

# 14

## Congruence of Triangles

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### 14.1 INTRODUCTION

You might have observed that leaves of different trees have different shapes, but leaves of the same tree have almost the same shape. Although they may differ in size. The geometrical figures which have same shape and same size are called congruent and the property is called congruence.

In this lesson you will study congruence of two triangles, some relations between their sides and angles in details.

### 14.2 OBJECTIVES

After studying this lesson, the learner will be able to :

- verify and explain whether two given figures are congruent or not.
- state the criteria for congruency of two triangles and apply them in solving problems.
- prove that angles opposite to equal sides of a triangle are equal.
- prove that sides opposite to equal angles of a triangle are equal.
- prove that if two sides of triangle are unequal, then the longer side has the greater angle opposite to it.
- state and verify inequalities in a triangle.
- solve problems based on the above results.

### 14.3 EXPECTED BACKGROUND KNOWLEDGE

- Recognition of plane geometric figures
  - Equality of lines and angles
  - Types of angles
  - Angle sum property of a triangle
  - Paper cutting and folding.
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## 14.4 CONCEPT OF CONGRUENCE

In our daily life you observe various figures or objects. These figures or objects can be categorised in terms of its shape and size in the following manner.

(i) Which have different shapes and sizes as shown in Fig. 14.1

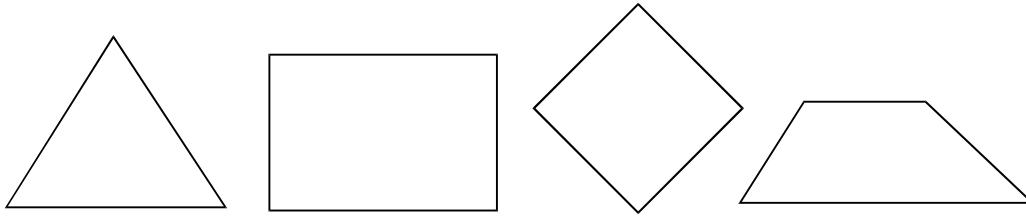


Fig. 14.1

(ii) Which have same shapes but different sizes as shown in Fig. 14.2



Fig. 14.2

(iii) Which have same shape and same size as shown in Figures 14.3, 14.4 and 14.5.

(a) two' one rupee coins



Fig. 14.3

(b) two postage stamps or post cards

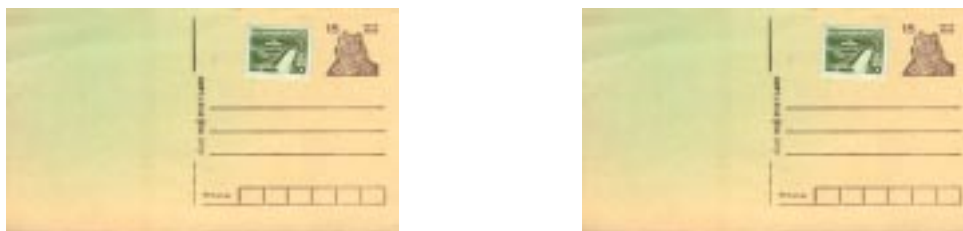


Fig. 14.4

(c) two photo prints of same size from the same negative.



Fig. 14.5

We will deal with the figures which have same shapes and same sizes.

**Two figures, which have the same shape and same size are called congruent figures and this property is called congruence.**

**14.4.1 ACTIVITY FOR YOU**

Take a sheet of paper, fold it in the middle and keep a carbon (paper) between the two folds. Now draw a figure of a leaf or a flower or any object which you like, on the upper part of the sheet. You will get a carbon copy of it, on the sheet below.

The figure you drew and its carbon copy are of the same shape and same size. Thus, these are congruent figures. Observe a butterfly folding its two wings. These appear to be one.

**14.4.2 CRITERIA FOR CONGRUENCE OF SOME FIGURES**

Congruent figures when placed one over the other, exactly coincide with one another or cover each other. In other words, two figures will be congruent, if parts of one figure are equal to the corresponding parts of the other. For example :

(1) Two line segments are congruent, when they are of equal length.



Fig. 14.6

(2) Two squares are congruent if their sides are equal.



Fig. 14.7

- (3) Two circles are congruent, if their radii are equal, implying their circumferences are also equal.

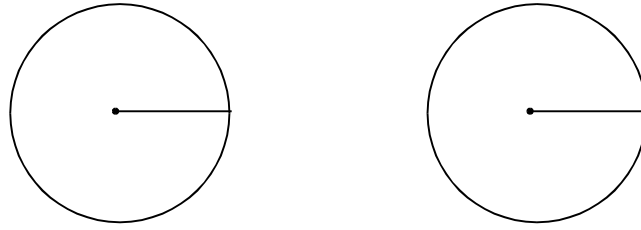


Fig. 14.8

### 14.5 CONGRUENCE OF TRIANGLES

Triangle is a basic rectilinear figure in geometry, having minimum number of sides. As such congruence of triangles plays a very important role in proving many useful results. Hence this needs a detailed study.

**Two triangles are congruent, if all the sides and all the angles of one are equal to the corresponding sides and angles of other.**

For example, in triangles PQR and XYZ in Fig. 14.9

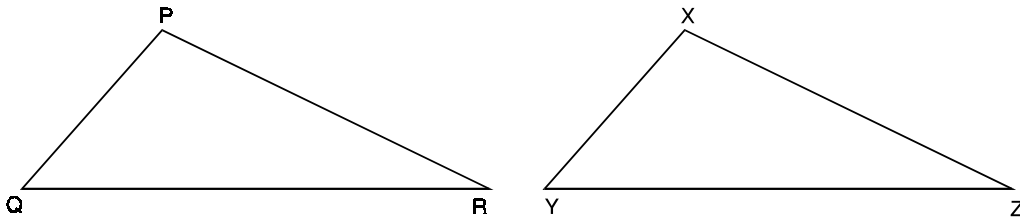


Fig. 14.9

$$PQ = XY, PR = XZ, QR = YZ$$

$$\angle P = \angle X, \angle Q = \angle Y \text{ and } \angle R = \angle Z$$

Thus we can say

$\Delta PQR$  is congruent to  $\Delta XYZ$  and we write

$$\Delta PQR \cong \Delta XYZ$$

Relation of congruence between two triangles is always written with corresponding or matching parts in proper order.

Here  $\Delta PQR \cong \Delta XYZ$

also means P corresponds to X, Q corresponds to Y and R corresponds to Z.

This congruence may also be written as  $\Delta QRP \cong \Delta YZX$  which means, Q corresponds to Y, R corresponds to Z and P corresponds to X. It also means corresponding parts, (elements) are equal, namely

$$QR = YZ, RP = ZX, QP = YX, \angle Q = \angle Y, \angle R = \angle Z$$

and  $\angle P = \angle X$ .

This congruence may also be written as

$$\Delta RPQ \cong \Delta ZXY$$

but **NOT** as  $\Delta PQR \cong \Delta YZX$ .

Or **NOT** as  $\Delta PQR \cong \Delta ZXY$ .

### 14.6 CRITERIA FOR CONGRUENCE OF TRIANGLES

In order to prove, whether two triangles are congruent or not, we need to know that all the six parts of one triangle are equal to the corresponding six parts of the other triangle. We shall now learn that it is possible to prove the congruence of two triangles, even if we are able to know the equality of **three** of their corresponding parts.

Consider a triangle ABC in Fig. 14.10

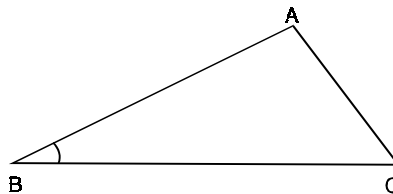


Fig. 14.10

Construct another triangle PQR such that  $QR = BC$ ,  $\angle Q = \angle B$  and  $PQ = AB$ .

(See Fig. 14.11).

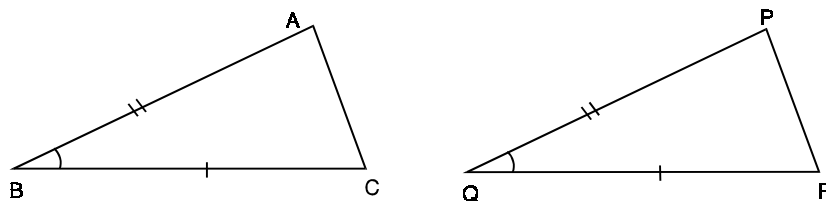


Fig. 14.11

If we trace or cut out triangle ABC and place it over triangle PQR, we will observe that one covers the other exactly. Thus, we may say they are congruent.

Alternatively, we can also measure the remaining parts, and observe that

$$AC = PR, \angle A = \angle P \text{ and } \angle C = \angle R$$

Showing that  $\Delta PQR \cong \Delta ABC$ .

It should be noted here that in constructing  $\Delta PQR$  congruent to  $\Delta ABC$  we used only two pairs of sides  $PQ = AB$ ,  $QR = BC$  and the included angle between them  $\angle Q = \angle B$ .

This means that equality of these three corresponding parts results in congruent triangles. Thus we have

**Criterion 1 : If any two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of the other triangle, the two triangles are congruent.**

This criterion is referred to as SAS (side angle side).

Again, consider  $\Delta ABC$  in Fig. 14.12.

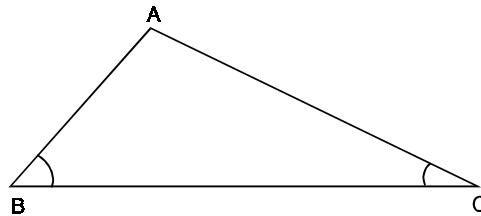


Fig. 14.12

Construct another  $\Delta PQR$  such that,  $QR = BC$ ,  $\angle Q = \angle B$  and  $\angle R = \angle C$ . (See Fig. 14.13)

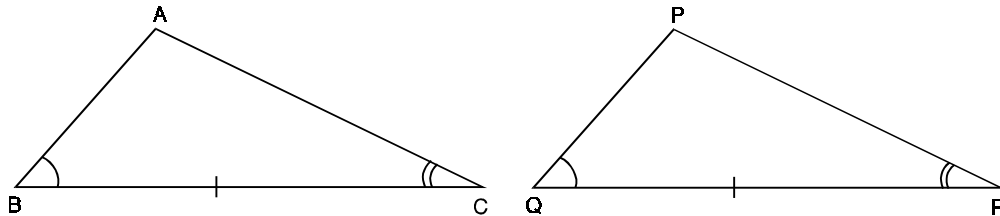


Fig. 14.13

By superimposition or by measuring the remaining corresponding parts, we observe that  $\angle P = \angle A$ ,  $PQ = AB$  and  $PR = AC$  establishing that  $\Delta PQR \cong \Delta ABC$ , which again means that equality of the three corresponding parts (two angles and the included side) of two triangles results in congruent triangles.

We also know that the sum of the three angles of a triangle is  $180^\circ$ , as such if two angles of one triangle are equal to the corresponding angles of another triangle, then the third angles will also be equal. Thus instead of included side we may have any pair of corresponding sides equal. Thus we have

**Criterion 2 : If any two angles and a side of one triangle are equal to corresponding angles and the side of the other triangle, then the two triangles are congruent.**

This criterion is referred to as ASA or AAS.

**14.6.1 ACTIVITY FOR YOU**

In order to explore another criterion we again take a triangle ABC (See Fig. 14.14)

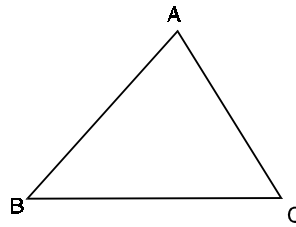


Fig. 14.14

Now take three thin sticks equal in lengths to sides AB, BC and CA of  $\Delta ABC$ . Place them in any order to form  $\Delta PQR$  or  $\Delta P'Q'R'$  near the  $\Delta ABC$  (Fig. 14.15).

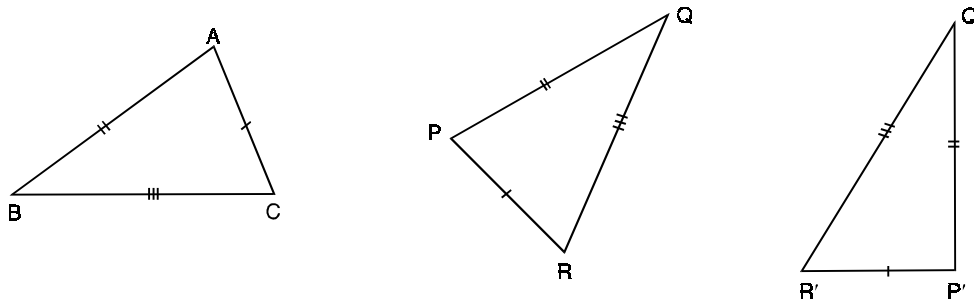


Fig. 14.15

By measuring the corresponding angles, we find that,  $\angle P = \angle P' = \angle A$ ,  $\angle Q = \angle Q' = \angle B$  and  $\angle R = \angle R' = \angle C$ , establishing that

$$\Delta PQR \cong \Delta P'Q'R' \cong \Delta ABC$$

which means that equality of the three corresponding sides of two triangles, results in congruent triangles. Thus we have

**Criterion 3 : If the three sides of one triangle are equal to the three corresponding sides of the other triangle, then the two triangles are congruent.**

This is referred to as SSS (side, side, side).

Similarly, we can establish one more criterion which will be applicable for two right triangles only.

**Criterion 4 : If the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle, then the two right triangles are congruent.**

This criterion is referred to as R.H.S. (Right Hypotenuse side).

Using these criteria we can easily prove, knowing three corresponding parts only, whether two triangles are congruent and establish the equality of remaining corresponding parts.

**Example 14.1 :** In which of the following criteria, two given triangles are **NOT** congruent.

- (a) All corresponding sides are equal
- (b) All corresponding angles are equal
- (c) Two corresponding sides and their included angles are equal
- (d) All corresponding angles and any pair of corresponding sides are equal.

**Ans.** (b)

**Example 14.2 :** Two rectilinear figures are congruent if they have only

- (a) all corresponding sides equal
- (b) all corresponding angles equal
- (c) the same area
- (d) all corresponding angles and all corresponding sides equal.

**Ans.** (d)

**Example 14.3 :** In Fig 14.16, PX and QY are perpendicular to PQ and  $PX = QY$ . Show that  $AX = AY$ .

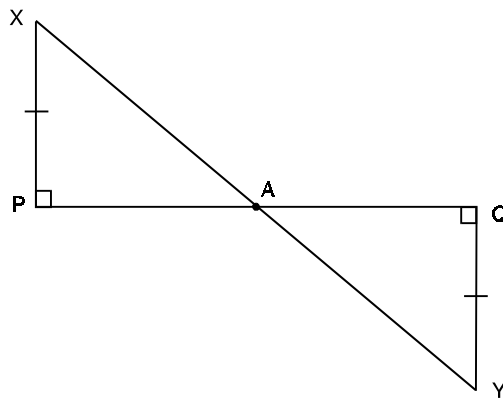


Fig. 14.16

**Solution :**

In  $\Delta PAX$  and  $\Delta QAY$

$$\angle XPA = \angle YQA$$

(Each is  $90^\circ$ )

$$\begin{aligned} & \angle PAX = \angle QAY && \text{(Vertically opposite angles)} \\ \text{and} & \quad \quad \quad PX = QY \\ \therefore & \quad \quad \quad \Delta PAX \cong \Delta QAY && \text{(AAS)} \\ \therefore & \quad \quad \quad AX = AY. \end{aligned}$$

**Example 14.4 :** In Fig. 14.17,  $\Delta ABC$  is a right triangle in which  $\angle B = 90^\circ$  and D is the mid point of AC.

Prove that  $BD = \frac{1}{2} AC$ .

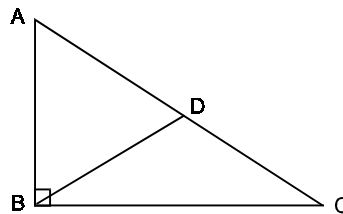


Fig. 14.17

**Solution :** Produce BD to E such that  $BD = DE$

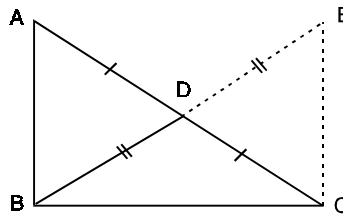


Fig. 14.18

In  $\Delta ADB$  and  $\Delta CDE$ ,

$$\begin{aligned} & AD = CD && \text{(D being mid point of AC)} \\ & DB = DE && \text{(By construction)} \\ \text{and} & \quad \quad \quad \angle ADB = \angle CDE && \text{(Vertically opposite angles)} \\ \therefore & \quad \quad \quad \Delta ADB \cong \Delta CDE && \dots(i) \\ \therefore & \quad \quad \quad AB = EC \\ \text{Also} & \quad \quad \quad \angle DAB = \angle DCE \end{aligned}$$

But they make a pair of alternate angles

$$\begin{aligned} \therefore & \quad \quad \quad AB \text{ is parallel to } EC \\ \therefore & \quad \quad \quad \angle ABC + \angle ECB = 180^\circ && \text{(Pair of interior angles)} \\ \therefore & \quad \quad \quad \angle 90^\circ + \angle ECB = 180^\circ \\ \therefore & \quad \quad \quad \angle ECB = 180^\circ - 90^\circ = 90^\circ \end{aligned}$$

Now in  $\triangle ABC$  and  $\triangle ECB$ ,

$$\begin{aligned} & AB = EC && \text{(From (i) above)} \\ & BC = CB && \text{(Common)} \\ \text{and} & \angle ABC = \angle ECB && \text{(Each } 90^\circ\text{)} \\ \therefore & \triangle ABC \cong \triangle ECB \\ \therefore & AC = EB \\ \text{But} & BD = \frac{1}{2} EB \\ \therefore & BD = \frac{1}{2} AC \end{aligned}$$



### CHECK YOUR PROGRESS 14.1

1. In  $\triangle ABC$  (Fig. 14.19) if  $\angle B = \angle C$  and  $AD \perp BC$ , then  $\triangle ABD \cong \triangle ACD$  by the criterion.

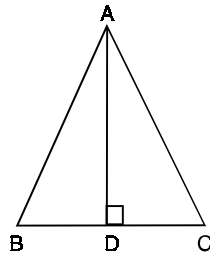


Fig. 14.19

- |         |         |
|---------|---------|
| (a) RHS | (b) ASA |
| (c) SAS | (d) SSS |

2. In Fig. 14.20,  $\triangle ABC \cong \triangle PQR$ . This congruence may also be written as

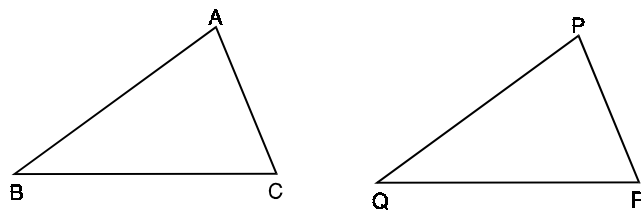


Fig. 14.20

- |   |   |
|---|---|
| (a) $\triangle BAC \cong \triangle RPQ$ | (b) $\triangle BAC \cong \triangle QPR$ |
| (c) $\triangle BAC \cong \triangle RQP$ | (d) $\triangle BAC \cong \triangle PRQ$ |

3. In order that two given triangles are congruent, along with equality of two corresponding angles, we must know the equality of
  - (a) No corresponding side
  - (b) Minimum one corresponding side
  - (c) Minimum two corresponding sides
  - (d) All the three corresponding sides
4. Two triangles are congruent, if
  - (a) All three corresponding angles are equal
  - (b) Two angles and a side of one are equal to two angles and a side of the other.
  - (c) Two angles and a side of one are equal to two angles and the corresponding side of the other
  - (d) One angle and two sides of one are equal to one angle and two sides of the other.
5. In Fig.14.21,  $\angle B = \angle C$  and  $AB = AC$ . Prove that  $\triangle ABE \cong \triangle ACD$ . Hence show that  $CD = BE$ .

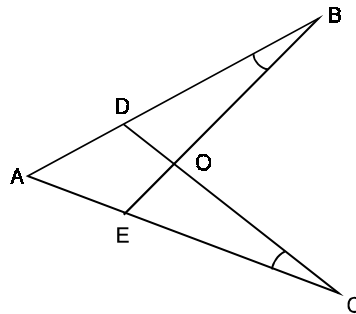


Fig. 14.21

6. In Fig. 14.22, AB is parallel to CD. If O is the mid point of BC, show that it is also the mid point of AD.

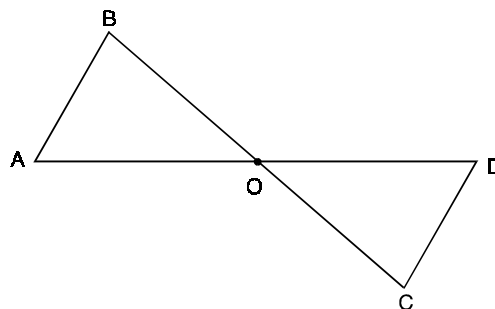


Fig. 14.22

7. In  $\triangle ABC$  (Fig. 14.23),  $AD$  is  $\perp BC$ ,  $BE$  is  $\perp AC$  and  $AD = BE$ . Prove that  $AE = BD$ .

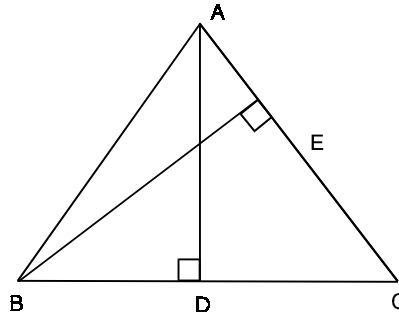


Fig. 14.23

8. From Fig. 14.24, show that the triangles are congruent and make pairs of equal angles.

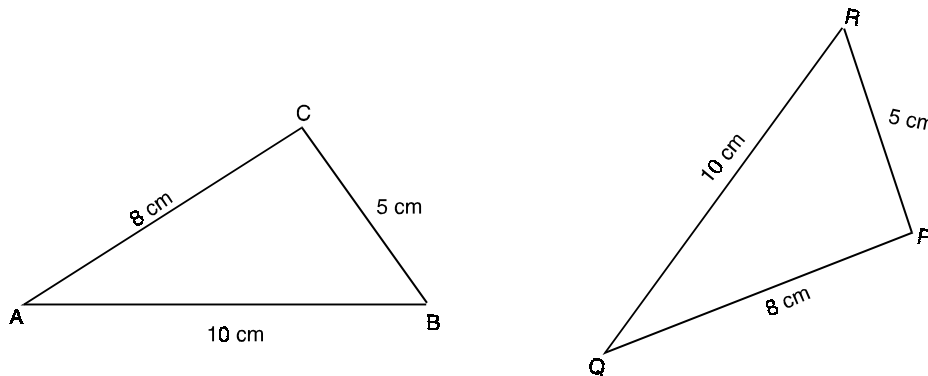


Fig. 14.24

### 14.7 ANGLES OPPOSITE TO EQUAL SIDES OF A TRIANGLE AND VICE VERSA

Using the criteria for congruence of triangles, we shall now prove some important theorems.

**Theorem :** The angles opposite to equal sides of a triangle are equal.

**Given :** A triangle  $ABC$  in which  $AB = AC$ .

**To prove :**  $\angle B = \angle C$ .

**Construction :** Draw bisector of  $\angle BAC$  meeting  $BC$  at  $D$ .

**Proof :** In  $\triangle ABD$  and  $\triangle ACD$ ,

$$\begin{array}{ll} AB = AC & \text{(Given)} \\ \angle BAD = \angle CAD & \text{(By construction)} \\ \text{and } AD = AD & \text{(Common)} \end{array}$$

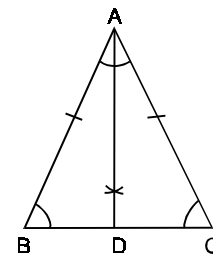


Fig. 14.25

$$\therefore \Delta ABD \cong \Delta ACD \quad (\text{SAS})$$

$$\text{Hence } \angle B = \angle C$$

The converse of the above theorem is also true. We prove it as a theorem.

### 14.7.1 The sides opposite to equal angles of a triangle are equal

**Given :** A triangle ABC in which  $\angle B = \angle C$

**To prove :**  $AB = AC$

**Construction :** Draw bisector of  $\angle BAC$  meeting BC at D.

**Proof :** In  $\Delta ABD$  and  $\Delta ACD$ ,

$$\angle B = \angle C \quad (\text{Given})$$

$$\angle BAD = \angle CAD \quad (\text{By construction})$$

$$\text{and } AD = AD \quad (\text{Common})$$

$$\therefore \Delta ABD \cong \Delta ACD \quad (\text{AAS})$$

$$\text{Hence } AB = AC$$

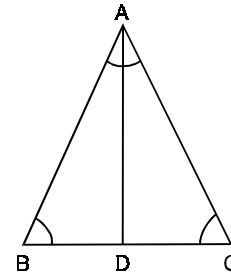


Fig. 14.26

Hence the theorem.

**Example 14.5 :** Prove that the three angles of an equilateral triangle are equal.

**Solution :**

**Given :** An equilateral  $\Delta ABC$

**To prove :**  $\angle A = \angle B = \angle C$

**Proof :**  $AB = AC$  (Given)

$$\therefore \angle C = \angle B \quad (\text{Angles opposite equal sides}) \quad \dots(i)$$

Also  $AC = BC$  (Given)

$$\therefore \angle B = \angle A \quad \dots(ii)$$

From (i) and (ii),

$$\angle A = \angle B = \angle C$$

Hence the result.

**Example 14.6 :** ABC is an isosceles triangle in which  $AB = AC$  (Fig. 14.28). If  $BD \perp AC$  and  $CE \perp AB$ , prove that  $BD = CE$ .

**Solution :** In  $\Delta BDC$  and  $\Delta CEB$

$$\angle BDC = \angle CEB \quad (\text{Each is } 90^\circ)$$

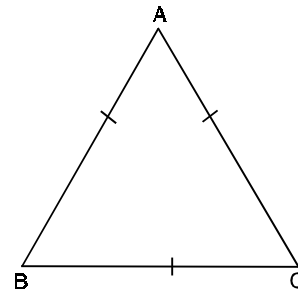


Fig. 14.27

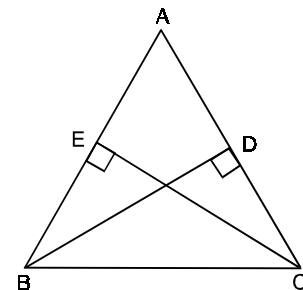


Fig. 14.28

$$\begin{aligned} & \angle DCB = \angle EBC \quad (\text{Angles opposite equal sides of a } \Delta) \\ \text{and} & \quad BC = CB \quad (\text{Common}) \\ \therefore & \quad \Delta BDC \cong \Delta CEB \quad (\text{AAS}) \\ \text{Hence} & \quad BD = CE \end{aligned}$$

This result can be stated in the following manner :

**Perpendiculars drawn to equal sides from opposite vertices (or altitudes) of an isosceles triangle are equal.**

The result can be extended to an equilateral triangle after which we can say that all the three altitudes of an equilateral triangle are equal.

**Example 14.7 :** In  $\Delta ABC$  (Fig. 14.29), D and E are mid points of AC and AB respectively. If  $AB = AC$ , then prove that  $BD = CE$ .

**Solution :**

$$BE = \frac{1}{2} AB$$

and  $CD = \frac{1}{2} AC$

$$\therefore BE = CD \quad \dots(i)$$

In  $\Delta BEC$  and  $\Delta CDB$ ,

$$BE = CD \quad [\text{By (i)}]$$

$$BC = CB \quad (\text{Common})$$

and  $\angle EBC = \angle DCB \quad (\because AB = AC)$

$$\therefore \Delta BEC \cong \Delta CDB$$

Hence  $CE = BD$

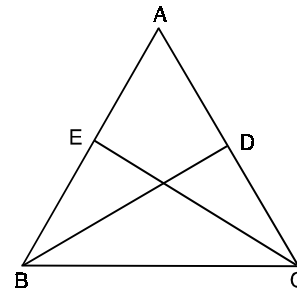


Fig. 14.29

**Example 14.8 :** In  $\Delta ABC$  (Fig. 14.30)  $AB = AC$  and  $\angle DAC = 124^\circ$ ; find the angles of the triangle.

**Solution.**

$$\angle BAC = 180^\circ - 124^\circ = 56^\circ$$

$$\angle B = \angle C$$

(Angles opposite to equal sides of a triangle)

Also  $\angle B + \angle C = 124^\circ$

$$\therefore \angle B = \angle C = \frac{124^\circ}{2} = 62^\circ$$

Hence  $\angle A = 56^\circ$ ,  $\angle B = 62^\circ$ , and  $\angle C = 62^\circ$ ,

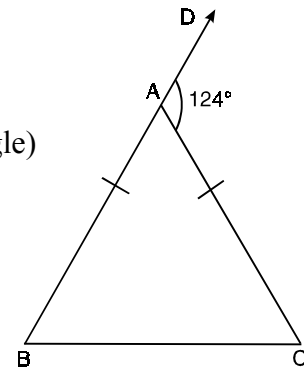


Fig. 14.30



**CHECK YOUR PROGRESS 14.2**

1. In Fig. 14.31,  $PQ = PR$  and  $SQ = SR$ . Prove that  $\angle PQS = \angle PRS$ .

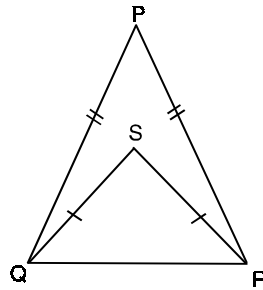


Fig. 14.31

2. Prove that  $\triangle ABC$  is an isosceles triangle, if the altitude  $AD$  bisects the base  $BC$  (Fig. 14.32).

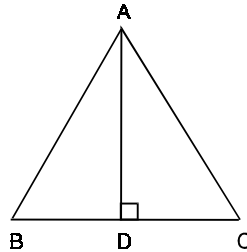


Fig. 14.32

3. If the line  $l$  in Fig. 14.33 is parallel to the base  $BC$  of the isosceles  $\triangle ABC$  find the angles.

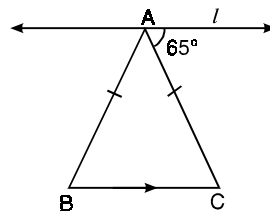


Fig. 14.33

4.  $\triangle ABC$  is an isosceles triangle such that  $AB = AC$ . Side  $BA$  is produced to a point  $D$  such that  $AB = AD$ . Prove that  $\angle BCD$  is a right angle.

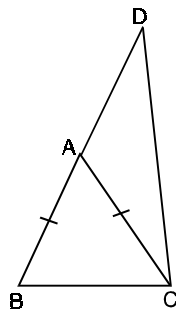


Fig. 14.34

5. In Fig. 14.35, D is the mid point of BC and perpendiculars DF and DE to sides AB and AC are equal in length. Prove that  $\triangle ABC$  is an isosceles triangle.

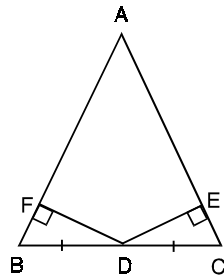


Fig. 14.35

6. In Fig. 14.36,  $PQ = PR$ . QS and RT are the angle bisectors of  $\angle Q = \angle R$  respectively. Prove that  $QS = RT$ .

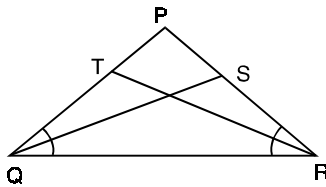


Fig. 14.36

7.  $\triangle PQR$  and  $\triangle SQR$  are isosceles triangles on the same base QR (Fig. 14.37). Prove that  $\angle PQS = \angle PRS$ .

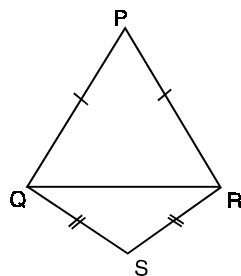


Fig. 14.37

8. In  $\triangle ABC$ ,  $AB = AC$  (Fig. 14.38). P is any point in the interior of the triangle such that  $\angle ABP = \angle ACP$ . Prove that AP bisects  $\angle BAC$ .

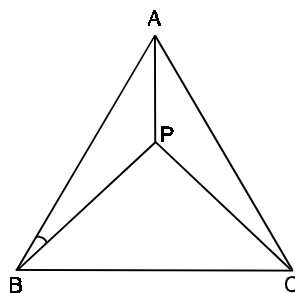


Fig. 14.38

### 14.8 INEQUALITIES IN A TRIANGLE

We have learnt the relationship between sides and angles of a triangle when they are equal. We shall now study some relations among sides and angles of a triangle, when they are unequal.

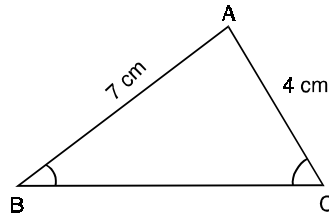


Fig. 14.39

In Fig. 14.39, triangle ABC has side AB longer than the side AC. Measure  $\angle B$  and  $\angle C$ . You will find that these angles are not equal and  $\angle C$  is greater than  $\angle B$ . If you repeat this experiment, you will always find that this observation is true. This can be proved easily, as follows.

#### 14.8.1 Theorem

If two sides of a triangle are unequal, then the longer side has the greater angle opposite to it.

**Given.** A triangle ABC in which  $AB > AC$ .

**To prove.**  $\angle ACB > \angle ABC$

**Construction.** Mark a point D on the side AB such that  $AD = AC$  and join DC.

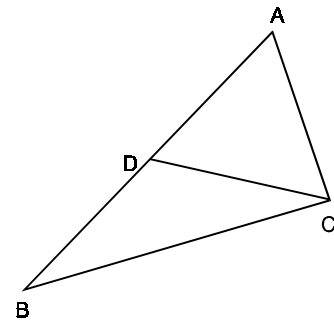


Fig. 14.40

**Proof :** In  $\triangle ADC$ ,

$$AD = AC$$

$$\therefore \angle ACD = \angle ADC \quad (\text{Angles opposite equal sides})$$

$$\text{But } \angle ADC > \angle ABC$$

(Exterior angle is greater than opposite interior angles)

$$\text{Again } \angle ACB > \angle ACD (\text{Point D lies in the interior of the } \angle ACB).$$

$$\therefore \angle ACB > \angle ABC$$

What can we say about the converse of this theorem. Let us examine.

In  $\triangle ABC$ , (Fig. 14.41) compare  $\angle C$  and  $\angle B$ . It is clear that  $\angle C$  is greater than  $\angle B$ . Now compare sides AB and AC opposite to these angles by measuring them. We observe that AB is longer than AC.

Again compare  $\angle C$  and  $\angle A$  and measure sides AB and BC opposite to these angles. We observe that  $\angle C > \angle A$  and  $AB > BC$ ; i.e. side opposite to greater angle is longer.

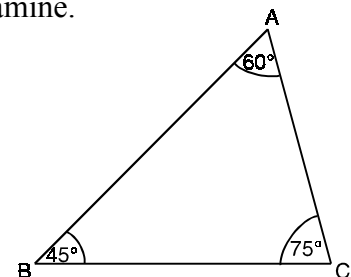


Fig. 14.41

Comparing  $\angle A$  and  $\angle B$ , we observe a similar result.  $\angle A > \angle B$  and  $BC > AC$ ; i.e. side opposite to greater angle is longer.

You can also verify this property by drawing any type of triangle, a right triangle or an obtuse triangle.

Measure any pair of angles in a triangle. Compare them and then compare the sides opposite to them by measurement. You will find the above result always true, which we state as a property.

**In a triangle, the greater angle has longer side opposite to it.**

Observe that in a triangle if one angle is right or an obtuse angle, then the side opposite to that angle is the longest.

You have already learnt the relationship among the three angles of a triangle i.e., the sum of the three angles of a triangle is  $180^\circ$ , we shall now study whether the three sides of a triangle are related in some way.

Draw a triangle ABC.

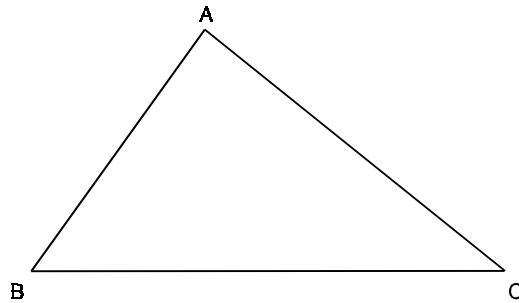


Fig. 14.42

Measure its three sides AB, BC and CA.

Now find the sum of different pairs  $AB + BC$ ,  $BC + CA$ , and  $CA + AB$  separately and compare each sum of a pair with the third side, we observe that

- (i)  $AB + BC > CA$
- (ii)  $BC + CA > AB$  and
- (ii)  $CA + AB > BC$ .

Thus we conclude that

**Sum of any two sides of a triangle is greater than the third side.**

### ACTIVITY FOR YOU

Fix three nails P, Q and R on a wooden board or any surface.

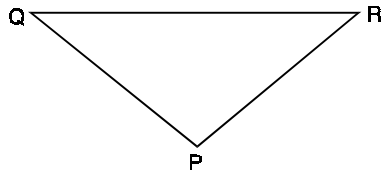


Fig. 14.43

Take a piece of thread equal in length to QR and another piece of thread equal in length QP + PR. Compare the two lengths, you will find that the length corresponding to QP + PR > the length corresponding to QR confirming the above property.

**Example 14.9 :** In which of the following four cases, is construction of a triangle possible from the given measurements :

- (a) 5 cm, 8 cm and 3 cm
- (b) 14 cm, 6 cm and 7 cm
- (c) 3.5 cm, 2.5 cm and 5.2 cm
- (d) 20 cm, 25 cm and 48 cm.

**Solution.** In (a)  $5 + 3 \not> 8$ ,                      in (b)  $6 + 7 \not> 14$   
 in (c)  $3.5 + 2.5 > 5.2$ ,  $3.5 + 5.2 > 2.5$  and  $2.5 + 5.2 > 3.5$  and  
 in (d)  $20 + 25 \not> 48$ .

**Ans.** (c)

**Example 14.10 :** In Fig. 14.44, AD is a median of  $\Delta ABC$ . Prove that  $AB + AC > 2AD$ .

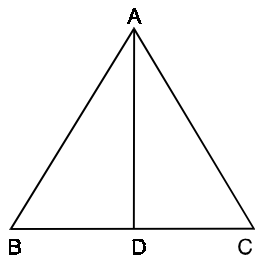


Fig. 14.44

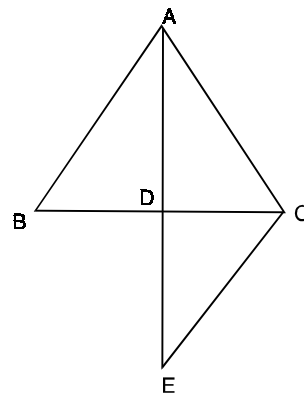


Fig. 14.45

**Solution :** Produce AD to E such that  $AD = DE$  and join C to E.

Consider  $\Delta ABD$  and  $\Delta ECD$

Here,                       $BD = CD$

$$\begin{aligned}\angle ADB &= \angle EDC \\ AD &= ED \\ \therefore \Delta ABD &\cong \Delta ECD \\ \therefore AB &= EC\end{aligned}$$

Now in  $\Delta ACE$ ,

$$\begin{aligned}EC + AC &> AE \\ \text{or } AB + AC &> 2AD \quad (\because AD = ED \Rightarrow AE = 2AD)\end{aligned}$$



### CHECK YOUR PROGRESS 14.3

1. PQRS is a quadrilateral in which diagonals PR and QS intersect at O. Prove that  $PQ + QR + RS + SP > PR + QS$
2. In a triangle ABC,  $AB = 5.7$  cm,  $BC = 6.2$  cm and  $CA = 4.8$  cm. Name the greatest and the smallest angle.
3. In Fig. 14.46, if  $\angle CBD > \angle BCE$  then prove that  $AB > AC$ .

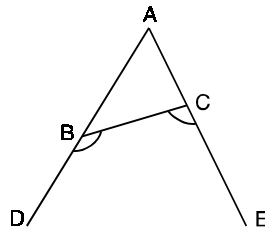


Fig. 14.46

4. In Fig. 14.47, D is any point on the base BC of a  $\Delta ABC$ . If  $AB > AC$  then prove that  $AB > AD$ .

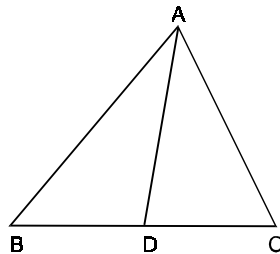


Fig. 14.47

5. Prove that the sum of the three sides of triangle is greater than the sum of its three medians. (Use Example 14.10)

6. In Fig. 14.48, if  $AB = AD$  then prove that  $BC > CD$ .

[Hint :  $\angle ADB = \angle ABD$ ].

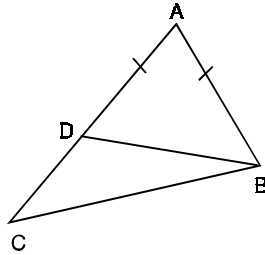


Fig. 14.48

7. In Fig. 14.49,  $AB$  is parallel to  $CD$ . If  $\angle A > \angle B$  then prove that  $BC > AD$ .

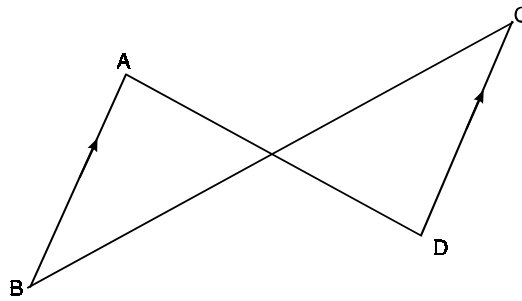


Fig. 14.49

### LET US SUM UP

- Figures which have the same shape and same size are called congruent figures.
- Congruent figures, when placed one over the other completely cover each other. All parts of one figure are equal to the corresponding parts of the other figure.
- To prove that two triangles are congruent we need to know the equality of only three corresponding parts. These corresponding parts must satisfy one of the four criteria
 

(i) SAS.	(ii) ASA or AAS
(iii) SSS	(iv) RHS
- Angles opposite to equal sides of a triangle are equal.
- Sides opposite to equal angles of a triangle are equal.
- If two sides of a triangle are unequal, then the longer side has the greater angle opposite to it.
- In a triangle, the greater angle has the longer side opposite to it.
- Sum of any two sides of a triangle is greater than the third side.

### TERMINAL EXERCISE

1. Two lines AB and CD bisect each other at O. Prove that  $CA = BD$  (Fig. 14.50)
2. In a  $\Delta ABC$ , if the median AD is perpendicular to the base BC then prove that the triangle is an isosceles triangle.

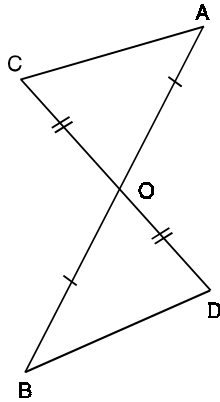


Fig. 14.50

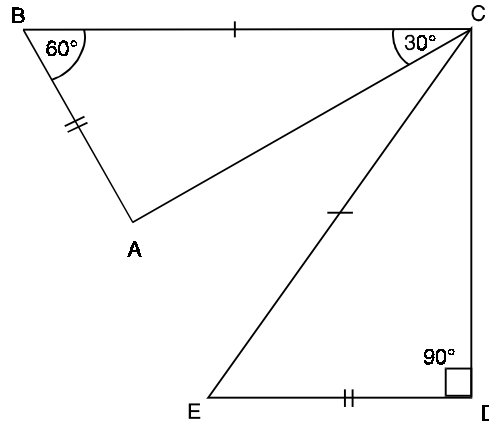


Fig. 14.51

3. In Fig. 14.51,  $\Delta ABC$  and  $\Delta CDE$  are such that  $BC = CE$  and  $AB = DE$ . If  $\angle B = 60^\circ$ ,  $\angle ACE = 30^\circ$  and  $\angle D = 90^\circ$ , then prove that the two triangles are congruent.
4. In Fig. 14.52, two sides AB and BC and the altitude AD of  $\Delta ABC$  are respectively equal to the sides PQ and QR and the altitudes PS. Prove that  $\Delta ABC \cong \Delta PQR$ .

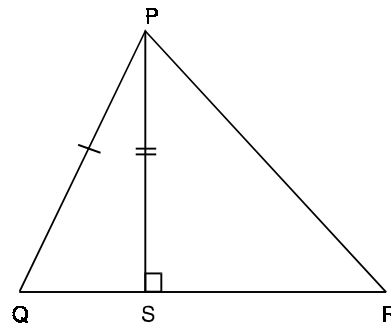
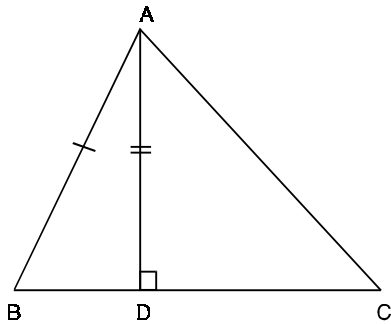


Fig. 14.52

5. In a right triangle, one of the acute angles is  $30^\circ$ . Prove that the hypotenuse is twice the side opposite to the angle of  $30^\circ$ .
6. Line segments AB and CD intersect each other at O such that O is the midpoint of AB. If AC is parallel to DB then prove that O is also the mid point of CD.
7. In Fig. 14.53, AB is the longest side and DC is the shortest side of a quadrilateral ABCD. Prove that  $\angle C > \angle A$  and  $\angle D > \angle B$ .

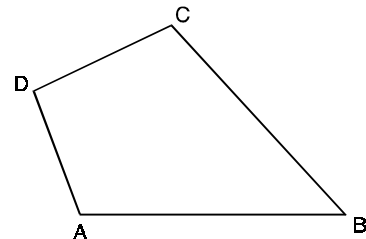


Fig. 14.53

[Hint : Join AC and BD].

8.  $ABC$  is an isosceles triangle in which  $AB = AC$  and  $AD$  is the altitude from  $A$  to the base  $BC$ . Prove that  $BD = DC$ .

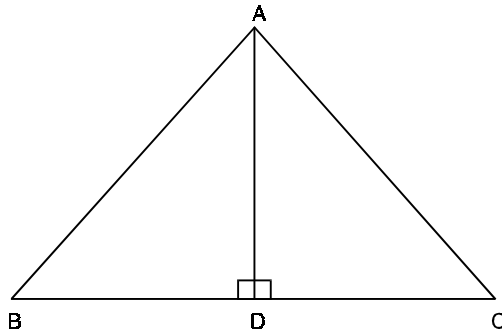


Fig. 14.54

9. Prove that the medians bisecting the equal sides of an isosceles triangle are also equal.  
[Hint : Show that  $\triangle DBC \cong \triangle ECB$ ]

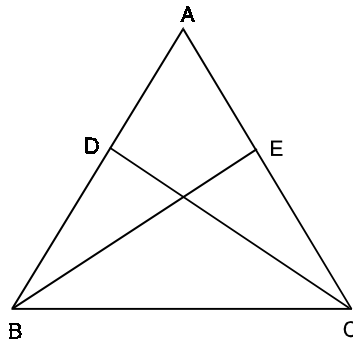


Fig. 14.55

**ANSWERS****Check Your Progress 14.1**

1. (a)
2. (b)
3. (b)
4. (c)
8.  $\angle P = \angle C$   $\angle Q = \angle A$  and  $\angle R = \angle B$ .

**Check Your Progress 14.2**

3.  $\angle B = \angle C = 65^\circ$ ;  $\angle A = 50^\circ$

**Check Your Progress 14.3**

2. Greatest angle is A and smallest angle is B.
-

