center of mass, makes our calculations extremely simple. Just imagine the amount of calculations we would have to do if we have to calculate the forces between individual particles a body is made of and then finding the resultant of all these forces.

You should remember that G and g represent different physical quantities. G **is the universal constant of gravitation** which remains the same everywhere, while g is **acceleration due to gravity,** which may change from place to place, as we shall see in the next section.

You may like to answer a few questions to check your progress.

INTEXT QUESTIONS 5.2

1. The mass of the earth is 5.97×10^{24} kg and its mean radius is 6.371×10^{6} m. Calculate the value of g at the surface of the earth.
2. Careful measurements show that the radius of the earth at the equator is 6378 km while at the poles it is 6357 km. Compare values of g at the poles and at the equator.
3. A particle is thrown up. What is the direction of g when (i) the particle is going up, (ii) when it is at the top of its journey, (iii) when it is coming down, and (iv) when it has come back to the ground?
4. The mass of the moon is 7.3×10^{22} kg and its radius is 1.74×10^{6} m. Calculate the gravitational acceleration at its surface.

5.3 VARIATION IN THE VALUE OF *G*

5.3.1 Variation with Height

The quantity R^2 in the denominator on the right hand side of Eqn. (5.7) suggests that **the magnitude of g decreases as square of the distance from the centre of the earth increases**. So, at a distance R from the surface, that is, at a distance $2R$ from the centre of the earth, the value of g becomes (1/4) th of the value of g at the surface. However, if the distance h above the surface of the earth, called **altitude**, is small compared with the radius of the earth, the value of g, denoted by g_h, is given by

$$g_h = \frac{\mathrm{G}M}{(R+h)^2}$$

$$= \frac{GM}{R^2\left(1+\dfrac{h}{R}\right)^2}$$

$$= \frac{g}{\left(1+\frac{h}{R}\right)^2} \qquad (5.9)$$

where $g = GM/R^2$ is the value of acceleration due to gravity at the surface of the earth. Therefore,

$$\frac{g}{g_h} = \left(1+\frac{h}{R}\right)^2 = 1 + \frac{2h}{R} + \left(\frac{h}{R}\right)^2$$

Since (h/R) is a small quantity, $(h/R)^2$ will be a still smaller quantity. So it can be neglected in comparison to (h/R). Thus

$$g_h = \frac{g}{\left(1+\frac{2h}{R}\right)} \qquad (5.10)$$

Let us take an example to understand how we apply this concept.

Example 5.2 : Modern aircrafts fly at heights upward of 10 km. Let us calculate the value of g at an altitude of 10 km. Take the radius of the earth as 6400 km and the value of g on the surface of the earth as 9.8 ms^{-2}.

Solution : From Eqn. (5.8), we have

$$g_h = \frac{g}{\left(1+\frac{2.(10)\ \text{km}}{6400\ \text{km}}\right)} = \frac{9.8\ \text{ms}^{-2}}{1.003} = 9.77\ \text{ms}^{-2}.$$

5.3.2 Variation of *g* with Depth

Consider a point P at a depth d inside the earth (Fig. 5.4). Let us assume that the earth is a sphere of uniform density ρ. The distance of the point P from the center of the earth is $r = (R - d)$. Draw a sphere of radius $(r - d)$. A mass placed at P will experience gravitational force from particles in (i) the shell of thickness d, and (ii) the sphere of radius r. It can be shown that the forces due to all the particles in the shell cancel each other. That is, the net force on the particle at P due to the matter in the shell is zero. Therefore, in calculating the acceleration due to gravity at P, we have to consider only the

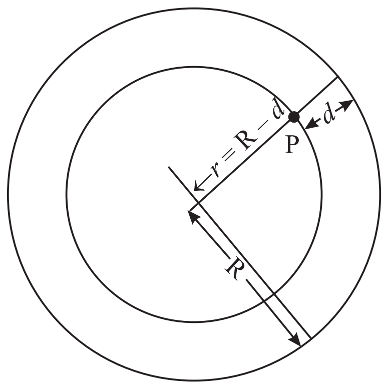


Fig. 5.4 : A point at depth d is at a distance $r = R - d$ from the centre of the earth

mass of the sphere of radius $(r - d)$. The mass M' of the sphere of radius $(r - d)$ is

$$M' = \frac{4\pi}{3} \rho (R - d)^3 \quad (5.10)$$

The acceleration due to gravity experienced by a particle placed at P is, therefore,

$$g_d = G\frac{M'}{(R-d)^2} = \frac{4\pi G}{3} \rho (R - d) \quad (5.11)$$

Note that as d increases, $(R - d)$ decreases. This means that **the value of g decreases as we go below the earth**. At $d = R$, that is, at the centre of the earth, the acceleration due to gravity will vanish. Also note that $(R - d) = r$ is the distance from the centre of the earth. Therefore, acceleration due to gravity is linearly proportional to r. The variation of g from the centre of the earth to distances far from the earth's surface is shown in Fig. 5.5.

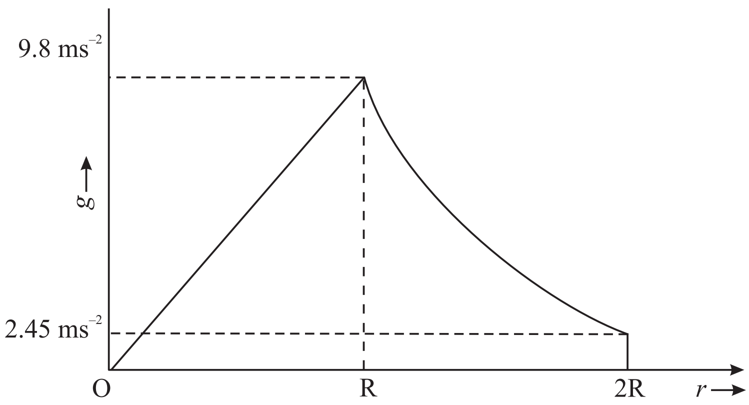


Fig. 5.5 : Variation of g with distance from the centre of the earth

We can express g_d in terms of the value at the surface by realizing that at $d = 0$, we get the surface value: $g = \frac{4\pi G}{3}\rho R$. It is now easy to see that

$$g_d = g\frac{(R-d)}{R} = g\left(1 - \frac{d}{R}\right), \ 0 \le d \le R \quad (5.12)$$

On the basis of Eqns. (5.9) and (5.12), we can conclude that g **decreases with both height as well as depth.**

Internal Structure of the Earth

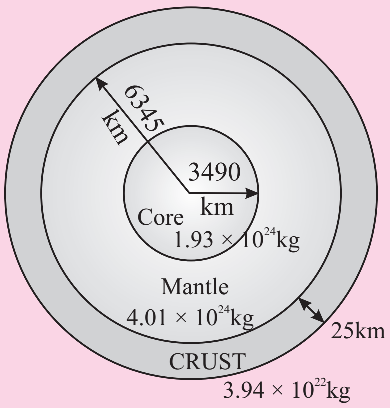


Fig. 5.6 : Structure of the earth (not to scale). Three prominent layers of the earth are shown along with their estimated masses.

Refer to Fig. 5.6 You will note that most of the mass of the earth is concentrated in its core. The top surface layer is very light. For very small depths, there is hardly any decrease in the mass to be taken into account for calculating g, while there is a decrease in the radius. So, the value of g increases up to a certain depth and then starts decreasing. It means that assumption about earth being a uniform sphere is not correct.

5.3.3 Variation of g with Latitude

You know that the earth rotates about its axis. Due to this, every particle on the earth's surface excecutes circular motion. In the absence of gravity, all these particles would be flying off the earth along the tangents to their circular orbits. Gravity plays an important role in keeping us tied to the earth's surface. You also know that to keep a particle in circular motion, it must be supplied centripetal force. A small part of the gravity force is used in supplying this centripetal force. As a result, the force of attraction of the earth on objects on its surface is slightly reduced. The maximum effect of the rotation of the earth is felt at the equator. At poles, the effect vanishes completely. We now quote the formula for variation in g with latitude without derivation. If g_λ denotes the value of g at latitude λ and g is the value at the poles, then

$$g_\lambda = g - R\omega^2 \cos\lambda, \qquad (5.13)$$

where ω is the angular velocity of the earth and R is its radius. You can easily see that at the poles, $\lambda = 90$ degrees, and hence $g_\lambda = g$.

Example 5.3 : Let us calculate the value of g at the poles.

Solution : The radius of the earth at the poles = 6357 km = 6.357×10^6 m

The mass of the earth = 5.97×10^{24} kg

Using Eqn. (5.7), we get

g at the poles = $[6.67 \times 10^{-11} \times 5.97 \times 10^{24} / (6.357 \times 10^{6})^{2}]$ ms^{-2}

$= 9.853$ ms^{-2}

Example 5.4 : Now let us calculate the value of g at $\lambda = 60°$, where radius of earth is 6371 km.

Solution : The period of rotation of the earth, $T = 24$ hours $= (24 \times 60 \times 60)$ s

$\therefore$ frequency of the earth's rotation $= 1/T$

angular frequency of the earth $\omega = 2\pi/T = 2\pi/(24 \times 60 \times 60)$

$= 7.27 \times 10^{-5}$

$\therefore R\omega^2 \cos\lambda = 6.371 \times 10^{6} \times (7.27 \times 10^{-5})^{2} \times 0.5 = 0.017$ ms^{-2}

Since $g_0 = g - R\omega^2 \cos\lambda$, we can write

g_λ (at latitude 60 degrees) $= 9.853 - 0.017 = 9.836$ ms^{-2}

INTEXT QUESTIONS 5.3

1. At what height must we go so that the value of g becomes half of what it is at the surface of the earth?

2. At what depth would the value of g be 80% of what it is on the surface of the earth?

3. The latitude of Delhi is approximately 30 degrees north. Calculate the difference between the values of g at Delhi and at the poles.

4. A satellite orbits the earth at an altitude of 1000 km. Calcultate the acceleration due to gravity acting on the satellite (i) using Eqn. (5.9) and (ii) using the relation g is proportional to $1/r^2$, where r is the distance from the centre of the earth. Which method do you consider better for this case and why?

5.4 WEIGHT AND MASS

The force with which a body is pulled towards the earth is called its weight. If m is the mass of the body, then its weight W is given by

$$W = mg \qquad (5.14)$$

Since weight is a force, its unit is newton. If your mass is 50 kg, your weight would be 50 kg $\times$ 9.8 ms^{-2} = 490 N.

Since g varies from place to place, weight of a body also changes from place to place.

The weight is maximum at the poles and minimum at the equator. This is because the radius of the earth is minimum at the poles and maximum at the equator. The weight decreases when we go to higher altitudes or inside the earth.

The mass of a body, however, does not change. Mass is an intrinsic property of a body. Therefore, it stays constant wherever the body may be situated.

Note: In everyday life we often use mass and weight interchangeably. Spring balances, though they measure weight, are marked in kg (and not in N).

5.4.1 Gravitational Potential and Potential energy

The Potential energy of an object under the influence of a conservative force may be defined as the energy stored in the body and is measured by the work done by an external agency in bringing the body from some standard position to the given position.

If a force F displaces a body by a small distance dr aganist the conservative force, without changing its speed, the small change in the potential energy dU is given by,

$$dU = -F.dr$$

In case of gravitational force between two masses M and m separated by a distance r,

$$F = \frac{GMm}{r^2}$$

$\therefore$ gravitational potential energy

$$dU = \frac{GMm}{r^2}dr$$

or
$$U = GMm\int_{\infty}^{r}\frac{1}{r^2}dr = -\frac{GMm}{r}$$

It shows that the gravitational potential energy between two particles of masses M and m separated by a distance r is given by

$$U = -\frac{GMm}{r} + \text{ a constant}$$

The gravitational potential energy is zero when r approaches infinity. So the constant is zero and $U = \frac{-GMm}{r}$

Gravitational Potential (V) of mass M is defined as the gravitational potential energy of unit mass. Hence,

Gravitational potential, $V = \frac{U}{m} = -\frac{GM}{r}$

It is a scalar quantity and its SI unit is J/kg.

ACTIVITY 5.2

Calculate the weight of an object of mass 50 kg at distances of 2R, 3R, 4R, 5R and 6R from the centre of the earth. Plot a graph showing the weight against distance. Show on the same graph how the mass of the object varies with distance.

Try the following questions to consolidate your ideas on mass and weight.

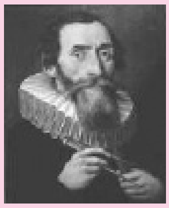
INTEXT QUESTIONS 5.4

1. Suppose you land on the moon. In what way would your weight and mass be affected?
2. Compare your weight at Mars with that on the earth? What happens to your mass? Take the mass of Mars = 6×10^{23} kg and its radius as 4.3×10^{6} m.
3. You must have seen two types of balances for weighing objects. In one case there are two pans. In one pan, we place the object to be weighed and in the other we place weights. The other type is a spring balance. Here the object to be weighed is suspended from the hook at the end of a spring and reading is taken on a scale. Suppose you weigh a bag of potatoes with both the balances and they give the same value. Now you take them to the moon. Would there be any change in the measurements made by the two balances?
4. State the SI unit of Gravitational potential.

5.5 KEPLER'S LAWS OF PLANETARY MOTION

In ancient times it was believed that all heavenly bodies move around the earth. Greek astronomers lent great support to this notion. So strong was the faith in the earth-centred universe that all evidences showing that planets revolved around the Sun were ignored. However, Polish Astronomer Copernicus in the 15th century proposed that all the planets revolved around the Sun. In the 16th century, Galileo, based on his astronomical observations, supported Copernicus. Another European astronomer, Tycho Brahe, collected a lot of observations on the motion of planets. Based on these observations, his assistant Kepler formulated **laws of planetary motion**.

Johannes Kepler



German by birth, Johannes Kepler, started his career in astronomy as an assistant to Tycho Brahe. Tycho religiously collected the data of the positions of various planets on the daily basis for more than 20 years. On his death, the data was passed on to Kepler who spent 16 years to analyse the data. On the basis of his analysis, Kepler arrived at the three laws of planetary motion.

He is considered as the founder of geometrical optics as he was the first person to describe the working of a telescope through its ray diagram.

For his assertion that the earth revolved around the Sun, Galileo came into conflict with the church because the Christian authorities believed that the earth was at the centre of the universe. Although he was silenced, Galileo kept recording his observations quietly, which were made public after his death. Interestingly, Galileo was freed from that blame recently by the present Pope.

Kepler formulated three laws which govern the motion of planets. These are:

1. The orbit of a planet is an ellipse with the Sun at one of the foci (Fig. 5.7). (An ellipse has two foci.)

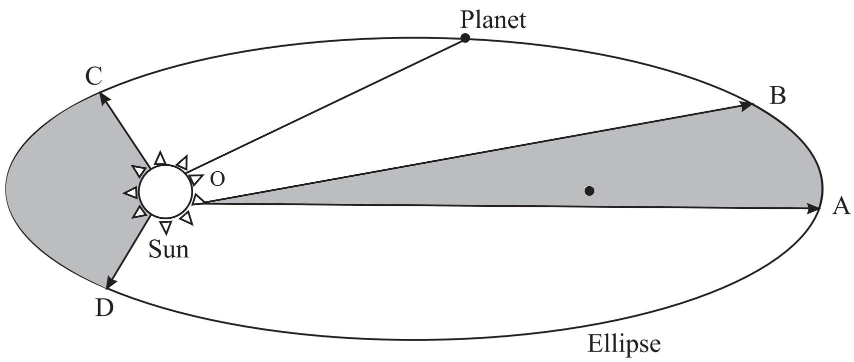


Fig. 5.7 : The path of a planet is an ellipse with the Sun at one of its foci. If the time taken by the planet to move from point A to B is the same as from point C to D, then according to the second law of Kepler, the areas AOB and COD are equal.

2. The area swept by the line joining the planet to the sun in unit time is constant through out the orbit (Fig 5.7)

3. The square of the period of revolution of a planet around the sun is proportional to the cube of its average distance from the Sun. If we denote the period by T and the average distance from the Sun as r, $T^2 \propto r^3$.

Let us look at the third law a little more carefully. You may recall that Newton used this law to deduce that the force acting between the Sun and the planets varied as $1/r^2$ (Example 5.1). Moreover, if T_1 and T_2 are the orbital periods of two planets and r_1 and r_2 are their mean distances from the Sun, then the third law implies that

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \tag{5.15}$$

The constant of proportionality cancels out when we divide the relation for one planet by the relation for the second planet. This is a very important relation. For example, it can be used to get T_2, if we know T_1, r_1 and r_2.

Example 5.5 : Calculate the orbital period of planet mercury, if its distance from the Sun is 57.9×10^9 m. You are given that the distance of the earth from the Sun is 1.5×10^{11} m.

Solution : We know that the orbital period of the earth is 365.25 days. So, T_1 = 365.25 days and $r_1 = 1.5 \times 10^{11}$ m. We are told that $r_2 = 57.9 \times 10^9$ m for mercury. Therefore, the orbital period of mercury is given by T_2

$$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$$

On substituting the values of various quantities, we get

$$T_2 = \sqrt{\frac{T_1^2 r_2^3}{r_1^3}} = \sqrt{\frac{(365.25)^2 \times (57.9 \times 10^9)^3 \text{m}^3}{(1.5 \times 10^{11})^3 \text{m}^3}} \text{ days}$$

$$= 87.6 \text{ days.}$$

In the same manner you can find the orbital periods of other planets. The data is given below. You can also check your results with numbers in Table 5.1.

Table 5.1: Some data about the planets of solar system

Name of the planet	Mean distance from the Sun (in terms of the distance of earth)	Radius ($\times 10^3$ km)	Mass (Earth Masses)
Mercury	0.387	2.44	0.53
Venus	0.72	6.05	0.815
Earth	1.0	6.38	1.00
Mars	1.52	3.39	0.107
Jupiter	5.2	71.40	317.8
Saturn	9.54	60.00	95.16
Uranus	19.2	25.4	14.50
Neptune	30.1	24.3	17.20
Pluto	39.4	1.50	0.002

Kepler's laws apply to any system where the force binding the system is gravitational in nature. For example, they apply to Jupiter and its satellites. They also apply to the earth and its satellites like the moon and artificial satellites.

Example 5.6 : A satellite has an orbital period equal to one day. (Such satellites are called geosynchronous satellites.) Calculate its height from the earth's surface, given that the distance of the moon from the earth is 60 R_E (R_E is the radius of the earth), and its orbital period is 27.3 days. [This orbital period of the moon is with respect to the fixed stars. With respect to the earth, which itself is in orbit round the Sun, the orbital period of the moon is about 29.5 day.]

Solution : A geostationary satellite has a period T_2 equal to 1 day. For moon T_1 = 27.3 days and $r_1 = 60\ R_E$, T_2 = 1 day. Using Eqn. (5.15), we have

$$r_2 = \left[\frac{r_1^3 T_2^2}{T_1^2}\right]^{1/3} = \left[\frac{(60^3\ R_E^3)(1^2\ \text{day}^2)}{27.3^2\ \text{day}^2}\right]^{1/3} = 6.6\ R_E.$$

Remember that the distance of the satellite is taken from the centre of the earth. To find its height from the surface of the earth, we must subtract R_E from 6.6 R_E. The required distance from the earth's surface is 5.6 R_E. If you want to get this distance in km, multiply 5.6 by the radius of the earth in km.

5.5.1 Orbital Velocity of Planets

We have so for talked of orbital periods of planets. If the orbital period of a planet is T and its distance from the Sun is r, then it covers a distance $2\pi r$ in time T. Its orbital velocity is, therefore,

$$v_{orb} = \frac{2\pi r}{T} \qquad (5.16)$$

There is another way also to calculate the orbital velocity. The centripetal force experienced by the planet is mv_{orb}^2/r, where m is its mass. This force must be supplied by the force of gravitation between the Sun and the planet. If M is the mass of the Sun, then the gravitational force on the planet is $\frac{G\,m\,M_s}{r^2}$. Equating the two forces, we get

$$\frac{mv_{orb}^2}{r} = \frac{G\ M_s}{r^2},$$

so that,

$$v_{orb} = \sqrt{\frac{G\ M_s}{r^2}} \qquad (5.17)$$

Notice that the mass of the planet does not enter the above equation. The orbital velocity depends only on the distance from the Sun. Note also that if you substitute υ from Eqn. (5.16) in Eqn. (5.17), you get the third law of Kepler.

INTEXT QUESTIONS 5.5

1. Many planetary systems have been discovered in our Galaxy. Would Kepler's laws be applicable to them?

2. Two artificial satellites are orbiting the earth at distances of 1000 km and 2000 km from the surface of the earth. Which one of them has the longer period? If the time period of the former is 90 min, find the time period of the latter.

3. A new small planet, named Sedna, has been discovered recently in the solar system. It is orbiting the Sun at a distance of 86 AU. (An AU is the distance between the Sun and the earth. It is equal to 1.5×10^{11} m.) Calculate its orbital period in years.

4. Obtain an expression for the orbital velocity of a satellite orbiting the earth.

5. Using Eqns. (5.16) and (5.17), obtain Kepler's third law.

5.6 ESCAPE VELOCITY

You now know that a ball thrown upwards always comes back due to the force of gravity. If you throw it with greater force, it goes a little higher but again comes back. If you have a friend with great physical power, ask him to throw the ball upwards. The ball may go higher than what you had managed, but it still comes back. You may then ask: Is it possible for an object to escape the pull of the earth? The answer is 'yes'. The object must acquire what is called the **escape velocity. It is defined as the minimum velocity required by an object to escape the gravitational pull of the earth.**

It is obvious that the escape velocity will depend on the mass of the body it is trying to escape from, because the gravitational pull is proportional to mass. It will also depend on the radius of the body, because smaller the radius, stronger is the gravitational force.

The escape velocity from the earth is given by

$$\upsilon_{esc} = \sqrt{\frac{2G\,M}{R}} \qquad (5.18)$$

where M is the mass of the earth and R is its radius. For calculating escape velocity from any other planet or heavenly body, mass and radius of that heavenly body will have to be substituted in the above expression.

It is not that the force of gravity ceases to act when an object is launched with escape velocity. The force does act. Both the velocity of the object as well as the force of gravity acting on it decrease as the object goes up. It so happens that the force becomes zero before the velocity becomes zero. Hence the object escapes the pull of gravity.

Try the following questions to grasp the concept.

INTEXT QUESTIONS 5.6

1. The mass of the earth is 5.97×10^{24} kg and its radius is 6371 km. Calculate the escape velocity from the earth.
2. Suppose the earth shrunk suddenly to one-fourth its radius without any change in its mass. What would be the escape velocity then?
3. An imaginary planet X has mass eight times that of the earth and radius twice that of the earth. What would be the escape velocity from this planet in terms of the escape velocity from the earth?

5.7 ARTIFICIAL SATELLITES

A cricket match is played in Sydney in Australia but we can watch it live in India. A game of Tennis played in America is enjoyed in India. Have you ever wondered what makes it possible? All this is made possible by artificial satellites orbiting the earth. You may now ask : How is an artificial satellite put in an orbit?

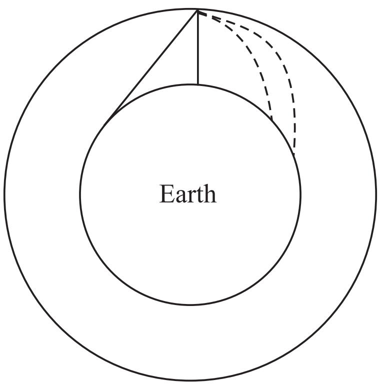


Fig. 5.8 : A projectile to orbit the earth

You have already studied the motion of a projectile. If you project a body at an angle to the horizontal, it follows a parabolic path. Now imagine launching bodies with increasing force. What happens is shown in Fig. 5.8. Projectiles travel larger and larger distances before falling back to the earth. Eventually, the projectile goes into an orbit around the earth. It becomes an **artificial satellite**. Remember that such satellites are man-made and launched with a particular purpose in mind. Satellites like the moon are natural satellites.

In order to put a satellite in orbit, it is first lifted to a height of about 200 km to minimize loss of energy due to friction in the atmosphere of the earth. Then it is given a horizontal push with a velocity of about 8 kms^{-1}.

The orbit of an artificial satellite also obeys Kepler's laws because the controlling force is gravitational force between the satellite and the earth. The orbit is elliptic in nature and its plane always passes through the center of the earth.

Remember that the orbital velocity of an artificial satellite has to be less than the escape velocity; otherwise it will break free of the gravitational field of the earth and will not orbit around the earth. From the expressions for the orbital velocity of a satellite close to the earth and the escape velocity from the earth, we can write

$$\upsilon_{orb} = \frac{\upsilon_{sec}}{\sqrt{2}} \qquad (5.19)$$

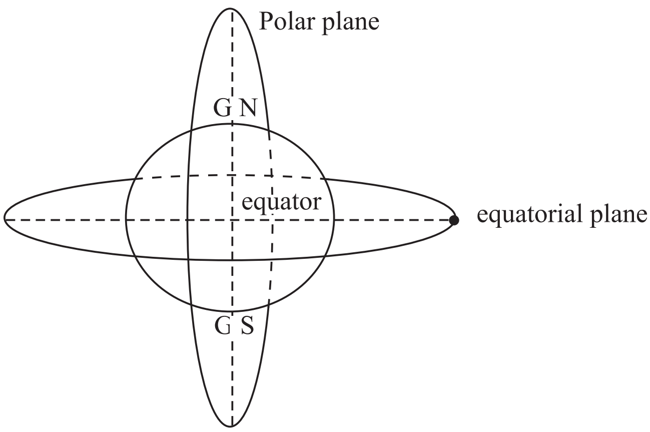


Fig. 5.9: Equitorial and polar orbits

Artificial satellites have generally two types of orbits (Fig. 5.9) depending on the purpose for which the satellite is launched. Satellites used for tasks such as remote sensing have **polar orbits**. The altitude of these orbits is about 800 km. If the orbit is at a height of less than about 300 km, the satellite loses energy because of friction caused by the particles of the atmosphere. As a result, it moves to a lower height where the density is high. There it gets burnt. The time period of polar satellites is around 100 minutes. It is possible to make a polar satellite **sun-synchronous**, so that it arrives at the same latitude at the same time every day. During repeated crossing, the satellite can scan the whole earth as it spins about its axis (Fig. 5.10). Such satellites are used for collecting data for weather prediction, monitoring floods, crops, bushfires, etc.

Satellites used for communications are put in equatorial orbits at high altitudes. Most of these satellites are **geo-synchronous, the ones which have the same orbital period as the period of rotation of the earth**, equal to 24 hours. Their height, as you saw in Example 5.6 is fixed at around 36000 km. Since their orbital period matches that of the earth, they appear to be hovering above the same spot on the earth. A combination of such satellites covers the entire globe, and signals can be sent from any place on the globe to any other place. Since a geo-synchronous satellite observes the same spot on the earth all the time, it can also be used for monitoring any peculiar happening that takes a long time to develop, such as severe storms and hurricanes.

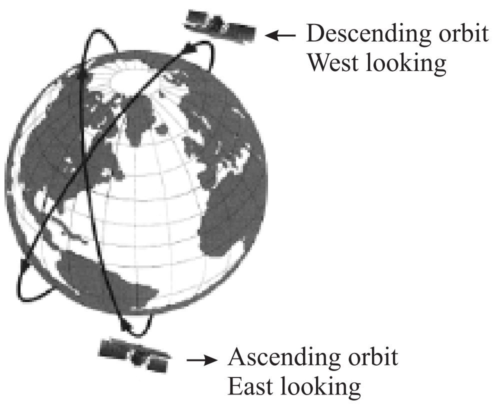


Fig. 5.10: A sun synchromous satellite scanning the earth

Applications of Satellites

Artificial satellites have been very useful to mankind. Following are some of their applications:

1. **Weather Forecasting :** The satellites collect all kinds of data which is useful in forecasting long term and short term weather. The weather chart that you see every day on the television or in newspapers is made from the data sent by these satellites. For a country like India, where so much depends on timely rains, the satellite data is used to watch the onset and progress of monsoon. Apart from weather, satellites can watch unhealthy trends in crops over large areas, can warn us of possible floods, onset and spread of forest fire, etc.

2. **Navigation :** A few satellites together can pinpoint the position of a place on the earth with great accuracy. This is of great help in locating our own position if we have forgotten our way and are lost. Satellites have been used to prepare detailed maps of large chunks of land, which would otherwise take a lot of time and energy.

3. **Telecommunication :** We have already mentioned about the transmission of television programmes from anywhere on the globe to everywhere became possible with satillites. Apart from television signals, telephone and radio signals are also transmitted. The communication revolution brought about by artificial satellites has made the world a small place, which is sometimes called a global village.

4. **Scientific Research :** Satellites can be used to send scientific instruments in space to observe the earth, the moon, comets, planets, the Sun, stars and galaxies. You must have heard of Hubble Space Telescope and Chandra X-Ray Telescope. The advantage of having a telescope in space is that light from distant objects does not have to go through the atmosphere. So there is hardly any reduction in its intensity. For this reason, the pictures taken by Hubble Space Telescope are of much superior quality than those taken by terrestrial telescopes.

   Recently, a group of Europeon scientists have observed an earth like planet out-side our solar system at a distance of 20 light years.

5. **Monitoring Military Activities :** Artificial satellites are used to keep an eye on the enemy troop movement. Almost all countries that can afford cost of these satellites have them.

Vikram Ambalal Sarabhai

Born in a family of industrialists at Ahmedabad, Gujarat, India. Vikram Sarabhai grew to inspire a whole generation of scientists in India. His initial work on time variation of cosmic rays brought him laurels in scientific fraternity. A founder of Physical Research Laboratory, Ahmedabad and a pioneer of space research in India, he was the first to realise the dividends that space research can bring in the fields of communication, education, metrology, remote sensing and geodesy, etc.

5.7.1 Indian Space Research Organization

India is a very large and populous country. Much of the population lives in rural areas and depends heavily on rains, particularly the monsoons. So, weather forecast is an important task that the government has to perform. It has also to meet the communication needs of a vast population. Then much of our area remains unexplored for minerals, oil and gas. Satellite technology offers a cost-effective solution for all these problems. With this in view, the Government of India set up in 1969 the Indian Space Research Organization (ISRO) under the dynamic leadership of Dr. Vikram Sarabhai. Dr. Sarabhai had a vision for using satellitis for educating the nation. ISRO has pursued a very vigorous programme to develop space systems for communication, television broadcasting, meteorological services, remote sensing and scientific research. It has also developed successfully launch vehicles for polar satellites ***(PSLV)*** (Fig. 5.11) and geo-synchronous satellites ***(GSLV)*** (Fig. 5.12). In fact, it has launched satellites for other countries like Germany, Belgium and Korea. and has joined the exclusive club of five countries. Its scientific programme includes studies of

(i) climate, environment and global change,

(ii) upper atmosphere,

(iii) astronomy and astrophysics, and

(iv) Indian Ocean.

Recently, ISRO launched an exclusive educational satillite EduSat, first of its kind in the world. It is being used to educate both young and adult students living in remote places.

It is now making preparation for a mission to the moon.

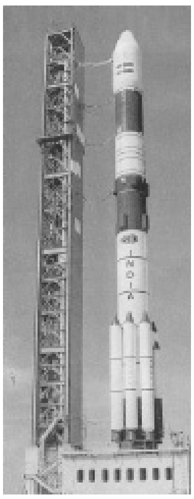


Fig. 5.11: PSLV

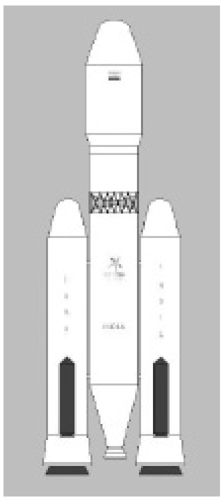


Fig. 5.12: GSLV

INTEXT QUESTIONS 5.7

1. Some science writers believe that some day human beings will establish colonies on the Mars. Suppose people living this desire to put in orbit a Mars synchronous satellite. The rotation period of Mars is 24.6 hours. The mass and radius of Mars are 6.4×10^{23} kg and 3400 km, respectively. What would be the height of the satellite from the surface of Mars?

2. List the advantages of having a telescope in space.

WHAT YOU HAVE LEARNT

- The force of gravitation exists between any two particles in the universe. It varies as the product of their masses and inversely as the square of distance between them.
- The constant of gravitation, G, is a universal constant.
- The force of gravitation of the earth attracts all bodies towards it.
- The acceleration due to gravity near the surface of the earth is 9.8 ms^{-2}. It varies on the surface of the earth because the shape of the earth is not perfectly spherical.
- The acceleration due to gravity varies with height, depth and latitude.
- The weight of a body is the force of gravity acting on it.
- Tthe gravitational potential energy between two particles of masses M and m separated by a distance r given by $U = -\dfrac{GMm}{r}$
- Kepler's first law states that the orbit of a planet is elliptic with sun at one of its foci.
- Kepler's second law states that the line joining the planet with the Sun sweeps equal areas in equal intervals of time.
- Kepler's third law states that the square of the orbital period of a planet is proportional to the cube of its mean distance from the Sun.
- A body can escape the gravitational field of the earth if it can acquire a velocity equal to or greater than the escape velocity.
- The orbital velocity of a satellite depends on its distance from the earth.

TERMINAL EXERCISE

1. You have learnt that the gravitational attraction is mutual. If that is so, does an apple also attract the earth? If yes, then why does the earth not move in response?
2. We set up an experiment on earth to measure the force of gravitation between two particles placed at a certain distance apart. Suppose the force is of magnitude F. We take the same set up to the moon and perform the experiment again. What would be the magnitude of the force between the two particles there?
3. Suppose the earth expands to twice its size without any change in its mass. What would be your weight if your present weight were 500 N?
4. Suppose the earth loses its gravity suddenly. What would happen to life on this plant?
5. Refer to Fig. 5.6 which shows the structure of the earth. Calculate the values of g at the bottom of the crust (depth 25 km) and at the bottom of the mantle (depth 2855 km).
6. Derive an expression for the mass of the earth, given the orbital period of the moon and the radius of its orbit.
7. Suppose your weight is 500 N on the earth. Calculate your weight on the moon. What would be your mass on the moon?
8. A polar satellite is placed at a height of 800 km from earth's surface. Calculate its orbital period and orbital velocity.

ANSWERS TO INTEXT QUESTIONS

5.1

1. Moon's time period $T = 27.3\text{d}$

$$= 27.3 \times 24 \times 3600 \text{ s}$$

Radius of moon's orbit $R = 3.84 \times 10^8$ m

$$\text{Moon's orbital speed } \upsilon = \frac{2\pi R}{T}$$

$$\text{Centripetal accleration} = \upsilon^2/R$$

$$= \frac{4\pi^2 R^2}{T^2}\cdot\frac{1}{R} = \frac{4\pi^2 R}{T^2}$$

$$= \frac{4\pi^2 \times 3.84 \times 10^8 \,\text{m}}{(27.3 \times 24 \times 3600)^2 \,\text{s}^2}$$

$$= \frac{4\pi^2 \times 3.84}{(27.3 \times 2.4 \times 3.6)^2} \times 10^{-2} \,\text{ms}^{-2}$$

$$= .00272 \,\text{ms}^{-2}$$

If we calculate centripetal acceleration on dividing g by 3600, we get the same value :

$$= \frac{9.8}{3600} \text{ms}^{-2}$$

$$= 0.00272 \,\text{ms}^{-2}$$

2. $F = \dfrac{G\, m_1\, m_2}{r^2}$

   F is force $\therefore\ G = \dfrac{\text{Force} \times r^2}{(\text{mass})^2} = \dfrac{\text{Nm}^2}{\text{kg}^2}$

3. $F = G\, \dfrac{m_1\, m_2}{r^2}$

   If m_1 = 1kg, m_2 = 1kg, r = 1m, then F = G

   or G is equal to the force between two masses of 1kg each placed at a distance of 1m from each other

4. (i) $F \propto 1/r^2$, if r is doubled, force becomes one-fourth.

   (ii) $F \propto m_1 m_2$, if m_1 and m_2 are both doubled then F becomes 4 times.

   (iii) $F \propto \dfrac{m_1\, m_2}{r^2}$,

   if each mass is doubled, and distance is also doubled, then

   F remains unchanged.

5. $F = G\, \dfrac{50 \,\text{kg} \times 60 \,\text{kg}}{1 \,\text{m}^2}$; $\qquad G = 6.68 \times 10^{-11} \dfrac{\text{Nm}^2}{\text{kg}^2}$

$$= 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \cdot \frac{3000 \,\text{kg}^2}{1 \,\text{m}^2}$$

$$= 6.67 \times 10^{-11} \times 3 \times 10^3 \,\text{N}$$

$$= 2 \times 10^{-7} \,\text{N}$$

5.2

1. $$g = \frac{GM}{R^2}$$

$$= 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \cdot \frac{5.97 \times 10^{24}\,\text{kg}}{(6.371 \times 10^6)^2\,\text{m}^2}$$

$$= \frac{6.97 \times 59.7}{6.371 \times 6.371} \frac{\text{N}}{\text{kg}} = 9.81 \text{ m s}^{-2}$$

2. g at poles

$$g_{\text{pole}} = \frac{GM}{R^2_{\text{pole}}}$$

$$= 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \cdot \frac{5.97 \times 10^{24}\,\text{kg}}{(6.371 \times 10^6)^2\,\text{m}^2}$$

$$= \frac{6.97 \times 59.7}{6.371 \times 6.371} \frac{\text{N}}{\text{kg}} = 9.81 \text{ ms}^{-2}$$

Similarly,

$$g_{\text{eguator}} = \frac{6.97 \times 59.7}{6.378 \times 6.378} \frac{\text{N}}{\text{kg}} = 9.79 \text{ ms}^{-2}$$

3. The value of g is always vertically downwards.

4. $$g_{\text{moon}} = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \times \frac{7.3 \times 10^{22}\,\text{kg}}{(1.74 \times 10^6)^2\,\text{m}^2}$$

$$= \frac{6.67 \times 7.3}{1.74 \times 1.74} \times 10^{-1} \frac{\text{N}}{\text{kg}} = 1.61 \text{ m s}^{-2}$$

5.3

1. Let g at distance r from the centre of the earth be called g_1.

Outside the earth,

then $$\frac{g}{g_1} = \frac{r^2}{\text{R}^2}$$

If $g_1 = g/2 \Rightarrow r^2 = 2R^2 \Rightarrow r = \sqrt{2}R = 1.412\ R$

$\therefore$ Height from earth's surface $= 1.4142\ R - R$

$= 0.4142\ R$

2. Inside the earth g varies as distance from the centre of the earth. Suppose at depth d, g is called g_d.

Then $\quad \dfrac{g_d}{g} = \dfrac{\mathrm{R} - d}{\mathrm{R}}$

If $\quad g_d = 80\%$, then

$$\frac{0.8}{1} = \frac{\mathrm{R} - d}{\mathrm{R}}$$

$\therefore \quad d = 0.2\ R$

3. In example 5.3, we calculated $\omega = 7.27 \times 10^{-5}$ rad s^{-1}

$\therefore$ $\mathrm{R}\omega^2 \cos 30^\circ = 6.37 \times 10^6 \times (7.27 \times 10^{-5})^2\ \mathrm{s}^{-2} \cdot \sqrt{3}/2 = 0.029\ \mathrm{ms}^{-2}$

g at poles $= 9.853$ m s^2

(Calculated in example 5.2)

$\therefore$ g at Delhi $= 9.853\ \mathrm{ms}^{-2} - 0.029\ \mathrm{ms}^{-2}$

$= 9.824\ \mathrm{ms}^{-2}$

4. Using formula (5.9),

$$g_h = \frac{g}{1 + \dfrac{2h}{R}} = \frac{9.81\ \mathrm{m\,s}^{-2}}{1 + \dfrac{2000\ \mathrm{km}}{6371\ \mathrm{km}}}$$

$$= \frac{9.81\ \mathrm{m\,s}^{-2}}{\dfrac{28371\ \mathrm{km}}{6371\ \mathrm{km}}} = 7.47\ \mathrm{m\ s}^{-2}$$

Using variation with r

$$g = \frac{GM}{(R+h)^2}$$

$$= 6.67 \times 10^{-11}\ \frac{\mathrm{Nm}^2}{\mathrm{kg}^2} \cdot \frac{5.97 \times 10^{24}\ \mathrm{kg}}{(7.371 \times 10^6)^2\ \mathrm{m}^2}$$

$= 7.33\ \mathrm{ms}^{-2}$

This gives more accurate results because formula (5.9) is for the case $h << R$. In this case h is not $<< R$.

5.4

1. On the moon the value of g is only 1/6th that on the earth. So, your weight on moon will become 1/6th of your weight on the earth. The mass, however, remains constant.

2. Mass of Mars = 6×10^{23} kg

   Radius of Mars = 4.3×10^{6} m

   $$\therefore \quad g_{\text{Mars}} = G\frac{M}{R^2} = 6.67 \times 10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2} \cdot \frac{6\times 10^{23}\text{kg}}{(4.3\times 10^{6})^2\text{m}^2} = 2.16$$

   $$\frac{\text{Weight on Mars}}{\text{Weight on Earth}} = \frac{\text{m}\,.\,2.16}{\text{m}\,.\,9.81} = 0.22$$

   So, your weight will become roughly 1/4th that on the earth. Mass remains constant.

3. Balances with two pans actually compare masses because g acts on both the pans and gets cancelled. The other type of balance, spring balance, measures weight. The balance with two pans gives the same reading on the moon as on the earth. Spring balance with give weight as 1/6th that on the earth for a bag of potatoes.

4. SI unit of Gravational potential is J/kg.

5.5

1. Yes. Wherever the force between bodies is gravitational, Kepler's laws will hold.

2. According to Kepler's third law

   $$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \quad \text{or} \quad T^2 \propto r^3 \Rightarrow T \propto r^{3/2}$$

   So, the satellite which is farther off has higher period.

   Let T_1 = 90 min, $\quad r_1$ = 1000 km + 6371 km

   r_2 = 2000 km + 6371 km

   [From the centre of the earth]

   $$\therefore \quad T_2^2 = \frac{T_1^2 . r_2^3}{r_1^3} = (90 \text{ min})^2 \left(\frac{8371\text{ km}}{7371\text{ km}}\right)^3$$

   T_2 = 108.9 min

3. According to Kepler's third law

$$\frac{T_{earth}^2}{T_{sedna}^2} = \frac{r_{earth}^3}{r_{sedna}^3} \qquad \text{[Distance from the Sun]}$$

$$T_{earth} = 1 \text{ yr.}, \ r_{earth} = 1 \text{ AU}$$

$$T_{sedna}^2 = \frac{(1\text{yr})^2 (86 \text{ AU})^3}{(1\text{AU})^3} (86)^3 \text{ yr}^2$$

$$\therefore \quad T_{sedna} = 797.5 \text{ yr}$$

4. If υ is the orbital velocity of the satellite of mass m at a distance r from the centre of the earth, then equating centripltal force with the gravitational force, we have

$$\frac{m\upsilon^2}{r} = \frac{GmM}{r^2} \Rightarrow \upsilon = \sqrt{\frac{GM}{r}}$$

where M is the mass of the earth.

5. From Eqs. (5.16) and (5.17),

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r} \Rightarrow T^2 = \frac{4\pi^2 . r^3}{GM.}$$

or $\quad T^2 \propto r^3$.

5.6

1. $$\upsilon_{esc} = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{2 \times 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} . \frac{5.97 \times 10^{24} \text{kg}}{6.371 \times 10^6 \text{ m}}}$$

$$= \sqrt{\frac{2 \times 6.67 \times 5.97 \times 10}{6.371}} \ 10^3 \text{ ms}^{-1}$$

$$= 11.2 \times 10^3 \text{ ms}^{-1} = 11.3 \text{ kms}^{-1}$$

2. $$\upsilon_{esc} \ \alpha \ \sqrt{\frac{1}{R}}$$

Notes

If R becomes 1/4th, υ_{esc} becomes double.

3. $\upsilon_{esc} \propto \sqrt{\dfrac{M}{R}}$

If M becomes eight times, and R twice,

then

$\upsilon_{esc} \propto \sqrt{4}$ or υ_{esc} becomes double.

5.7

1. $(R+h)\dfrac{4\pi^2}{T^2} = \dfrac{GM}{(R+h)^2}$

$$\Rightarrow (R+h)^3 = \frac{GM}{4\pi^2}T^2$$

$$= \frac{6.67\times10^{-11}\times6.4\times10^{23}\times(14.6\times3600)^2}{4\times(3.14)^2}$$

$$= 8370 \times 10^{18}\ \text{m}$$

$$R + h = 20300\ \text{km}$$

$$h = 26900\ \text{km}$$

2. (a) Images are clearer

(b) x-ray telescopy etc. also work.

Answers to Terminal Problems

3. 125 N

5. □ g, 5.5 ms^{-2}

7. Weight $= \dfrac{500}{6}$ N, mass 50 kg on moon as well as on earth

8. T □ $1\dfrac{1}{2}$ h, $\upsilon = 7.47$ km s^{-1}