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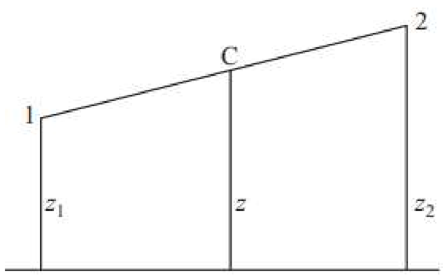
# Motion of a Rigid Body

## Rigid Body

- A rigid body is one in which the separation between the constituent particles does not change with its motion.

**CENTRE OF MASS (C.M.) OF A RIGID BODY**




- The potential energies of particles 1 and 2 are mgz1 and mgz2 , respectively. The potential energy of the particle at C is 2mgz.

$$2\,mgz = mgz_1 + mgz_2$$

$$z = \frac{z_1 + z_2}{2}$$

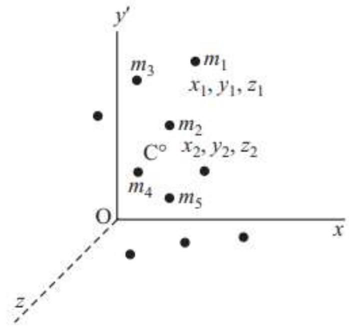
$$(m_1 + m_2)gz = m_1gz_1 + m_2gz_2$$

$$z = \frac{m_1z_2 + m_1z_2}{(m_1 + m_2)}$$

- The point C is called the centre of mass (CM) of the system. As such, it is a mathematical tool and there is no physical point as CM.

If the particle with mass m1 has coordinates (x1 , y1 , z1 ) with respect to some coordinate system, mass m2 has coordinates (x2 , y2 , z2 ) and so on the coordinates of CM are given by

$$x = \frac{m_1x_1+m_2x_2+m_3x_3+m_4x_4\ldots\ldots\ldots}{m_1+m_2\ldots\ldots\ldots}$$



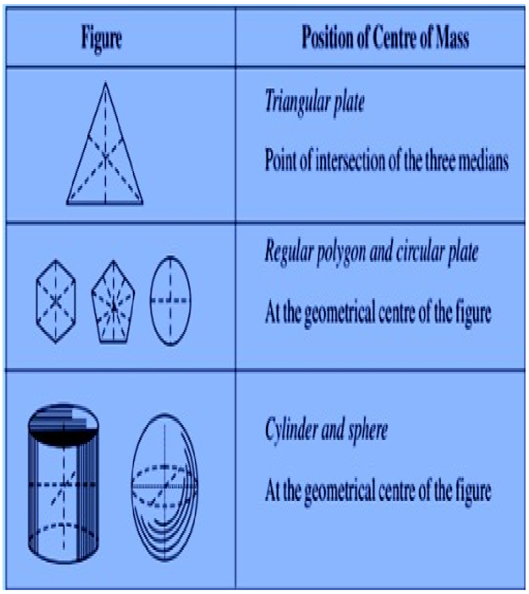

$$x = \frac{\sum_i^N m_ix_i}{\sum_{i=1}^N m_i}$$

$$Y = \frac{\sum_i^N m_iy_i}{\sum_{i=1}^N m_i}$$

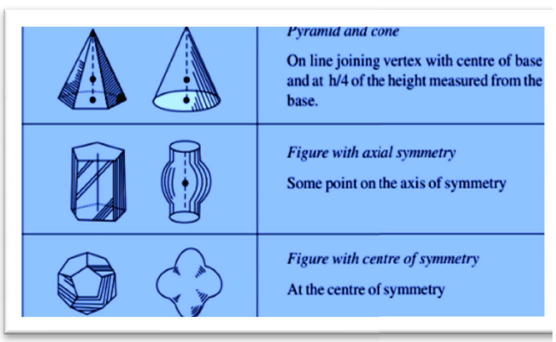
$$z = \frac{\sum_i^N m_iz_i}{\sum_{i=1}^N m_i}$$

- The forces acting on a body can be of two kinds. Some forces can be due to sources outside the body. These forces are called the external forces.
- A familiar example is the force of gravity.
- Some other forces arise due to the interaction among the particles of the body. These are called internal forces.
- A familiar example is cohesive force
- The CM of a body moves as though the entire mass of the body were located at that point and it was acted upon by the sum of all the external forces acting on the body.

## CM of Some Bodies



| Figure | Position of Centre of Mass |
|---|---|
| | *Triangular plate* Point of intersection of the three medians |
| | *Regular polygon and circular plate* At the geometrical centre of the figure |
| | *Cylinder and sphere* At the geometrical centre of the figure |



| | |
|---|---|
| | *Pyramid and cone* On line joining vertex with centre of base and at h/4 of the height measured from the base. |
| | *Figure with axial symmetry* Some point on the axis of symmetry |
| | *Figure with centre of symmetry* At the centre of symmetry |

**TRANSLATIONAL AND ROTATIONAL MOTION OF A RIGID BODY**

- When a rigid body moves in such a way that all its particles move along parallel paths its motion is called translational motion
- The motion of a rigid body in which all its constituent particles describe concentric circular paths is known as rotational motion.

## Moment of Inertia

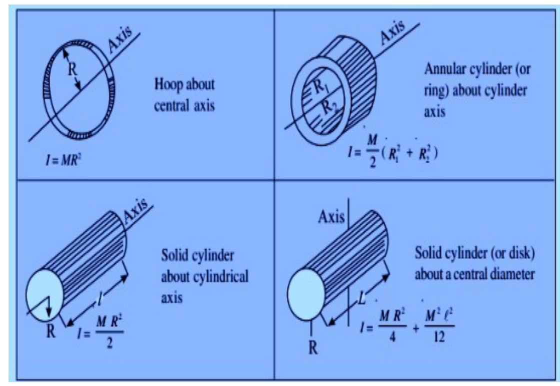
$$\mathbf{T = (1/2)}m_1v_1^2 + m_2v_2^2$$

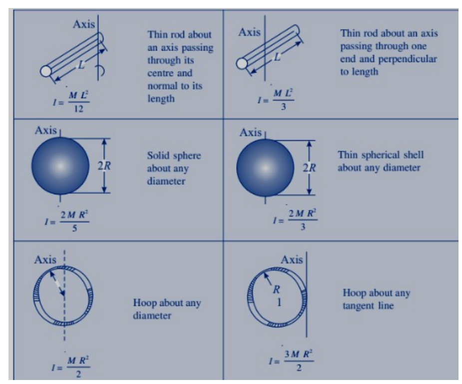
$$=\sum_{i=1}^{N}\left(\frac{1}{2}\right)m_iv_i^2$$

$$\mathbf{T}=\sum_{i=1}^{N}\left(\frac{1}{2}\right)m_i\left(r_i^2\ \ \omega^2\right)$$

$$\mathbf{I} = \sum m_i\, r_i^2$$

I is called the moment of inertia of the body.



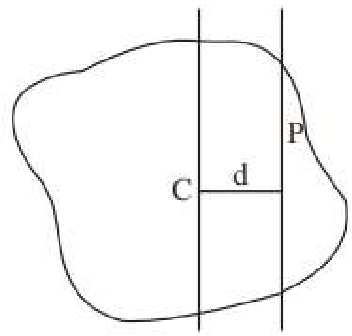

## Equations of motion for a uniformly rotating rigid body

- $\theta = \omega\, t$
- $\omega_f = \omega_i + \alpha\, t$
- $\theta = \omega_i\, t + \frac{1}{2}\alpha t^2$
- $\omega_f^2 = \omega_i^2 + 2\alpha\theta$

## Theorems of moment of inertia

## Theorem of parallel axes

Theorem of parallel axis states that the moment of inertia about an axis parallel to the axis passing through its centre of mass is equal to the moment of inertia about its centre of mass plus the product of mass and square of the perpendicular distance between the parallel axes

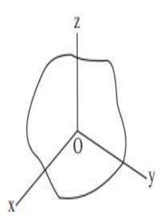


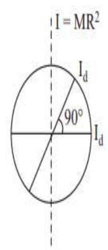

$$I = I_c + Md^2$$

## Theorem of perpendicular axes

The sum of the moments of inertia about x and y axes is equal to the moment of inertia about the z– axis.

$$I_z = I_x + I_y$$




**Torque and Couple**

The turning effect of a force is called torque. Its magnitude is given by

$\tau = F\ s = Fr \sin\theta$

## ANGULAR MOMENTUM

- . The product of linear momentum and the distance from the axis is called angular momentum, denoted by L.
- $L=\sum_i \omega\, m_i r_i^2$
- $L=I\omega$

## Conservation of angular momentum

If there is no net torque acting on the body

$$\frac{dL}{dt} = 0.$$

- This means that there is no change in angular momentum, i.e. the angular momentum is constant.
- This is the principle of conservation of angular momentum.

Check your progress

1. Position of center of mass of uniform solid sphere.
   a) Center of Sphere
   b) Radius of sphere
   c) Diameter of sphere
   d) N.A
2. Which one of the following is correct?
   A. $\tau = r.F$
   B. $\tau = r \times F$
   C. $\tau = r/F$
   D. $\tau = F/r$
3. Dimension of angular velocity
   a) $M^0L^0T^{-2}$
   b) $M^0L^0T^{-1}$
   c) $M^1L^0T^{-1}$
   d) $M^0L^1T^{-1}$
4. Moment of inertia for a solid sphere of radius R
   a) $2/5\ MR^2$
   b) $2/3MR^2$
   c) $1/2MR^2$
   d) $1/4MR^2$
5. For which of the following does the center of mass lie outside of body
   a) Pencil
   b) Dice
   c) Bangle
   d) Shotput

Check your strength

1. Can a body in translator motion have angular momentum explain?

2. Can a body in translatory motion have angular momentum?
3. State the two theorem of M.I.
4. In a molecule of CO the nuclei of the two atoms are $1.13 \times 10^{-10}$ m apart. Locate the center of mass of the molecule
5. Discuss the physical meaning of angular momentum.

Answer to Check Yourself

1A) 2B) 3 B) 4A) 5C)