13

SIMPLE HARMONIC MOTION

PERIODIC MOTION

A motion which repeats itself after a fixed interval of time is called periodic motion

There are two types of periodic motion

: (i) non–oscillatory, and (ii) oscillatory

an oscillatory motion is normally periodic but a periodic motion is not necessarily oscillatory

a motion which repeats itself in equal intervals of time is periodic and if it is about a mean position, it is oscillatory.

Displacement as a Function of Time

Periodic Motion

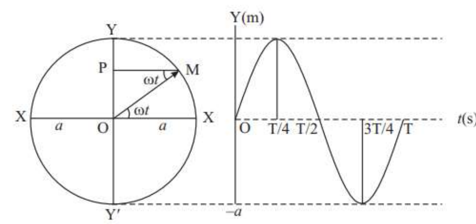
When an object repeats its motion after a definite interval of time, its motion is said to be periodic.

the changes in the position or displacement of the object can be expressed as a function of time: $x = af(t + T)$ where a is a constant and T is the time after which the value of x is repeated

SIMPLE HARMONIC MOTION : CIRCLE OF REFERENCE

A particle is said to execute simple harmonic motion if it moves to and fro about a fixed point periodically, under the action of a force F which is directly proportional to its displacement x from the fixed point and the direction of the force is opposite to that of the displacement.

$F = - kx$



Basic Terms Associated with SHM

Displacement

is the distance of the harmonic oscillator from its mean (or equilibrium) position at a given instant.

Amplitude

is the maximum displacement of the oscillator on either side of its mean position.

Time period

is the time taken by the oscillator to complete one oscillation.

Frequency

is the number of oscillations completed by an oscillator in one second.

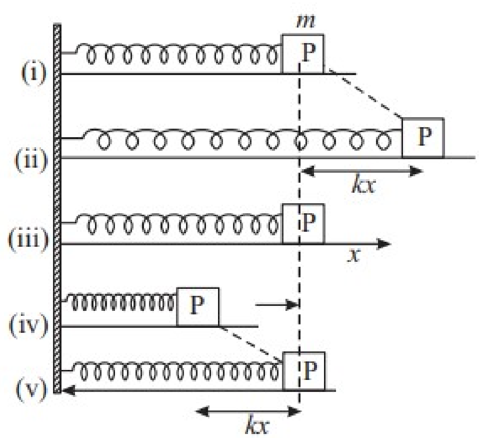
Phase

φ is the angle whose sine or cosine at a given instant indicates the position and direction of motion of the oscillator.

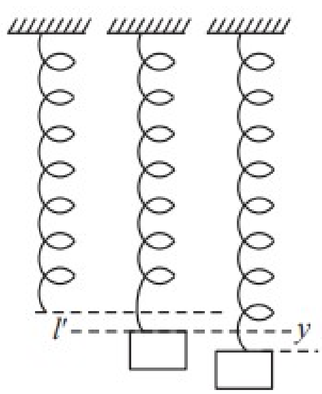
Angular Frequency

ω describes the rate of change of phase angle. It is expressed in radian per second.

Horizontal Oscillations of a Spring-Mass System

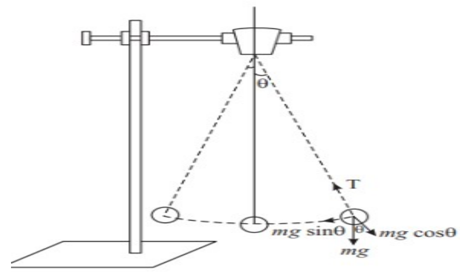


Vertical Oscillations of a Spring–Mass System



Simple Pendulum

A simple pendulum is a small spherical bob suspended by a long cotton thread held between the two halves of a clamped split cork in a stand,



$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/l}{m}} = \sqrt{\frac{g}{l}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

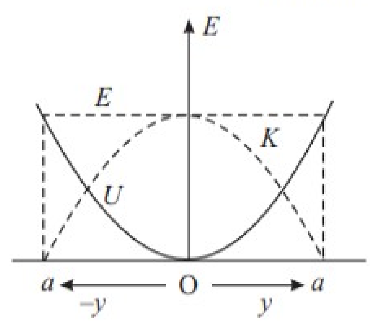
$$T = 2\pi\sqrt{\frac{l}{g}}$$

ENERGY OF SIMPLE HARMONIC OSCILLATOR

$$E = U + K$$

$$= \frac{1}{2}m\omega^2a^2\ (\sin^2\omega t + \cos^2\omega t)$$

$$= \frac{1}{2}ma^2\omega^2$$



DAMPED HARMONIC OSCILLATIONS

Every oscillating system normally has a viscous medium surrounding it.

As a result in each oscillation some of its energy is dissipated as heat. As the energy of oscillation decreases the amplitude of oscillation also decreases.

The amplitude of oscillations of a pendulum in air decreases continuously. Such oscillations are called damped oscillations.

FREE AND FORCED VIBRATIONS : RESONANCE

When an oscillatroy system vibrates on its own, its vibrations are said to be free. If, however, an oscillatory system is driven by an external system, its vibrations are said to be forced vibrations. And if the frequency of the driver equals to the natural frequency of the driven, the phenomenon of resonance is said to occur

Check Your self

1. A particle in simple harmonic motion while passing through mean position will have

   A. Maximum kinetic energy and maximum potential energy
   B. Maximum kinetic energy and minimum potential energy
   C. Minimum kinetic energy and maximum potential energy
   D. Minimum kinetic energy and minimum potential energy

2. For a body moving with simple harmonic motion, the number of cycles per second, is known as its

   A. Oscillation
   B. Amplitude
   C. Periodic time
   D. Frequency

3. In a simple harmonic motion, acceleration of a particle is proportional to

   A. Rate of change of velocity
   B. Displacement
   C. Velocity
   D. Direction

4. The motion of a particle moving with simple harmonic motion, from an extremity to the other, constitutes

   A. Half an oscillation
   B. One an oscillation
   C. Two oscillations
   D. None of the above

5. The periodic time of a body moving in simple harmonic motion is

   A. Directly proportional to its angular velocity
   B. Directly proportional to the weight of the body
   C. Directly proportional to the momentum of swinging body
   D. Inversely proportional to the angular velocity

Stretch Yourself

1. What is phase angle. How is it related to angular frequency
2. A simple pendulum oscillates with amplitude 0.04 m .If its time period is 10s calculate its maximum velocity
3. Show graphically how the potential energy, kinetic energy and the total energy E of a simple harmonic oscillator vary with displacement from equilibrium position.

Answer to check yourself

1B 2 D 3 B 4 A 5 D