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GRAVITATION

LAW OF GRAVITATION

- Every particle attracts every other particle in the universe with a force which varies as the product of their masses and inversely as the square of the distance between them.

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2}$$

- In vector form

$$\boldsymbol{F_{12}} = G\frac{m_1 m_2}{r_{12}^2}\widehat{r_{12}}$$

$$\boldsymbol{F_{21}} = -G\frac{m_1 m_2}{r_{12}^2}\widehat{r_{12}}$$

$$\boldsymbol{F_{12}} = -\boldsymbol{F_{21}}$$

- The constant of proportionality G , is called the universal constant of gravitation.
- Value of G is $6.67 \times 10^{-11} Nm^2 kg^{-2}$
- Its value remains the same between any two objects everywhere

ACCELERATION DUE TO GRAVITY

F = ma(By Newton's second law)

F = $G\frac{mM}{r^2}$(By law of gravitation)

$$a = g = G\frac{M}{r^2}$$

VARIATION IN THE VALUE OF G

- **Variation with Height**

the magnitude of g decreases as square of the distance from the centre of the earth increases

$$g_h = \frac{GM}{R^2(1+\frac{h}{R})^2}$$

- **Variation of g with Depth**

The value of g decreases as we go below the earth

$$g_d \frac{4\pi G}{3}\rho(R-d)$$

- **Variation of g with Latitude**

$$\boldsymbol{g_\lambda = g - R\omega^2 \cos\lambda}$$

Where g_λ is gravity at latitude, g value of gravity at pole, ω angular velocity of earth ,R radius of earth.

WEIGHT AND MASS

- The force with which a body is pulled towards the earth is called its weight. If m is the mass of the body, then its weight W is given by W = mg
- It's unit is newton.
- Since g varies from place to place, weight of a body also changes from place to place

- The weight is maximum at the poles and minimum at the equator

- The mass of a body, however, does not change. Mass is an intrinsic

property of a body. Therefore, it stays constant wherever the body may be situated

Gravitational Potential and Potential energy

Gravitational Potential (V) of mass M is defined as the gravitational potential energy of unit mass. Hence,

$$dU = - F.\ dr$$

$$dU = \frac{GmM}{r^2} dr$$

$$U = GMm \int_{\infty}^{r} \frac{1}{r^2} dr$$

Kepler's Laws Of Planetary Motion

1: The orbit of a planet is an ellipse with the Sun at one of the foci (An ellipse has two foci.)

2: The area swept by the line joining the planet to the sun in unit time is constant throughout the orbit.

3: The square of the period of revolution of a planet around the sun is proportional to the cube of its average distance from the Sun. If we denote the period by T and the average distance from the Sun as r,

$$T^2 \propto r^3$$

Orbital Velocity of Planets

$$\boldsymbol{v_{orb} = \frac{2\pi r}{T}}$$

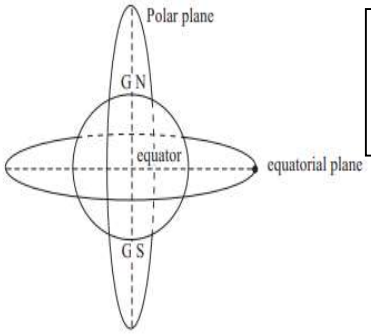
$$\boldsymbol{v_{orb} = \sqrt{\frac{GM_s}{r^2}}}$$

ESCAPE VELOCITY

It is defined as the minimum velocity required by an object to escape the gravitational pull of the earth

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

ARTIFICIAL SATELLITES



$$v_{orb} = \frac{v_{sec}}{\sqrt{2}}$$

Polar Orbit	Equatorial orbit
Satellite used for remote sensing	Satellite used for communication
Altitude is 800 km	Height is fixed at around 3600 km
Time period is 100 min	Time period of rotation is 24 hour

CHECK YOURSELF

1. Dimension of gravitational constant G.
   A. $M^{-1}L^3T^{-2}$
   B. $M^{-1}L^3T^{-1}$
   C. $M^{-1}L^2T^{-2}$
   D. ML^3T^{-2}

2. Value of g at $\lambda = 60^0$, radius of earth is 6371 km
   A. 9.836 ms^{-2}
   B. 9.8ms^{-2}
   C. 9.7ms^{-2}
   D. 9.836ms^{-2}
3. S.I. unit of gravitational potential
   A. Jkg^{-1}
   B. Jkg
   C. $J^{-1}kg$
   D. $(Jkg)^{-2}$
4. Mass of earth is $5.97x10^{24}$kg and its radius is 6371km escape velocity from earth is

A. 11.3 ms^{-1}
B. 11.3 kms^{-1}
C. 11.9 kms^{-1}
D. 11.9ms^{-1}

5. The atmosphere around the earth is held by
   A. Gravity
   B. Winds
   C. Clouds
   D. None of the above

STRETCH YOURSELF

1. Why is gravitational potential energy always negative? Explain
2. A boy is weightless at the center of earth why?
3. At what depth would the value of g be 50% of what it is on the surface of the earth?
4. Obtain an expression for the orbital velocity of a satellite orbiting the earth.
5. A polar satellite is placed at a height of 1000 km from earth surface. Calculate its orbital period and orbital velocity.

Answer to check yourself

1A) 2A) 3A) 4B) 5A)