312en06

6

WORK ENERGY AND POWER

You know that motion of objects arises due to application of force and is described by Newton's laws of motion. You also know how the velocity (speed and direction) of an object changes when a force acts on it. In this lesson, you will learn the concepts of work and energy. Modern society needs large amounts of energy to do many kinds of work. Primitive man used muscular energy to do work. Later, animal energy was harnessed to help people do various kinds of tasks. With the invention of various kinds of machines, the ability to do work increased greatly. Progress of our civilization now critrcally depends the on the availability of usable energy. Energy and work are, therefore, closely linked.

From the above discussion you will appreciate that the rate of doing work improved with newer modes, i.e. as we shifted from humans $\rightarrow$ animals $\rightarrow$ machines to provide necessary force. The rate of doing work is known as **power**.

OBJECTIVES

After studying this lesson, you should be able to:

- *define work done by a force and give unit of work;*
- *calculate the work done by an applied force;*
- *state work-energy theorem;*
- *define power of a system;*
- *calculate the work done by gravity when a mass moves from one point to another;*
- *explain the meaning of energy;*
- *obtain expressions for gravitational potential energy and elastic potential energy;*

- *apply the principle of conservation of energy for physical system; and*
- *apply the laws of conservation of momentum and energy in elastic collisions.*

6.1 WORK

The word 'work' has different meaning for different people. When you study, you do mental work. When a worker carries bricks and cement to higher floors of a building, he is doing physical work against the force of gravity. But in science, work has a definite meaning. The technical meaning of work is not always the same as the common meaning. The work is defined in the following way :

Let us suppose that a **constant force F** acting on an object results in displacement **d** i.e. moves it by a distance d along a straight line on a horizontal surface, as shown in Fig. 6.1. *The work done by a force is the product of the magnitude of force component in the direction of displacement and the displacement of this object.*

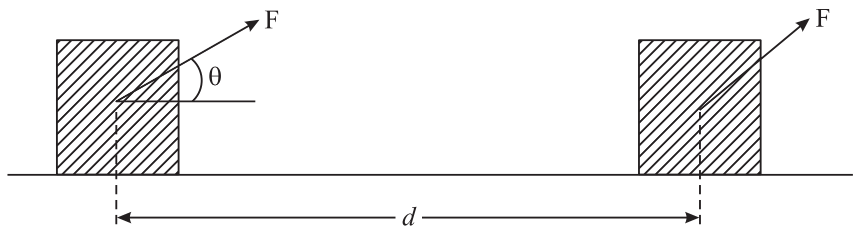


Fig 6.1 : A force F on a block moves it by a horizontal distance d. The direction of force makes an angle θ with the horizontal direction.

If force **F** is acting at angle θ with respect to the displacement **d** of the object, its component along **d** will be $F \cos \theta$. Then work done by force **F** is given by

$$W = F \cos\theta . d \tag{6.1}$$

In vector form, the work done is given by:

$$W = F . \mathbf{d} \tag{6.2}$$

Note that if $d = 0$, $W = 0$. That is, no work is done by a force, whatever its magnitude, if there is no displacement of the object. Also note that though both force and displacement are vectors, work is a scalar.

ACTIVITY 6.1

You and your friends may try to push the wall of a room. Irrespective of the applied force, the wall will not move. Thus we say that no work is done.

The unit of work is defined using Eqn.(6.2). If the applied force is in newton and displacement is in metre, then the unit of work is joule.

(Unit of Force)×(Unit of displacement) = newton . metre = Nm (6.3)

This unit is given a special name, **joule,** and is denoted by J.

One joule is defined, as the work done by a force of one newton when it produces a displacement of one metre. Joule is the SI unit of work.

Example 6.1 : Find the dimensional formula of work.

Solution :

$$W = \text{Force} \times \text{Distance}$$

$$= \text{Mass} \times \text{Acceleration} \times \text{distance}$$

$$\text{Dimension of work} = [M] \times [LT^{-2}] \times [L]$$

$$= [ML^2T^{-2}]$$

In electrical measurements, kilowatt-hour (kW h) is used as unit of work. It is related to joule as

$$1\text{kW h} = 3.6 \times 10^6 \text{ J}$$

You will study the details of this unit later in this lesson.

Example 6.2 : A force of 6 N is applied on an object at an angle of 60° with the horizontal. Calculate the work done in moving the object by 2m in the horizontal direction.

Solution : From Eqn. (6.2) we know that

$$W = Fd\cos\theta$$

$$= 6\times2 \times \cos 60^\circ$$

$$= 6\times2 \times (1/2)$$

$$= 6 \text{ J}$$

Example 6.3 : A person lifts 5 kg potatoes from the ground floor to a height of 4m to bring it to first floor. Calculate the work done.

Solution : Since the potatoes are lifted, work is being done against gravity. Therefore, we can write

$$\text{Force} = mg$$

$$= 5 \text{ kg} \times 9.8 \text{ m s}^{-2}$$

$$= 49 \text{ N}$$

$$\text{Work done} = 49 \times 4 \text{ (N m)}$$

$$= 196 \text{ J}$$

6.1.1 Positive and Negative Work

As you have seen, work done is defined by Eqn.(6.2), where the angle θ between the force and the displacement is also important. In fact, it leads us to the situation in which work becomes a positive or a negative quantity. Consider the examples given below:

Fig. 6.2 (a) shows a car moving in + x direction and a force F is applied in the same direction. The speed of the car keeps increasing. The force and the displacement both are in the same direction, i.e. $\theta = 0°$. Therefore, the work done is given by

$$W = Fd \cos 0°$$

$$= Fd \qquad (6.4)$$

The work is this case is positive.

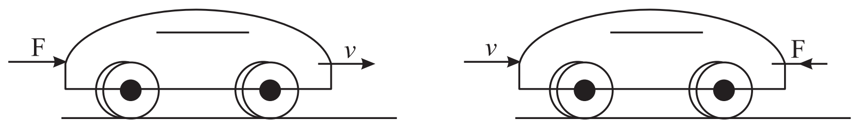


Fig. 6.2 : A car is moving on a horizontal road. a) A force F is applied in the direction of the moving car. It gets accelerated. b) A force F is applied in opposite direction so that the car comes to rest after some distance.

Figure 6.2 (b) shows the same car moving in the +x direction, but the force **F** is applied in the opposite direction to stop the car. Here, angle $\theta = 180°$. Therefore,

$$W = Fd \cos 180°.$$

$$= -Fd \qquad (6.5)$$

Hence, the work done by the force is negative. In fact, the work done by a force shall be negative for θ lying between 90° and 270°.

From the above examples, we can conclude that

a) When we press the accelerator of the car, the force is in the direction of motion of the car. As a result, we increase the speed of the car. The work done is positive.

b) When we apply brakes of a car, the force is applied in a direction opposite to its motion. The car loses speed and may finally come to rest. Negative work is said to have been done.

c) In case the applied force and displacement are as right angles, i.e. $\theta = 90°$, no work is said to be done.

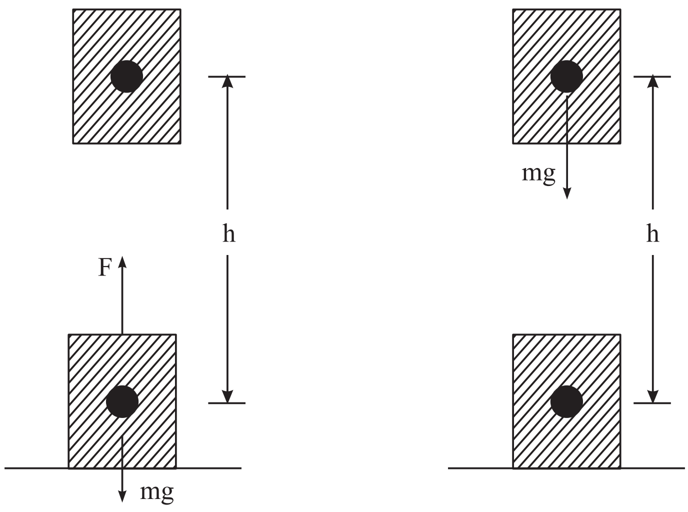
6.1.2 Work Done by the Force of Gravity

Fig.6.3(a) shows a mass m being lifted to a height h and Fig. 6.3(b) shows the same mass being lowered by a distance h. The weight of the object is mg in both cases. You may recall from the previous lesson that weight is a force.

In Fig. 6.3 (a), the work is done against the force mg (downwards) and the displacement is upward ($\theta = 180°$). Therefore,

$$W = Fd \cos 180°$$

$$= - mgh$$



(a) (b)

Fig 6.3 : (a) The object is lifted up against the force of gravity, (b) The object is lowered towards the earth.

In the Fig. 6.3(b), the mass is being lowered. The force mg and the displacement d are in the same direction ($\theta = 0°$). Therefore, the work done

$$W = Fd \cos 0°$$

$$= + mgh \qquad (6.6)$$

You must be very careful in interpreting the results obtained above. When the object is lifted up, the work done by the gravitational force is **negative** but the work done by the person lifting the object is **positive.** When the object is being lowered, the work done by the gravitational force is **positive** but the workdone by the person lowering the object is **negative**. In both of these cases, it is assumed that the object is being moved without acceleration.

INTEXT QUESTIONS 6.1

1. When a particle rotates in a circle, a force acts on the particle. Calculate the work done by this force on the particle.
2. Give one example of each of the following. Work done by a force is
   (a) zero
   (b) negative
   (c) positive
3. A bag of grains of mass 2 kg. is lifted through a height of 5m.
   (a) How much work is done by the lift force?
   (b) How much work is done by the force of gravity?
4. A force $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ N produces a displacements $d = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ m. Calculate the work done.
5. A force $\mathbf{F} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ acts on a particle to give a displacement $\mathbf{d} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$m
   (a) Calculate the magnitude of displacement
   (b) Calculate the magnitude of force.
   (c) How much work is done by the force?

6.2 WORK DONE BY A VARIABLE FORCE

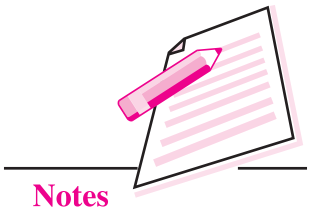
You have so far studied the cases where the force acting on the object is constant. This may not always be true. In some cases, the force responsible for doing work may keep varying with time. Let us now consider a case in which the magnitude of force F(x) changes with the position x of the object. Let us now calculate the work done by a variable force. Let us assume that the displacement is from x_i to x_f, where x_i and x_f are the initial and final positions. In such a situation, work is calculated over a large number of small intervals of displacements Δx. In fact, Δx is taken so small that the force F(x) can be assumed to be constant over each such interval. The work done during a small displacements Δx is given by

$$\Delta W = F(x)\, \Delta x \qquad (6.7)$$

$F(x)\, \Delta x$ is numerically equal to the small area shown shaded in the Fig. 6.4(a). The total work done by the force between x_i and x_f is the sum of all such areas (area of all strips added together):

$$W = \Sigma \Delta W$$

$$= \Sigma F(x)\, \Delta x \qquad (6.8)$$



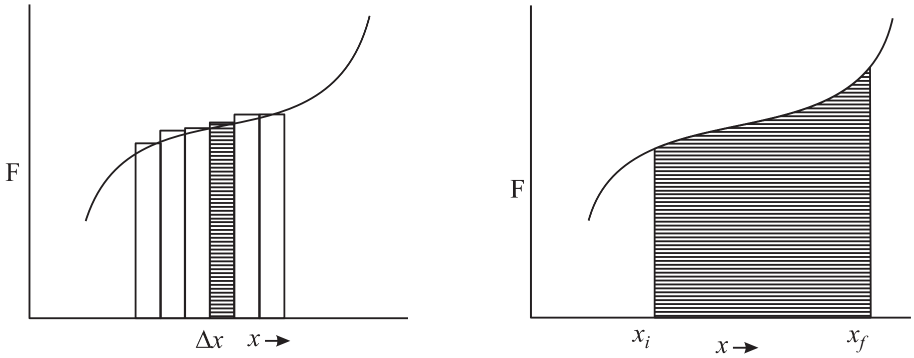


Fig 6.4 : A varying force F moves the object from the initial position x_i to final position x_f. The variation of force with distance is shown by the solid curve (arbitrary) and work done is numerically equal to the shaded area.

The width of the strips can be made as small as possible so that the areas of all strips added together are equal to the total area enclosed between x_i and x_f. It will give the total work done by the force between x_i and x_f :

$$W = \sum_{\lim \Delta x \to 0} F(x)\,\Delta x \tag{6.9}$$

6.2.1 Work done by a Spring

A very simple example of a variable force is the force exerted by a spring. Let us derive the expression for work done in this case.

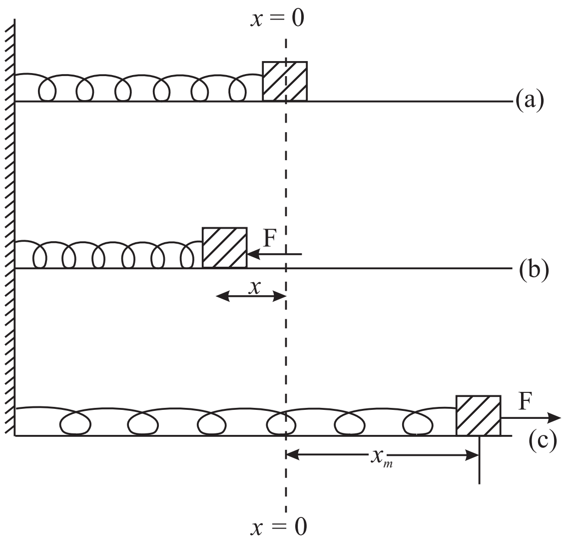


Fig. 6.5 : A spring-mass system whose one end is rigidly fixed and mass m, rests on a smooth horizontal surface. (a) The relaxed position of the spring's, free end at x = 0; (b) The spring is compressed by applying external force F and (c) Pulled or elongated by an external force F. The maximum compression/ elongation is x_m.

Fig. 6.5(a) shows the equilibrium position of a light spring whose one end is attached to a rigid wall and the other end is attached to a block of mass m. The system is placed on a smooth horizontal table. We take x-axis along the horizontal direction. Let mass m be at position $x = 0$. The spring is now compressed (or elongated) by an external force **F.** An internal force $\mathbf{F}_s$ is called into play in the spring due to its elastic property. This force $\mathbf{F}_s$ keeps increasing with increasing x and becomes equal to **F** when the compression (or elongation) is maximum at $x = x_m$.

According to Hooke's law (true for small x only), $|\mathbf{F}_s| = kx$, where k is known as spring constant. Since the direction of $\mathbf{F}_s$ is always opposite to compression (or extension), it is written as :

$$\mathbf{F} = \mathbf{F}_s = -k\mathbf{x} \tag{6.10}$$

Let us now calculate the work done and also examine, if it is positive or negative. In the event of compression of the spring, the external force **F** is directed towards left and the displacement **x** is also towards left. Hence, the work done by the **external force** is positive. However, for the same direction of displacement, the restoring force generated in the spring is towards right, i.e. **F** and **x** are oppositely directed. The work done by the **spring force** is negative. You can yourself examine the case of extension of the spring and arrive at the same result: "***the work done by the external force is positive but the work done by the spring force is negative and its magnitude is*** (½) kx_m^2"

A simple calculation can be done to derive an expression for the work done. At $x = 0$, the force $\mathbf{F}_s = 0$. As x increases, the force $\mathbf{F}_s$ increases and becomes equal to **F** when $x = x_m$. Since the variation of the force is linear with displacement, the average force during compression (or extension) can be approximated to $\left(\dfrac{0 + \mathbf{F}_s}{2}\right)$ $= \dfrac{\mathbf{F}_s}{2}$. The work done by the force is given by

$$W = \text{force . displacement}$$

$$= \frac{\mathbf{F}_s}{2} \cdot \mathbf{x},$$

But $|F_s| = k\,|\,x_m|$. Hence

$$W = \frac{1}{2} k\, x_m \times x_m$$

$$= \frac{1}{2} k\, x^2_m \tag{6.11}$$

The work done can also be obtained graphically. It is shown in Fig. 6.6.

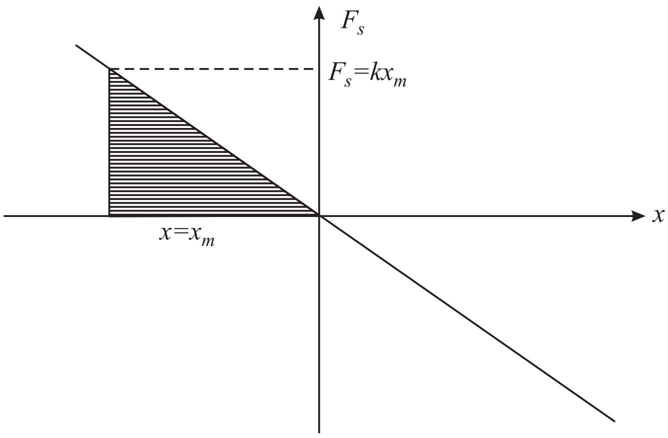


Fig. 6.6: The work done is numerically equal to the area of the shaded triangle.

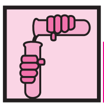
The area of the shaded triangle is:

$$= \frac{1}{2} \text{ base} \times \text{height}$$

$$W = \frac{1}{2} x_m \times kx_m$$

$$= \frac{1}{2} kx_m^{\ 2} \qquad (6.12)$$

This is the same as that obtained analytically in Eqn. (6.11)



ACTIVITY 6.2

Measuring spring constant

Suspend the spring vertically, as shown in Fig. 6.7 (a). Now attach a block of mass *m* to the lower end of the spring. On doing so, the spring extends by some distance. Measure the extension. Suppose it is *s*, as shown in Fig 6.7 (b). Now think why does the spring not extend further. This is because the spring force (restoring force) acting upwards balances the weight *mg* of the block in equilibrium state. You can calculate the spring constant by putting the values in

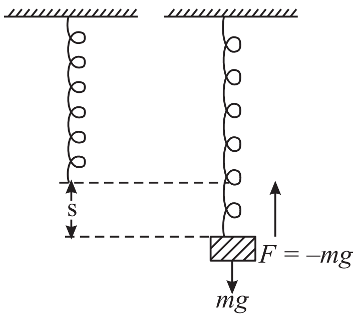


Fig. 6.7 : Extension in a spring under a load.

$$F_s = k.s$$

or $$mg = k.s$$

Thus, $$k = \frac{mg}{s} \qquad (6.13)$$

Example 6.4: A mass of 2 kg is attached to a light spring of force constant $k = 100$ Nm^{-1}. Calculate the work done by an external force in stretching the spring by 10 cm.

Solution:

$$W = \frac{1}{2}kx^2$$

$$= \frac{1}{2} \times 100 \times (0.1)^2$$

$$= 50 \times 0.01 = 0.5 \text{ J}$$

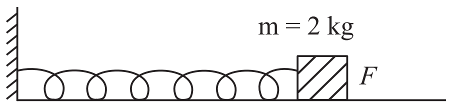


Fig. 6.8: A mass m = 2 kg is attached to a spring on a horizontal surface.

As explained earlier, the work doen by the restioning force in the spring = – 0.5 J.

INTEXT QUESTIONS 6.2

1. Define spring constant. Give its SI unit.
2. A force of 10 N extends a spring by 1cm.How much force is needed to extend this spring by 5 cm? How much work will be done by this force?

6.3 POWER

You have already learnt to calculate the work done by a force. In such calculations, we did not consider whether the work is done in one second or in one hour. In our daily life, however, the time taken to perform a particular work is important. For example, a man may take several hours to load a truck with cement bags, whereas a machine may do this work in much less time. Therefore, it is important to know the rate at which work is done. **The rate at which work is done is called power.**

If ΔW work is done in time Δt, the average power is defined as

$$\text{Average Power} = \frac{\text{Work done}}{\text{time taken}}$$

Mathematically, we can write

$$P = \frac{\Delta W}{\Delta t} \tag{6.14}$$

If the rate of doing work is not constant, this rate may vary. In such cases, we may define instantaneous power P

$$P = \underset{\Delta t \to 0}{\text{limit}} \left(\frac{\Delta W}{\Delta t}\right) = \frac{dW}{dt} \tag{6.15}$$

The definition of power helps us to determine the SI unit of power:

$$P = \frac{\Delta W}{\Delta t}$$

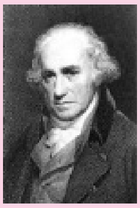
$$= \text{joule/ second} = \text{watt}$$

Thus, the SI unit of power is watt. It is abbreviated asW.

The power of an agent doing work is 1W, if one joule of work is done by it in one second. The more common units of power are kilowatt (kW) and megawatt (MW).

$$1 \text{ kW} = 10^3 \text{ W}, \quad \text{and} \quad 1 \text{ MW} = 10^6 \text{W}$$

James Watt
(1736–1819)



Scottish inventor and mechanical engineer, James Watt is renowned for improving the efficiency of a steam engine. This paved the way for industrial revolution.

He, introduced horse power as the unit of power. SI unit of power watt is named in his honour. Some of the important inventions by James Watt are : a steam locomotive and an attachment that adapted telescope to measure distances.

Example 6.5 : Determine the dimensions of power.

Solution : Since $P = \frac{\text{Work}}{\text{Time}}$

$$= \text{Force} \times \frac{\text{Distance}}{\text{Time}}$$

$$\therefore \text{Dimension of } P = [\text{Mass}] \times [\text{Acceleration}] \times \frac{[\text{Distance}]}{[\text{Time}]}$$

$$= [\text{M}] \times \left[\frac{\text{L}}{\text{T}^2}\right] \times \left[\frac{\text{L}}{\text{T}}\right]$$

$$= [\text{ML}^2\text{T}^{-3}]$$

You may have heard electricians discussing the power of a machine in terms of the horse power, abbreviated as hp. This unit of power was under British system. It is a larger unit:

$$1\text{hp} = 746 \text{ W} \tag{6.16}$$

The unit of power is used to define a new unit of work (energy). One such unit of work is **kilowatt hour.** This unit is commonly used in electrical measurement.

$$\begin{aligned} \text{kilowatt. hour (kWh)} &= 1 \text{ kW} \times 1 \text{ hour} \\ &= 10^3 \text{ W} \times 3600 \text{ s} \\ &= \frac{10^3 \text{ J}}{1\text{s}} \times 3600 \text{ s} \\ &= 36{,}00{,}000 \text{ J} = 3.6 \times 10^6 \text{ J} \end{aligned}$$

Or $$1 \text{ kW h} = 3.6 \text{ MJ (mega joules)} \tag{6.17}$$

The electrical energy that is consumed in homes is measured in kilowatt-hour. In common man's language : 1kW h = 1 Unit of electrical energy consumption.

INTEXT QUESTIONS 6.3

1. A body of mass 100 kg is lifted through a distance of 8 m in 10 s. Calculate the power of the lifter.
2. Convert 10 horse power into kilowatt.

6.4 WORK AND KINETIC ENERGY

As you know, the capacity to do work is called energy. If a system (object) has energy, it has ability to do work. An automobile moving on a road uses chemical energy of fuel (CNG, petrol, diesel). It can push an object which comes on its way to some distance. Thus it can do work. All moving objects possess energy because they can do work before they come to rest. We call this kind of energy as **kinetic energy**. Kinetic energy is the energy of an object because of its motion.

Let us consider an object of mass m moving along a straight line when a constant force of magnitude F acts on it along the direction of motion. This force produces a uniform acceleration a such that $F = ma$. Let υ_1 be the speed of the object at time t_1. This speed becomes υ_2 at another instant of time t_2. During this interval of time $t = (t_2 - t_1)$, the object covers a distance, s. Using Equations of Motion, we can write

$$\upsilon_2^2 = \upsilon_1^2 + 2as$$

or $$a = \frac{\upsilon_2^2 - \upsilon_1^2}{2s} \qquad (6.18)$$

Combining this result with Newton's second law of motion, we can write

$$F = m \times \frac{\upsilon_2^2 - \upsilon_1^2}{2s}$$

We know that work done by the force is given by

$$W = Fs$$

Hence, $$W = m \times \frac{\upsilon_2^2 - \upsilon_1^2}{2s} s$$

$$= \frac{1}{2}m\upsilon_2^2 - \frac{1}{2}m\upsilon_1^2$$

$$= K_2 - K_1 \qquad (6.19)$$

where $K_2 = \frac{1}{2}m\upsilon_2^2$ and $K_1 = \frac{1}{2}m\upsilon_1^2$ respectively denote the final and initial kinetic energies.

$(K_2 - K_1)$ denotes the change in kinetic energy, which is equal to the work done by the force.

Kinetic Energy is a scalar quantity. It depends on the product of mass and the square of the speed. It does not matter which one of the two (m and υ) is small and which one is large. It is the total value $\frac{1}{2}m\upsilon^2$ that determines the kinetic energy.

Work-Energy Theorem

The work-energy theorem states that the work done by the resultant of all forces acting on a body is equal to the change in kinetic energy of the body.

Example 6.6 : A body of mass 10 kg is initially moving with a speed of 4.0 m s^{-1}. A force of 30 N is now applied on the body for 2 seconds.

(i) What is the final speed of the body after 2 seconds?

(ii) How much work has been done during this period?

(iii) What is the initial kinetic energy?

(iv) What is the final kinetic energy?

(v) What is the distance covered during this period?

(vi) Show that the work done is equal to the change in kinetic energy?

Solution :

(i) Force (F) $= ma$

or $a = F/m$

$= 30/10$

$= 3 \text{ m s}^{-2}$

The final speed $\upsilon_2 = \upsilon_1 + at$

$= 4 + (3 \times 2) = 10 \text{ m s}^{-1}$

(ii) The distance covered in 2 seconds:

$$s = ut + \frac{1}{2}at^2$$

$$= (4\times2) + \frac{1}{2}(3\times4)$$

$$= 8 + 6 = 14 \text{ m}$$

Work done $W = F \times S$

$= 30 \times 14 = 420 \text{ J}$

(iii) The initial Kinetic Energy

$$K_1 = \frac{1}{2}m\upsilon_1^2$$

$$= \frac{1}{2}(10 \times 16) = 80 \text{ J}$$

(iv) The final kinetic energy

$$K_2 = \frac{1}{2}m\upsilon_2^2$$

$$= \frac{1}{2}(10 \times 100) = 500 \text{ J}$$

(v) The distance covered as calculated above = 14m

(vi) The change in kinetic energy is:

$$K_2 - K_1 = (500 - 80) = 420 \text{ J}$$

As may be seen, this is same as wok done.

INTEXT QUESTION 6.4

1. Is it possible for a particle to have a negative value of kinetic energy? Why?
2. What happens to the kinetic energy of a particle if
   (a) The speed v of the particle is made $2v$.
   (b) The mass m of the particle is made $m/2$?
3. A particle moving with a kinetic energy 3.6 J collides with a spring of force constant 180 N m^{-1}. Calculate the maximum compression of the spring.
4. A car of mass 1000 kg is moving at a speed of 90 km h^{-1}. Brakes are applied and the car stops at a distance of 15 m from the braking point. What is the average force applied by brakes? If the car stops in 25 s after braking , calculate the average power of the brakes?
5. If an external force does 375 J of work in compressing a spring, how much work is done by the spring itself?

6.5 POTENTIAL ENERGY

In the previous section we have discussed that a moving object has kinetic energy associated with it. Objects possess another kind of energy due to their position in space. This energy is known as **Potential Energy.** Familiar example is the **Gravitational Potential Energy possessed by a body in Gravitational Field**. Let us understand it now.

6.5.1 Potential Energy in Gravitational Field

Suppose that a person lifts a mass m from a given height h_1 to a height h_2 above the earth's surface. Let us also assume that the value of acceleration due to gravity remains constant. The mass has been displaced by a distance $h = (h_2 - h_1)$ against the force of gravity. The magnifude of this force is mg and it acts downwards. Therefore, the work done by the person is

$$W = \text{force} \times \text{distance}$$

$$= mgh \qquad (6.20)$$

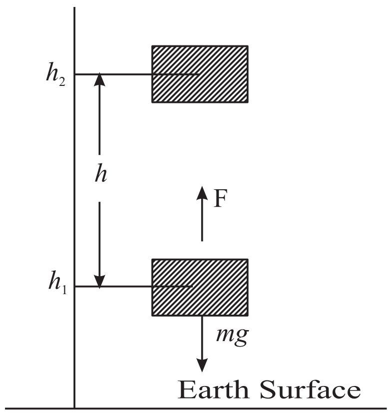


Fig. 6.9 : Object of mass m originally at height h_1 above the earth's surface is moved to a height h_2.

The work is positive and is stored in mass m as energy. This energy by virtue of the position in space is called **gravitational potential energy**. It has capacity to do work. If this mass is left free, it will fall down and during the fall it can be made to do work. For example, it can lift another mass if properly connected by a string, which is passing over a pulley.

The selection of the initial height h_1 is arbitrary. The important concept is the change in height, i.e. $(h_2 - h_1)$. We, therefore, say that the point of **zero potential energy** is arbitrary. Any point in space can be chosen as a point of zero potential energy. Normally, a point on the surface of the earth is assumed to be the reference point with zero potential energy.

Example 6.7 : A truck is loaded with sugar bags. The total mass of the load and the truck together is 100,000 kg. The truck moves on a winding path up a mountain to a height of 700 m in 1 hour. What average power must the engine produce to lift the material?

Solution :

$$W = mgh$$

$$= (100,000 \text{ kg}) \times (9.8 \text{ m s}^{-2} \times 700 \text{ m})$$

$$= 9.8 \times 7 \times 10^7 \text{ J}$$

$$= 68.6 \times 10^7 \text{J}$$

$$\text{Time taken} = 1 \text{ hour} = 60 \times 60 \text{ s}$$

$$= 3600 \text{ s}$$

$$\text{Average Power, } P = W / t$$

$$= \frac{68.6 \times 10^7 \text{ J}}{3600 \text{ s}}$$

$$= 1.91 \times 10^5 \text{ W}$$

We know that 746 W = 1 hp

$$\therefore \qquad P = \frac{1.91 \times 10^5}{746} = 2.56 \times 10^2 = 256 \text{ hp.}$$

Example 6.8 : Hydroelectric power generation uses falling water as a source of energy to turn turbine blades and generate electrical power. In a power station, 1000×10^3 kg water falls through a height of 51 m in one second.

(i) Calculate the work done by the falling water?

(ii) How much power can be generated under ideal conditions?

Solution :

(i) The potential energy of the water at the top = mgh

$$\text{P.E.} = (1000 \times 10^3 \text{ kg}) \times (9.8 \text{ m s}^{-2}) \times (51 \text{ m}).$$

$$= 9.8 \times 51 \times 10^6 \text{ J}$$

$$= 500 \times 10^6 \text{ J}$$

Water loses all its potential energy. The same is converted into work in moving the turbine blades. Therefore

$$W = \text{Force} \times \text{distance}$$

$$= mg \times h$$

$$= 1000 \times 10^3 \times 9.8 \times 51 \text{ J}$$

$$= 500 \times 10^6 \text{ J}$$

$$= 500 \text{ M J}$$

(ii) The work done per second is given by

$$P = W/t$$

$$= \frac{500 \text{ M J}}{1 \text{ s}}$$

$$= 500 \text{ MW}$$

Ideal conditions mean that there is no loss of energy due to frictional forces. In practice, there is the always some loss in machines. Such losses can be minimized but can never be eliminated.

6.5.2 Potential energy of springs

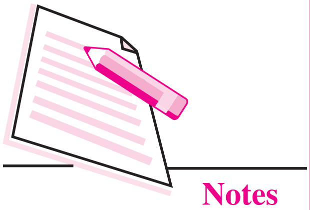
You now know that an external force is required to compress or stretch a given spring. These situations are shown in Fig. 6.5. Let there be a spring of force constant k. This spring is compressed by a distance x. From Eqn.(6.11) we recall that work done by the external force to compress the spring is given by

$$W = \frac{1}{2} kx^2$$

This work is stored in the spring as elastic potential energy. When the spring is left free, it bounces back and the elastic potential energy of the spring is converted into kinetic energy of the mass m.

6.5.3 Conservation of Energy

We see around us various forms of energy but we are familiar with some forms more than others. Examples are Electrical Energy, Thermal Energy, Gravitational Energy, Chemical Energy and Nuclear Energy etc. These forms of energy are very closely related in the sense that one can be changed to another. There is a very fundamental law about energy. It is known as **Law of Conservation of Energy**. It states, " **The total energy of an isolated system always remains constant.**" The energy may change its form. It can be converted from one form to other. But the total energy of the system remains unchanged. In an isolated



system, if there is any loss of energy of one form, there is a gain of an equal amount of another form of energy. Thus energy is neither created nor destroyed. The universe is also an isolated system as there is nothing beyond this. It is therefore said that the total energy of the universe always remains constant in spite of the fact that variety of changes are taking place in the universe every moment. It is a law of great importance. It has led to many new discoveries in science and it has not been found to fail.

In a Thermal Power Station, the chemical energy of coal is changed into electrical energy. The electrical energy runs machines. In these machines, the electrical energy changes into mechanical energy, light energy or thermal energy.

The law of conservation of energy is more general than we can think of. It applies to systems ranging from big planets and stars to the smallest nuclear particles.

(a) Conservation of mechanical energy during the free fall of a body

We now wish to test the validity of the law of conservation of energy in case of mechanical energy, which is of immediate interest.

Let us suppose that an object of mass m lying on the ground is lifted to a height h. The work done is mgh, which is stored in the object as potential energy. This object is now allowed to fall freely. Let us calculate the energy of this object when it has fallen through a distance h_1. The height of the object now above the earth surface is $h_2 = h - h_1$ (Fig 6. 10). At this point P, the potential energy = mgh_2.

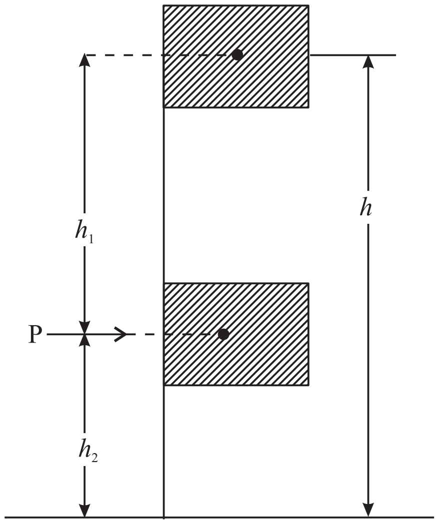


Fig. 6.10 : Mass m is lifted to a height h from earth's surface. It is then lowered to a height h_2 at point P. The total energy at P is same as that at the highest point.

When the object falls freely, it gets accelerated and gains in speed. We can calculate the speed of the object when it has fallen through a height h_1 from the top positions using the equation

$$v^2 = u^2 + 2gs \tag{6.21}$$

where u is the initial speed at the height h_1, i.e. $u = 0$ and $s = h_1$. Then, we have

$$v^2 = 2gh_1$$

The kinetic energy at point P is given by

$$\text{K.E} = \frac{1}{2}mv^2$$

$$= \frac{m}{2} \times 2gh_1$$

$$= mgh_1 \tag{6.22}$$

The total energy at the point P is

Kinetic Energy + Potential Energy = $mgh_1 + mgh_2$

$$= mgh \tag{6.23}$$

This is same as the potential energy at the highest point. **Thus, the total Energy is conserved.**

(b) Conservation of Mechanical Energy for a Mass Oscillating on a Spring

Fig. 6.11 shows a spring whose one end is fixed to a rigid wall and the other end is connected to a wooden block lying on a smooth horizontal table. This free end is at x_0 in the relaxed position of the spring. A block of mass m moving with speed v along the line of the spring collides with the spring at the free end, and compresses it by x_m. This is the maximum compression. At x_0, the total energy of the spring-mass system is $\frac{1}{2}mv^2$. It is the kinetic energy of the mass. The potential energy of the spring is zero. At the point of extreme compression, the potential energy of the spring is $\frac{1}{2}k\,x_m^2$ and the kinetic energy of the mass is zero. The total energy now is $\frac{1}{2}k\,x_m^2$. Obviously, this means that

$$\frac{1}{2}k\,x_m^2 = \frac{1}{2}mv^2 \tag{6.24}$$

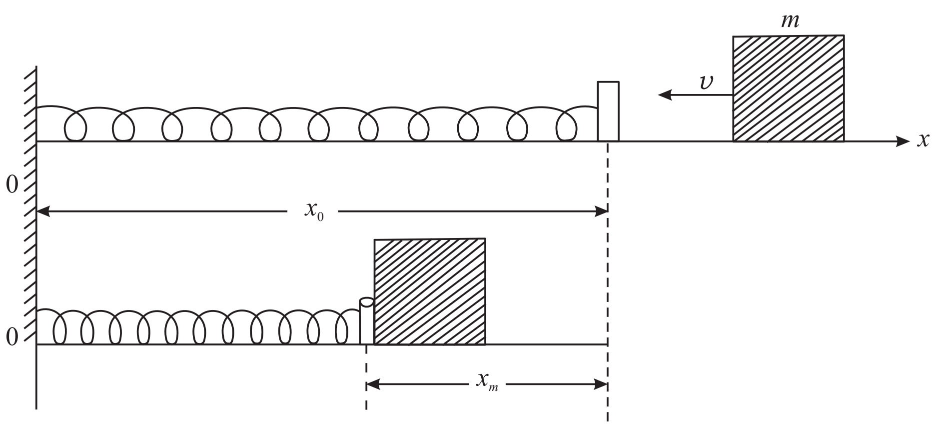


Fig. 6.11 : A block of mass m moving with velocity v on a horizontal surface collides with the spring. The maximum compression is x_m.

K.E + P.E (Before collision) = K.E. + P.E. (After collision)

Notes

$$\frac{1}{2}mv^2 + 0 = 0 + \frac{1}{2}kx_m^2 \qquad (6.25)$$

i.e., *the total energy is conserved.*

Conservation of mass-energy in nuclear reactions

Nuclear energy is different from other forms of energy in the sense that it is not obtained by the transformation of some other form of energy. On the contrary, it is obtained by transformation of mass into energy.

Hence, in nuclear reacions, the law of conservation of mass and the law of conservation of energy merge into a single law of conservation of mass-energy.

Example 6.9 : A block of mass 0.5 kg slides down a smooth curved surface and falls through a vertical height of 2.5m to reach a horizontal surface at B (Fig 6.12). On the basis of energy conservtion, calculate, i) the energy of the block at point A, and ii) the speed of the block at point B.

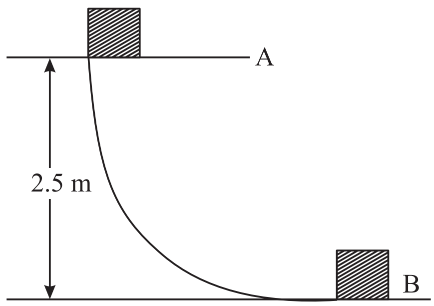


Fig. 6.12 : A block slides on a curved surface. The total energy at A (Potential only) gets converted into total energy at B (kinetic only).

Solution :

i) Potential energy at

$A = mgh = (0.5) \times (9.8) \times 2.5 \text{ J}$

$= 4.9 \times 2.5 \text{ J}$

$= 12.25 \text{ J}$

The kinetic energy at A = 0 and

Total Energy = 12.25 J

ii) The total energy of the block at A must be the same as the total energy at B.

The total energy (P.E. + K.E.) at A = 12.25 J

The total energy (P.E. + K.E.) at B = $\frac{1}{2}mv^2$

Since P.E. at B is zero, the total energy is only K.E.

$\therefore \qquad \frac{1}{2}mv^2 = 12.25$

$$v^2 = \frac{12.25 \times 2}{0.5}$$

$$= 12.25 \times 4$$

$$v^2 = 49.00$$

Hence $\quad v = 7.0 \text{ ms}^{-1}$

Note: This can also be obtained from the equations of motion:

$$v^2 = v_0^2 + 2gx$$

$$= 0 + 2 \times 9.8 \times 2.5$$

$$v^2 = 49$$

$$v = 7 \text{ ms}^{-1}$$

6.5.4 Conservative and dissipative (Non conservative) Forces

(a) Conservative forces

We have seen that the work done by the gravitational force acting on an object depends on the product of the weight of the object and its vertical displacement. If an object is moved from a point A to a point B under gravity, (Fig 6.13), the work done by gravity depends on the vertical separation between the two points. It does not depend on the path followed to reach B starting from A. When a force obeys this rule, it is called **a conservative force**. Some of the examples of conservative forces are gravitational force, elastic force and electrostatic force.

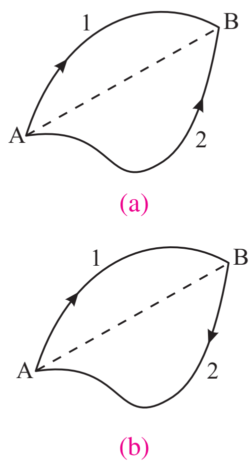


Fig. 6.13 : a) The object is moved from A to B along two different paths. b) It is taken from A to B along path 1 and brought back to A along path 2.

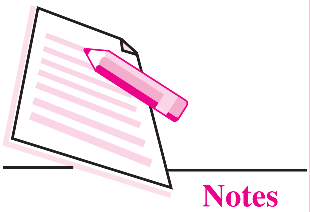
A conservative force has a property that *the work done by a conservative force is independent of path. In* Fig 6.13 (*a*)

$$W_{AB} \text{ (along 1)} = W_{AB} \text{ (along 2)}$$

Fig. 6.13 (b) shows the same two positions of the object. The object moves from A to B along the path 1 and returns back to A along the path 2. By definition, the work done by a conservative force along path 1 is equal and opposite to the work done along the path 2.

$$W_{AB} \text{ (along 1)} = -W_{BA} \text{ (along 2)}$$

or $$W_{AB} + W_{BA} = 0 \qquad (6.27)$$



Notes

This result brings out an important property of the conservative force in ***that the work done by a conservative force on an object is zero when the object moves around a closed path and returns back to its starting point.***

(b) Non-conservative Forces

The force of friction is a good example of a non-conservative force. Fig. 6.14 shows a rough horizontal surface. A block of mass m is moving on this surface with a speed υ at the point A.

After moving a certain distance along a straight line, the block stops at the point B. The block had a kinetic energy $E = \frac{1}{2}m\upsilon^2$ at the point A. It has neither kinetic energy nor potential energy at the point B. It has lost all its energy. Do you know where did the energy go? It has changed its form. Work has been done against the frictional force or we can say that force of friction has done negative work on the block. The kinetic energy has changed to thermal energy of the system. The block with the same kinetic energy E is now taken from A to B through a longer path 2. It may not even reach the point B. It may stop much before reaching B. This obviously means that more work has to be done along this path. Thus, it canbe said that the work done depends on the path.

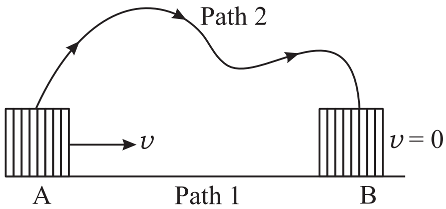


Fig. 6.14: A block which is given an initial speed v on a rough horizontal surface, moves along a straight line path 1 and comes to rest at B. It starts with the same speed υ at A but now moves along a different path 2.

INTEXT QUESTIONS 6.5

1. ABC is a triangle where AB is horizontal and BC is vertical. The length of the sides AB = 3m, BC = 4m and AC=5m. A block of mass 2 kg is at A. What is the change in potential energy of the block when

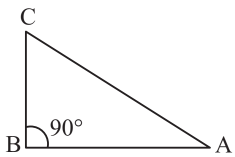


Fig. 6.15

   a) it is taken from A to B
   b) from B to C
   c) from C to A
   d) How much work is done by gravitational force in moving the mass form B to C (positive or Negative work)?

2. A ball of mass 0.5 kg is at A at a height of 10m above the ground. Solve the following questions by applying work-energy principle. In free fall
   a) What is the speed of the ball at B?
   b) What is the speed of the ball at the point C?
   c) How much work is done by gravitational force in bringing the ball from A to C (give proper sign)?

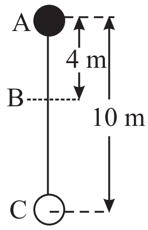


Fig. 6.16

3. A block at the top of an inclined plane slides down. The length of the plane BC = 2m and it makes an angle of 30° with horizontal. The mass of the block is 2 kg. The kinetic energy of the block at the point B is 15.6 J. How much of the potential energy is lost due to non-conservative forces (friction). How much is the magnitude of the frictional force?

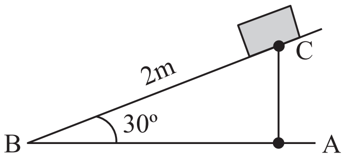


Fig. 6.17

4. The Figure shows two curves A and B between energy E and displacement x of the bob of a simple pendulum. Which one represents the P.E. of the bob and why?

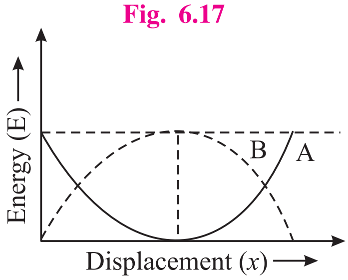


Fig. 6.18

5. When non- conservative forces work on a system, does the total mechanical energy remain constant?

6.6 ELASTIC AND INELASTIC COLLISIONS

Let us consider a system of two bodies. The system is a closed system which implies that no external force acts on it. The system may consist of two balls or two springs or one ball and one spring and so on. When two bodies interact, it is termed as ***collision***. There are no external forces acting on the system.

Let us start with a collision of two balls and to make the analysis simpler, let there be a "**head-on**" or "**central collision**". In such collisions, colliding bodies move along the line joining their centres. The collisions are of two kinds :

(i) Perfectly Elastic Collision: If the forces of interaction between the two bodies are **conservative,** the total kinetic energy is conserved i.e. the total kinetic energy before collision is same as that after the collision. Such collisions are termed as **completely elastic collisions**.

(ii) Perfectly Inelastic collision: When two colliding bodies stick together after the collision and move as one single unit, it is termed as **perfectly inelastic collision.** It is like motion of a bullet embedded in a target.

You should remember that the momentum is conserved in all types of collisions. Why? But kinetic energy is conserved in elastic collisions only.

6.6.1 Elastic Collision (Head-on)

Let two balls A and B having masses m_A and m_B respectively collide "head-on", as shown in Fig. (6.19). Let υ_{Ai} and υ_{Bi} be the velocities of the two balls before collision and υ_{Af} and υ_{Bf} be their velocities after the collision .

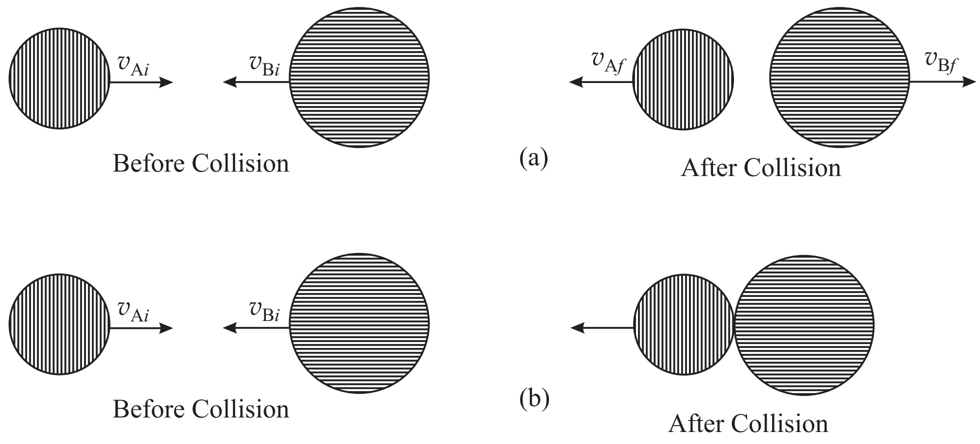


Fig. 6.19 : Schematic representation of Head-on collision (a) Elastic collision; (b) In elastic collision

Now applying the laws of conservation of momentum and kinetic energy, we get

For conservation of momentum

$$m_A \upsilon_{Ai} + m_A \upsilon_{Bi} = m_A \upsilon_{Af} + m_B \upsilon_{Bf} \tag{6.28}$$

For conservation of kinetic energy

$$\frac{1}{2} m_A \upsilon_{Ai}^2 + \frac{1}{2} m_{Bi} = \frac{1}{2} m_A \upsilon_{Af}^2 + \frac{1}{2} m_B \upsilon_{Bf}^2 \tag{6.29}$$

There are only two unknown quantities (velocities of the balls after collision) and there are two independent equations [Eqns. (6.28) and (6.29)]. The solution is not difficult, but a lengthy one. Therefore, we quote the results only

$$(\upsilon_{Bf} - \upsilon_{Af}) = -(\upsilon_{Bi} - \upsilon_{Ai}) \tag{6.30}$$

$$\upsilon_{Af} = \frac{2m_B \upsilon_{Bi}}{m_A + m_B} + \frac{\upsilon_{Ai}(m_A - m_B)}{m_A + m_B} \tag{6.31}$$

$$\upsilon_{Bf} = -\frac{2m_A \upsilon_{Ai}}{m_A + m_B} + \frac{(m_B - m_A)\,\upsilon_{Bi}}{(m_A + m_B)} \tag{6.32}$$

Notes

We now discuss some special cases.

CASE I : Suppose that the two balls colliding with each other are identical i.e. $m_A = m_B = m$. Then the second term in Eqns. (6.31 and (6.32) will drop out resulting in

$$v_{Af} = v_{Bi} \tag{6.33}$$

and

$$v_{Bf} = v_{Ai} \tag{6.34}$$

That is, if two identical balls collide "head-on", their velocities after collision get interchanged.

After collision:

i) the velocity of A is same as that of B before collision.

ii) the velocity of B is same as that of A before collision.

Now, think what would happen if one of the balls is at rest before collision?

Let B be at rest so that $v_{Bi} = 0$. Then $v_{Af} = 0$ and $v_{Bf} = v_{Ai}$

After collision, A comes to rest and B moves with the velocity of A before collision.

Similar conclusion can be drawn about the kinetic energy of the balls after collision. Complete loss of kinetic energy or partial loss of kinetic energy $(m_A \# m_B)$ by A is same as the gain in the kinetic energy of B. These facts have very important applications in nuclear reactors in slowing down neutrons.

CASE II : The second interesting case is that of collision of two particles of unequal masses.

i) Let us assume that m_B is very large compared to m_A and particle B is initially at rest :

$$m_B >> m_A \text{ and } v_{Bi} = 0$$

Then, the mass m_A can be neglected in comparison to m_B. From Eqns. (6.31) and (6.32), we get

$$v_{Af} \approx -v_{Ai}$$

and

$$v_{Bf} \approx 0$$

After collision, the heavy particle continues to be at rest. The light particle returns back on its path with a velocity equal to its the initial velocity.

This is what happens when a child hits a wall with a ball.

These results find applications in Physics of atoms, as for example in the case where an α – particle hits a heavy nucleus such as uranium.

INTEXT QUESTIONS 6.6

1 Two hard balls collide when one of them is at rest.

a) Is it possible that both of them remain at rest after collision?

b) Is it possible that one of them remains at rest after collision?

2. There is a system of three identical balls A B C on a straight line as shown here. B and C are in contact and at rest. A moving with a velocity υ collides "head-on" with B. After collision, what will be the velocities of A, B and C separately? Explain.

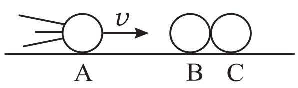


Fig. 6.20

3. Ball A of mass 2 kg collides head-on with ball B of mass 4 kg. A is moving in + x direction with speed 50 m s^{-1} and B is moving in $-x$ direction with speed 40 m s^{-1}. What are the velocities of A and B after collision? The collision is elastic.

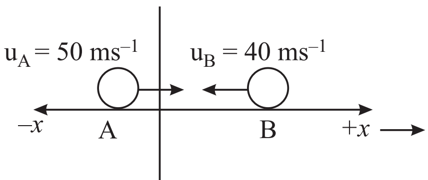


Fig. 6.21

4. A bullet of mass 1 kg is fired and gets embedded into a block of wood of mass 1 kg initially at rest. The velocity if the bullet before collision is 90m/s.

a) What is the velocity of the system after collision.

b) Calculate the kinetic energies before and after the collision?

c) Is it an elastic collision or inelastic collision?

d) How much energy is lost in collision?

5. In an elastic collision between two balls, does the kinetic energy of each ball change after collision?

WHAT YOU HAVE LEARNT

- Work done by a constant force F is

$$W = \mathbf{F.d} = Fd \cos\theta$$

Where θ is the angle between F and d. The unit of work is joule. Work is a scalar quantity.

- Work is numerically equal to the area under the F versus x graph.

- Work done by elastic force obeying Hooke's law is $W = \frac{1}{2}kx^2$ where k is force constant of the elastic material (spring or wire). The sign of W is **positive** for the external force acting on the spring and **negative** for the restoring force offered by spring. x is compression or elongation of the spring.

- The unit of k is newton per metre (N m^{-1}.)
- Power is the time rate of doing work. $P = W/t$ its unit is J/s i.e., watt (W)
- Mechanical energy of a system exists in two forms (i) kinetic energy and (ii) Potential energy.
- Kinetic energy of mass m moving with speed v is $E = \frac{1}{2}mv^2$. It is a scalar quantity.
- The **Work-Energy Theorem** states that the work done by all forces is equal to the change in the kinetic energy of the object.

$$W = K_f - K_i = \Delta K$$

- Work done by a conservative force on a particle is equal to the change in mechanical energy of the particle, that is change in the kinetic energy + the change in potential energy. In other words the mechanical energy is conserved under conservative forces.

$$\Delta E = (E_f - E_i) + (E_f - E_i)$$
$$= (\Delta E)_P + (\Delta E)_k$$

- Work done by a conservative force on an object is zero for a round trip of the object (object returning back to its starting point).
- Work done by a conservative force does not depend on the path of the moving object. It depends only on its initial and final positions.
- Work done is path dependent for a non-conservative force. The total mechanical energy is not conserved.
- The potential energy of a particle is the energy because of its position in space in a conservative field.
- Energy stored in a compressed or stretches spring is known as elastic potential energy. It has a value $\frac{1}{2}kx^2$, where k is spring constant and x is diplacement.
- The energy stored in a mass m near the earth's surface is mgh. It is called the gravitational potential energy. Here h is change in vertical co-ordinate of the mass. The reference level of zero potential energy is arbitrary.
- Energy may be transformed from one kind to another in an isolated system, but it can neither be created nor destroyed. The total energy always remains constant.

- Laws of conservation of momentum always hold good in any type of collision.
- The kinetic energy is also conserved in elastic collision while it is not conserved in inelastic collision.

TERMINAL EXERCISE

1. If two particles have the same kinetic energy, are their momenta also same? Explain.
2. A particle in motion collides with another one at rest. Is it possible that both of them are at rest after collision?
3. Does the total mechanical energy of a system remain constant when dissipative forces work on the system?
4. A child throws a ball vertically upwards with a velocity 20 m s^{-1}.

   (a) At what point is the kinetic energy maximum?

   (b) At what point is the potenital energy maximum?
5. A block of mass 3 kg moving with a velocity 20m s^{-1} collides with a spring of force constant 1200 N m^{-1}. Calculate the maximum compression of the spring.
6. What will be the compression of the spring in question 5 at the moment when kinetic energy of the block is equal to twice the elastic potential energy of the spring?
7. The power of an electric bulb is 60 W. Calculate the electrical energy consumed in 30 days if the bulb is lighted for 12 hours per day.
8. 1000 kg of water falls every second from a height of 120 m. The energy of this falling water is used to generate electricity. Calculate the power of the generator assuming no losses.
9. The speed of a 1200 kg car is 90 km h^{-1} on a highway. The driver applies brakes to stop the car. The car comes to rest in 3 seconds. Calculate the average power of the brakes.
10. A 400 g ball moving with speed 5 m s^{-1} has elastic head-on collision with another ball of mass 600 g initially at rest. Calculate the speed of the balls after collision.
11. A bullet of mass 10 g is fired with an initial velocity 500 m s^{-1}. It hits a 20 kg wooden block at rest and gets embedded into the block.

    (a) Calculate the velocity of the block after the impact

    (b) How much energy is lost in the collision?

12. An object of mass 6 kg. is resting on a horizontal surface. A horizontal force of 15 N is constantly applied on the object. The object moves a distance of 100m in 10 seconds.

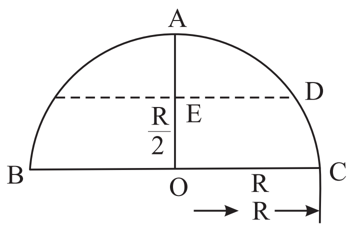
    (a) How much work does the applied force do?

    (b) What is the kinetic energy of the block after 10 seconds?

    (c) What is the magnitude and direction of the frictional force (if there is any)?

    (d) How much energy is lost during motion?

13. A, B, C and D are four point on a hemispherical cup placed inverted on the ground. Diameter BC = 50 cm. A 250 g particle at rest at A, slide down along the smooth surface of the cup. Calculate it's

    (a) Potential energy at A relative to B.

    (b) Speed at the point B (Lowest point).

    (c) Kinetic and potential energy at D.

    Do you find that the mechanical energy of the block is conserved? Why?

14. The force constant of a spring is 400 N/m. How much work must be done on the spring to stretch it (a) by 6.0 cm (b) from $x = 4.0$ cm to $x = 6.0$ cm, where $x = 0$ is the relaxed position of the spring.

15. The mass of a car is 1000 kg. It starts from rest and attains a speed of 15 m s^{-1} in 3.0 seconds. Calculate

    (a) The average power of the engine.

    (b) The work done on the car by the engine.

ANSWERS TO INTEXT QUESTIONS

6.1

1. The force always works at right angle to the motion of the particle. Hence no work is done by the force.

2. (a) Work done is zero (i) when there is no displacement of the object. (ii) When the angle between force and the displacement is 90º.

    When a mass moves on a horizontal plane the work done by gravitation force is zero.

(b) When a particle is thrown verically upwards, the work done by gravitational force is **negative**.

(c) When a particle moves in the direction of force, the work done by force is **positive.**

3. (a) $W = mgh = 2 \times 9.8 \times 5 = +\,98$ J

   (b) The work done by gravity is –98 J

4. $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ $\quad \mathbf{d} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$

   $W = \mathbf{F.d} \qquad = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \,.\, (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$

   $-2 + 6 = \mathbf{4}$

5. $\mathbf{F} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ $\quad \mathbf{d} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

   (a) $|\,\mathbf{d}\,| = \sqrt{9+16} = \sqrt{25} = \mathbf{5}$ m

   (b) $|\,\mathbf{F}\,| = \sqrt{25+9} = \sqrt{34} = 5.83$

   (c) $W = \mathbf{F.d} \qquad = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \,.\, (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

   $= 15 + 12 = 27$ J

6.2

1. Spring constant is defined as the restoring force per unit displacement. Thus, it is measured in N m^{-1}.

2. $k = \dfrac{10\text{N}}{1\text{cm}} = \dfrac{10\text{N}}{1/100\text{m}} = 100$ N m

As $F = kx$ for $x = 50$ cm. $F = \left(100\dfrac{\text{N}}{\text{m}}\right)(0.5\text{ m})$

$= 50$ N.

$$W = \frac{1}{2}kx^2 = \frac{1}{2} \times \frac{100\text{N}}{\text{m}} \times \left(\frac{5}{100} \times \frac{5}{100}\right)\text{m}^2$$

$= 1.25$ N m $= 1.25$ J.

6.3

1. $P = \dfrac{\text{mgh}}{\text{t}} = \dfrac{(100 \times 9.8 \times 8)}{10\text{ s}}\text{J} = 784$ W.

2. 10 H.P $= (10 \times 746)$ W $= \dfrac{10 \times 746}{1000}$ W

   $= 7.46$ kW

6.4

1. k.E. = $\frac{1}{2}m\upsilon^2$. It is never negative because

   (i) m is never negative

   (ii) υ^2 is always positive.

2. (a) K.E = $\frac{1}{2}m\upsilon^2 = E$

   When υ is made 2υ, K.E becomes 4 times and E becomes $4E$

   (b) When m becomes $\frac{m}{2}$, E becomes $\frac{E}{2}$

3. P.E. of spring = $\frac{1}{2}kx^2 = 3.6$ J

   $\therefore \quad x^2 = \frac{2\times3.6}{\text{k}} = \frac{2\times3.6}{180} = 0.04$ m

   and $\quad x = 0.2$ m = 20 cm.

4. $\upsilon^2 = u^2 - 2as$ Final velocity is zero and initial velocity is $\frac{90\text{ km}}{\text{h}} = 25\text{m s}^{-1}$

   $\therefore \quad \frac{u^2}{2s} = a = \frac{25\times25}{2\times15} = 20.83\text{ m s}^{-2}$

   $F = ma = 1000 \times 20.83 = 20830$ N.

   Power = $\frac{W}{t} = \frac{20830\times15}{25} = 12498$ W

5. Work done by external force = 375 J

   Work done by spring = – 375 J

6.5

1. (a) O, no change in P.E.

   (b) Change in P.E. = $mgh = 2 \times 9.8 \times 4 = 78.4$ J

   (c) Change in P.E. = 78.4 J.

   (d) – 78.4 J.

2. (a) Change in P.E. from $= mgh = 0.5 \times 9.8 \times 4 = 19.6$ J

K.E. at B $= \frac{1}{2}mv^2 = 19.6$ J

$v^2 = \frac{19.6 \times 2}{0.5}$

$v^2 = 78.4 \Rightarrow v = 8.85$ m s^{-1}

(b) $v = 14$ m s^{-1}

(c) $mgh = 0.5 \times 9.8 \times 10 = 49.0$ J (+ positive)

W = +49 J

3. BC = 2m

$\frac{AC}{BC} = \sin 30^{\circ}$

AC = BC sin30°

$= 2 \times \frac{1}{2} = 1$

Change in P.E. from C to B = mgh $= 2 \times 9.8 \times 1 = 19.6$ J

But the K.E. at B is = 15.6 J

Energy lost = 19.6 – 15.6 = 4J

This loss is due to frictional force

$4J = F \times d = F \times 2$

$F = 2N$

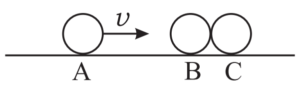
4. When the bob of a simple pendulum oscillates, its K.E. is max at $x = 0$ and min at $x = x_m$. The P.E. is min at $x = 0$ and max at $x = x_m$. Hence A represents the P.E. curve.

5. No.

6.6

1. (a) No, because, it will go against the low of conservation of linear momentum.

(b) yes.

2. 

$v_A = 0, v_B = 0, v_C = v$

∵ This condition only satisfies the laws of conservation of (i) linear momentum and (ii) total kinetic energy.

Notes

3. $$v_{Af} = \frac{2m_B v_{Bi}}{m_A + m_B} + \frac{v_{Ai}(m_A - m_B)}{m_A + m_B}$$

$$= \frac{2 \times 4 \times (-40)}{6} - \frac{50(-2)}{6}$$

$$= -\frac{320}{6} + \frac{100}{6}$$

$$= -\frac{220}{6}$$

$$= -35 \text{ms}^{-1}.$$

$$v_{Bf} = -\frac{2m_A v_{Ai}}{m_A + m_B} + \frac{(m_B - m_A)\, v_{Bi}}{(m_A + m_B)}$$

$$= \frac{2 \times 2 \times 50}{6} + \frac{(-40)(4-2)}{6}$$

$$= \frac{200}{6} - \frac{80}{6}$$

$$= \frac{120}{6} = 20 \text{ ms}^{-1}.$$

Thus ball A returns back with a velocity of 35 m s^{-1} and ball B moves on with a velocity of 20 m s^{-1}.

4. (a) 1.76 ms^{-1}.

   (b) 81 J and 1.58 J

   (c) Inelastic collision

   (d) 79.42 J

5. yes, but the total energy of both the balls together after collision is the same as it was before collision.

Answers to Terminal Problem

5. 1 m.
6. 0.707 m
7. 21.6 kW

8. 1.2 mega watt

9. 125 kW

10. $\frac{1}{4}$ m s^{-1}, $\frac{19}{6}$ m s^{-1}

11. (a) 0.25 m s^{-1} (b) 1249.4 J

12. (a) 1500 J (b) 1200 J
    (c) 3 N opposite to the direction of motion
    (d) 300 J

13. (a) 0.625 J (b) $\sqrt{5}$ m s^{-1} (c) 0.313 J

14. (a) 0.72 J (b) 0.4 J

15. (a) 37.5 kW (b) 1.125×10^5 J