

## 7

211en07

## ARITHMETIC PROGRESSIONS

In your daily life you must have observed that in nature, many things follow patterns such as petals of flowers, the holes of a honey-comb, the spirals on a pine apple etc. In this lesson, you will study one special type of number pattern called Arithmetic Progression (AP). You will also learn to find general term and the sum of first $n$ terms of an arithmetic progression.

## OBJECTIVES

After studying this lesson, you will be able to

- identify arithmetic progression from a given list of numbers;
- determine the general term of an arithmetic progression;
- find the sum of first $n$ terms of an arithmetic progression.


## PREVIOUS BACKGROUND KNOWLEDGE

- Knowledge of number system
- Operations on number system


### 7.1 SOME NUMBER PATTERNS

Let us consider some examples:
(i) Rita deposits ₹ 1000 in a bank at the simple interest of $10 \%$ per annum. The amount at the end of first, second, third and fourth years, in rupees will be respectively
$1100,1200,1300,1400$
Do you observe any pattern? You can see that amount increases every year by a fixed amount of ₹ 100 .
(ii) The number of unit squares in a square with sides $1,2,3,4, \ldots$ units are respectively $1,4,9,16, \ldots$.


Can you see any pattern in the list of these numbers? You can observe that

$$
1=1^{2}, 4=2^{2}, 9=3^{2}, 16=4^{2}, \ldots
$$

i.e., these are squares of natural numbers.

Now consider some more lists of numbers and try to recognise a pattern if possible:

$$
\begin{align*}
& 1,3,5,7,9 \ldots . .  \tag{1}\\
& 2,4,6,8,10 \ldots  \tag{2}\\
& 1,4,7,10,13 \ldots  \tag{3}\\
& 5,3,1,-1,-3 \ldots  \tag{4}\\
& 1,3,9,27,81, \ldots  \tag{5}\\
& 2,3,5,7,11,13 \ldots \tag{6}
\end{align*}
$$

You can observe that numbers in the list (1) are odd natural numbers. The first number is 1 , second number is 3 , third number is 5 , etc. All these numbers follow a pattern. The pattern is that all these numbers, except the first is obtained by adding 2 to its previous number.

In lists (2), (3) and (4), each number except the first is obtained by adding 2,3 , and -2 respectively to its previous number.

In (5), each number, except the first is obtained by multiplying 3 to its previous number. In the list (6), you can see that it is the list of prime numbers and it is not possible to give any rule till date, which gives the next prime number.

The numbers in a list are generally denoted by

$$
\begin{aligned}
& \quad \mathrm{a}_{1}, \mathrm{a}_{2}, \mathrm{a}_{3}, \ldots, \mathrm{a}_{\mathrm{n}}, \ldots \\
& \text { or } \\
& \mathrm{t}_{1}, \mathrm{t}_{2}, \mathrm{t}_{3}, \ldots, \mathrm{t}_{\mathrm{n}}, \ldots
\end{aligned}
$$

which are respectively called first, second, third and nth term in the list of numbers. We sometimes call each of these lists as sequence or pattern of numbers.

### 7.2 ARITHMETIC PROGRESSION

You have seen different type of patterns. Some patterns follow definite mathematical rules to generate next term in the pattern. You will now study one particular type of pattern of
numbers.Recall the following patterns.

$$
\begin{align*}
& 1,3,5,7,9, \ldots  \tag{1}\\
& 2,4,6,8,10, \ldots  \tag{2}\\
& 1,4,7,10,13, \ldots \tag{3}
\end{align*}
$$

You have observed that in (1) and (2), each term except first is obtained by adding 2 to its previous number (term). In (3), each term except first is obtained by adding 3 to its previous term. The numbers appearing in a number pattern are called its terms. As already stated these terms are usually denoted by
or $\quad t_{1}, t_{2}, t_{3}, \ldots ., t_{n}, \ldots$ etc
The suffix denotes the position of the term in the pattern. Thus, $\mathrm{a}_{\mathrm{n}}$ or $\mathrm{t}_{\mathrm{n}}$ denotes ' n 'th term of the pattern.

A particular type of pattern in which each term except the first is obtained by adding a fixed number (positive or negative) to the previous term is called an Arithmetic Progression (A.P.). The first term is usually denoted by ' $a$ ' and the common difference is denoted by $d$. Thus, standard form of an Arithmetic Progression would be:

$$
a, a+d, a+2 d, a+3 d, \ldots
$$

Example 7.1: In the following list of numbers, find which are Arithmetic Progressions. In case of AP, find their respective first terms and common differences.
(i) $2,7,12,17,22, \ldots$.
(ii) $4,0,-4,-8,-12 \ldots$
(iii) $3,7,12,18,25 \ldots$
(iv) $2,6,18,54,162 \ldots$

## Solution:

(i) It is an arithmetic progression (AP).

Since $7-2=5,12-7=5,17-12=5$ and $22-17=5$
Thus, each term except first is obtained by adding 5 to its previous term. Hence, first term $\mathrm{a}=2$ and common difference $\mathrm{d}=5$.
(ii) We observe that
$0-4=-4,-4-0=-4,-8-(-4)=-4,-12-(-8)=-4$
Thus, it is an AP with first term a $=4$
and common difference $\mathrm{d}=-4$.

## Arithmetic Progressions

(iii) You can see that in the list
$3,7,12,18,25, \ldots$
$7-3=4,12-7=5,18-12=6,25-18=7$
Thus, difference of two consecutive terms is not the same. Hence, it is not an AP.

(iv) In the list of numbers
$2,6,18,54,162, \ldots$
$6-2=4,18-6=12$
Therefore, difference of two consecutive terms is not the same. Hence, it is not an AP.

## Q. <br> CHECK YOUR PROGRESS 7.1

Which of the following are AP ? If they are in AP , find their first terms and common differences:

1. $-5,-1,3,7,11, \ldots$.
2. $6,7,8,9,10, \ldots$
3. $1,4,6,7,6,4, \ldots$.
4. $-6,-3,0,3,6,9, \ldots$.

### 7.3 GENERAL (nth) TERM OF AN AP

Let us consider an AP whose first term is ' $a$ ' and common difference in ' $d$ '. Let us denote the terms of AP as $t_{1}, t_{2}, t_{3}, \ldots, t_{n}$, where $t_{n}$ denotes the nth term of the AP. Since first term is a , second term is obtained by adding d to a i.e., $\mathrm{a}+\mathrm{d}$, the third term will be obtained by adding ' $d$ ' to $\mathrm{a}+\mathrm{d}$. So, third term will be $(\mathrm{a}+\mathrm{d})+\mathrm{d}=\mathrm{a}+2 \mathrm{~d}$ and so on.

With this

$$
\begin{array}{ll}
\text { First term, } \mathrm{t}_{1}=\mathrm{a} & =a+(1-1) \mathrm{d} \\
\text { Second term, } \mathrm{t}_{2}=\mathrm{a}+\mathrm{d} & =a+(2-1) \mathrm{d} \\
\text { Third term, } \mathrm{t}_{3}=\mathrm{a}+2 \mathrm{~d} & =a+(3-1) \mathrm{d} \\
\text { Fourth term, } \mathrm{t}_{4}=\mathrm{a}+3 \mathrm{~d} & =a+(4-1) \mathrm{d}
\end{array}
$$

Can you see any pattern? We observe that each term is a+(term number -1 ) d. What will be 10th term, say:

$$
\mathrm{t}_{10}=\mathrm{a}+(10-1) \mathrm{d}=\mathrm{a}+9 \mathrm{~d}
$$

Can you now say "what will be the nth term or general term?"
Clearly $\mathrm{t}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$

Example 7.2: Find the 15th and nth terms of the AP

$$
16,11,6,1,-4,-9, \ldots
$$

Solution: $\quad$ Here $\mathrm{a}=16$ and $\mathrm{d}=11-16=-5$
Thus, $\quad \mathrm{t}_{15}=\mathrm{a}+(15-1) \mathrm{d}=\mathrm{a}+14 \mathrm{~d}$

$$
=16+14(-5)=16-70
$$

$$
=-54
$$

Therefore, 15 th term i.e., $\mathrm{t}_{15}=-54$
Now

$$
\begin{aligned}
\mathrm{t}_{\mathrm{n}} & =\mathrm{a}+(\mathrm{n}-1) \mathrm{d} \\
& =16+(\mathrm{n}-1) \times(-5)=16-5 n+5 \\
& =21-5 n
\end{aligned}
$$

Thus, nth term, i.e., $\mathrm{t}_{\mathrm{n}}==21-5 \mathrm{n}$
Example 7.3: The first term of an AP is -3 and 12th term is 41. Determine the common difference.

Solution: Let first term of APbe a and commong difference be d.
Therefore, $\quad \mathrm{t}_{12}=\mathrm{a}+(12-1) \mathrm{d}=41$
or $\quad-3+11 \mathrm{~d}=41 \quad[$ Since $\mathrm{a}=-3]$
or $\quad 11 d=44$
or $\quad d=4$
Therefore, common difference is 4 .
Example 7.4: The common difference of an AP is 5 and 10th term is 43 . Find its first term.

Solution: We have:

$$
\mathrm{t}_{10}=a+(10-1) d
$$

So,

$$
43=a+9 \times 5 \quad[\text { Since } d=5]
$$

or $\quad 43=a+45$
Hence, $\quad a=-2$
Therefore, first term is -2 .
Example 7.5: The first term of an AP is -2 and 11th term is 18 . Find its 15 th term.
Solution: To find 15th term, you need to find d.

$$
\text { Now } \quad \mathrm{t}_{11}=\mathrm{a}+(11-1) \mathrm{d}
$$

So, $\quad 18=-2+10 \mathrm{~d}$
or $\quad 10 \mathrm{~d}=20$
or $\quad \mathrm{d}=2$
Now $\quad t_{15}=a+14 d$

$$
=-2+14 \times 2=26
$$



Therefore, $\mathrm{t}_{15}=26$.
Example 7.6: If p times the pth term of an AP is equal to q times the qth term, prove that its $(p+q)$ th term is zero, provided $p \neq q$.
Solution: We have:

$$
\begin{array}{ll} 
& \mathrm{t}_{\mathrm{p}}=\mathrm{a}+(\mathrm{p}-1) \mathrm{d} \\
& \mathrm{t}_{\mathrm{q}}=\mathrm{a}+(\mathrm{q}-1) \mathrm{d} \\
\text { Since } & \mathrm{pt}_{\mathrm{p}}=\mathrm{qt}_{\mathrm{q}} \text {, therefore, } \\
& \mathrm{p}[\mathrm{a}+(\mathrm{p}-1) \mathrm{d}]=\mathrm{q}[\mathrm{a}+(\mathrm{q}-1) \mathrm{d}] \\
\text { or } & \mathrm{pa}+\mathrm{p}(\mathrm{p}-1) \mathrm{d}-\mathrm{qa}-\mathrm{q}(\mathrm{q}-1) \mathrm{d}=0 \\
\text { or } & (\mathrm{p}-\mathrm{q}) \mathrm{a}+\left(\mathrm{p}^{2}-\mathrm{q}^{2}\right) \mathrm{d}-\mathrm{pd}+\mathrm{qd}=0 \\
\text { or } & (\mathrm{p}-\mathrm{q}) \mathrm{a}+\left(\mathrm{p}^{2}-\mathrm{q}^{2}\right) \mathrm{d}-(\mathrm{p}-\mathrm{q}) \mathrm{d}=0 \\
\text { or } & (\mathrm{p}-\mathrm{q}) \mathrm{a}+(\mathrm{p}-\mathrm{q})(\mathrm{p}+\mathrm{q}) \mathrm{d}-(\mathrm{p}-\mathrm{q}) \mathrm{d}=0 \\
\text { or } & (\mathrm{p}-\mathrm{q})[\mathrm{a}+(\mathrm{p}+\mathrm{q}) \mathrm{d}-\mathrm{d}]=0 \\
\text { or } & \mathrm{a}+(\mathrm{p}+\mathrm{q}) \mathrm{d}-\mathrm{d}=0 \quad[\text { as } p-q \neq 0] \\
\text { or } & \mathrm{a}+(\mathrm{p}+\mathrm{q}-1) \mathrm{d}=0
\end{array}
$$

Since, LHS is nothing but $(\mathrm{p}+\mathrm{q})$ th term, therefore,

$$
\mathrm{t}_{\mathrm{p}+\mathrm{q}}=0
$$

## CHECK YOUR PROGRESS 7.2

1. The first term of an AP is 4 and common difference is -3 , find its 12 th term.
2. The first term of an AP is 2 and 9 th term is 26 , find the common difference.
3. The 12th term of an AP is -28 and 18th term is -46 . Find its first term and common difference.
4. Which term of the AP $5,2,-1, \ldots$. is -22 ?
5. If the $p$ th, $q$ th and $r$ th terms of an AP are $\mathrm{x}, \mathrm{y}$ and z respectively, prove that: $\mathrm{x}(q-\mathrm{r})+\mathrm{y}(\mathrm{r}-p)+\mathrm{z}(p-q)=0$

### 7.4 SUM OF FIRST n TERMS OF AN AP

Carl Friedrich Gauss, the great German mathematician, was in elementary school, when his teacher asked the class to find the sum of first 100 natural numbers. While the rest of the class was struggling with the problem, Gauss found the answer within no time. How Gauss got the answer? Probably, he did as follows:

$$
\begin{equation*}
S=1+2+3+\ldots+99+100 \tag{1}
\end{equation*}
$$

Writing these numbers in reverse order, we get

$$
\begin{equation*}
S=100+99+98+\ldots+2+1 \tag{2}
\end{equation*}
$$

Adding (1) and (2), term by term, we get

$$
\begin{aligned}
2 \mathrm{~S} & =101+101+101+\ldots+101+101(100 \text { times }) \\
& =100 \times 101 \\
\text { or } \mathrm{S} & =\frac{100 \times 101}{2}=5050
\end{aligned}
$$

We shall use the same method to find the sum of first ' $n$ ' terms of an AP.
The first ' $n$ ' terms of an AP are

$$
a, a+d, a+2 d, \ldots, a+(n-2) d, a+(n-1) d
$$

Let us denote the sum of n terms by $\mathrm{S}_{\mathrm{n}}$. Therefore,

$$
\begin{equation*}
S_{n}=a+(a+d)+(a+2 d)+\ldots .+[a+(n-2) d]+[a+(n-1) d] \tag{3}
\end{equation*}
$$

Writing these terms in reverse order, we get

$$
\begin{equation*}
S_{n}=[a+(n-1) d]+[a+(n-2) d]+\ldots+(a+d)+a \tag{4}
\end{equation*}
$$

We now add (3) and (4), term by term. We can see that the sum of any term in (3) and the corresponding term in (4) is $2 a+(n-1) d$. We get

$$
\begin{aligned}
& 2 \mathrm{~S}_{\mathrm{n}}=[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]+[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]+\ldots+[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]+[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}], \mathrm{n} \text { times } \\
& \text { or } 2 \mathrm{~S}_{\mathrm{n}}=n[2 a+(n-1) d] \\
& \text { or } \mathrm{S}_{\mathrm{n}}=\frac{n}{2}[2 a+(n-1) d],
\end{aligned}
$$

which gives general formula for finding the sum of first ' $n$ ' terms of an AP.
This can be rewritten as

$$
\begin{aligned}
\mathrm{S}_{\mathrm{n}} & =\frac{n}{2}[a+\{a+(n-1) d\}] \\
& =\frac{n}{2}\left(a+t_{\mathrm{n}}\right), \quad\left[\text { as } n^{\text {th }} \text { term } t_{\mathrm{n}}=a+(n-1) d\right]
\end{aligned}
$$

Sometimes, $n$th term is named as last term and is denoted by ' $l$ '. Thus:

$$
\begin{equation*}
\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}(\mathrm{a}+l) \tag{4}
\end{equation*}
$$

Example 7.7: Find the sum of the first 12 terms of the following AP
(i) $11,16,21,26 \ldots$
(ii) $-151,-148,-145,-142$

Solution: (i) The given AP is

$$
11,16,21,26 \ldots .
$$

Here, $\quad \mathrm{a}=11, \mathrm{~d}=16-11=5$ and $\mathrm{n}=12$.
You know that sum of first $n$ terms of an AP is given by

$$
S_{n}=\frac{n}{2}[2 a+(n-1) d]
$$

Therefore, $\quad S_{12}=\frac{12}{2}[2 \times 11+(12-1) 5]$

$$
=6[22+55]=6 \times 77=462
$$

Hence, required sum is 462.
(ii) The given AP is

$$
-151,-148,-145,-142
$$

Here, $\quad \mathrm{a}=-151, \mathrm{~d}=-148-(-151)=3$ and $\mathrm{n}=12$.
We know that

$$
\mathrm{S}_{\mathrm{n}}=\frac{n}{2}[2 a+(n-1) d]
$$

Hence, sum of first 12 terms is

$$
\begin{aligned}
\mathrm{S}_{12} & =\frac{12}{2}[2 \times(-151)+(12-1) 3] \\
& =6[-302+33]=6 \times(-269) \\
& =-1614
\end{aligned}
$$

Therefore, required sum is -1614 .

Example 7.8: How may terms of the AP $2,4,6,8,10 \ldots$ are needed to get sum 210 ?
Solution: For the given AP, $\mathrm{a}=2, \mathrm{~d}=2$ and $\mathrm{S}_{\mathrm{n}}=210$.

$$
\begin{aligned}
\text { We have: } & S_{n}=\frac{n}{2}[2 a+(n-1) d] \\
& \\
\text { or } & 210=\frac{n}{2}[2 \times 2+(n-1) 2] \\
\text { or } & 420=n[2 n+2] \\
\text { or } & 420=2 n^{2}+2 n \\
\text { or } & 2 n^{2}+2 n-420=0 \\
\text { or } & n^{2}+n-210=0 \\
\text { or } & n^{2}+15 n-14 n-210=0 \\
\text { or } & n(n+15)-14(n+15)=0 \\
\text { or } & (n+15)(n-14)=0 \\
\text { or } & n=-15 \text { or } n=14
\end{aligned}
$$

Since, $n$ cannot be negative, so, $\mathrm{n}=14$
Therefore, first 14 terms are needed to get the sum 210 .
Example 7.9: Find the following sum

$$
2+5+8+11+\ldots .+59
$$

Solution: Here 2, 5, 8, 11, $\ldots$ are in AP and $\mathrm{a}=2, \mathrm{~d}=3$ and $\mathrm{t}_{\mathrm{n}}=59$.
To find the sum, you need to find the value of $n$.
Now, $\quad t_{\mathrm{n}}=a+(n-1) d$
So, $\quad 59=2+(n-1) 3$
or $\quad 59=3 n-1$
or $\quad 60=3 n$
Therefore, $\quad \mathrm{n}=20$
Now, $\quad \mathrm{S}_{\mathrm{n}}=\frac{n}{2}[2 a+(n-1) d]$
or $\quad \mathrm{S}_{20}=\frac{20}{2}[2 \times 2+(20-1) 3]$

## Arithmetic Progressions

$$
\text { or } \quad S_{20}=10[4+57]=610
$$

Therefore, required sum is 610 .
Example 7.10: Find the sum of all natural numbers between 1 and 1000 which are divisible by 7 .

Solution: Here, the first number which is divisible by 7 is 7 and last number, which is divisible by 7 is 994 . Therefore, the terms to be added are

$$
7,14,21, \ldots ., 994
$$

Here $\quad a=7, d=7, t_{n}=994$
Now $\quad t_{n}=a+(n-1) d$
or $\quad 994=7+(n-1) 7$
or $\quad 994=7 n$
This gives $\mathrm{n}=142$.
Now, $\quad \mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[\mathrm{a}+l]$

$$
\begin{aligned}
& =\frac{142}{2}[7+994]=71 \times 1001 \\
& =71071
\end{aligned}
$$

Therefore, required sum is 71071 .
Example 7.11: The sum of first three terms of an AP is 36 and their product is 1620 . Find the AP.

Solution: We can take three terms of the AP as $\mathrm{a}, \mathrm{a}+\mathrm{d}$ and $\mathrm{a}+2 \mathrm{~d}$. However, the product will be rather difficult and solving the two equations simultaneoulsy will be time consuming. The elegant way is to assume the first three terms as $a-d$, a and $a+d$, so that the sum of three terms becomes 3a.

Let first three terms of the APba-d, a and a + d
Therefore, $\quad a-d+a+a+d=36$
or $3 \mathrm{a}=36$,
which gives $\quad \mathrm{a}=12$
Now, since product is 1620 , we have:

$$
(a-d) a(a+d)=1620
$$

or $\quad(12-d) 12(12+d)=1620$
or $\quad 12^{2}-d^{2}=135$

or

$$
144-d^{2}=135
$$

or $\quad d^{2}=9$
Therefore, $\quad d=3$ or -3
If $d=3$, the numbers are $12-3,12$ and $12+3$
i.e. $9,12,15($ Since $a=12)$

If $d=-3$, the numbers are 15,12 and 9
Therefore, the first three terms of the AP 9,12, 15 and 15, 12, 9
satisfy the given conditions.

## CHECK YOUR PROGRESS 7.3

1. Find the sum of first 15 terms of the following APs:
(i) $11,6,1,-4,-9 \ldots$
(ii) $7,12,17,22,27 \ldots$
2. How many terms of the AP $25,28,31,34, \ldots$. are needed to give the sum 1070 ?
3. Find the following sum:
$1+4+7+10+\ldots .+118$
4. Find the sum of all natural numbers upto 100 which are divisible by 3 .
5. The sum of any three consecutive terms of an AP is 21 and their product is 231 . Find the three terms of the AP.
6. Of the 1, $a, n, d$ and $S_{n}$, determine the ones that are missing for each of the following arithmetic progression
(i) $a=-2, d=5, S_{n}=568$.
(ii) $l=8, n=8, S_{8}=-20$
(iii) $\mathrm{a}=-3030, l=-1530, n=5$
(iv) $d=\frac{2}{3}, l=10, n=20$

## LET US SUM UP

- A progression in which each term, except the first, is obtained by adding a constant to the previous term is called an AP.
- The first term of an AP is denoted by a and common difference by d .


## Arithmetic Progressions

- The ' $n$ 'th term of an AP is given by $t_{n}=a+(n-1) d$.
- The sum of first $n$ terms of an AP is given by $S_{n}=\frac{n}{2}[2 a+(n-1) d]$
- The sum of an AP whose first term is a and last term is $l$ and number of terms is $n$ is
given by $\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}(\mathrm{a}+l)$


## TERMINAL EXERCISE

1. Which of the following patterns are arithmetic progressions?
(i) $2,5,8,12,15, \ldots$.
(ii) $-3,0,3,6,9 \ldots$....
(iii) $1,2,4,8,16, \ldots$.
2. Write the nth term of each of the following arithmetic progressions:
(i) $5,9,13,17, \ldots$.
(ii) $-7,-11,-15,-19$
3. The fourth term of an AP is equal to three times its first term and seventh term exceeds twice the third term by 1 . Find the first term and common difference.
4. The 5th term of an AP is 23 and 12th term is 37 . Find the first term and common difference.
5. The angles of a triangle are in AP. If the smallest angle is one-third the largest angle, find the angles of the triangle.
6. Which term of AP
(i) $100,95,90,85, \ldots$, is -25 ?
(ii) $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4} \ldots .$. is $\frac{25}{4}$ ?
7. The nth term of an $A P$ is given by $t_{n}=a+b n$. Show that it is an AP. Find its first term and common difference.
8. If 7 times the 7 th term of an $A P$ is equal to 11 times the 11 th term, show that the 18th term is zero.
9. Each term of an AP whose first term is a and common difference is d , is doubled. Is the resulting pattern an AP ? If so, find its first term and common difference.
10. If $k+2,4 k-6$ and $3 k-2$ are three consecutive terms of an AP, find $k$.
11. How many terms of the AP:
(i) $1,4,7,10, \ldots$ are needed to get the sum 715 ?
(ii) $-10,-7,-4,-1, \ldots$ are needed to get the sum 104 ?
12. Find the sum of first 100 odd natural numbers.
13. In an $A P, a=2$ and sum of the first five terms is one-fourth the sum of the next five terms. Show that its 20th term is -12 .
[Hint: If AP is $a, a+d, a+2 d, \ldots$, then $S_{5}=\frac{5}{2}[a+(a+4 d)]$
In the next five terms, the first term is $\mathrm{a}+5 \mathrm{~d}$ and last term is $\mathrm{a}+9 \mathrm{~d}$.
14. If sum of first $n$ terms of an AP is $2 n+3 n^{2}$, find rth term of the A.P. [Hint $\left.t_{r}=S_{r}-S_{r-1}\right]$ 15. Find the sum of all 3-digit numbers which leave the remainder 1, when divided by 4 .
[Hint: First term = 101, last term = 997]

7.1
15. $a=-5, d=4$
16. $a=6, d=1$
17. Not an AP
18. $a=-6, d=3$
7.2
19. -29
20. 3
21. $5,-3$
22. $10^{\text {th }}$ term
7.3
23. (i) -360 (ii) 630
24. 20
25. 2380
26. 1689
27. $3,7,11$ or $11,7,3$
28. (i) $n=16, l=73$
(ii) $a=-3, d=3$
(iii) $d=375, S_{n}=-11400$
(iv) $a=-\frac{3}{8}, S_{n}=\frac{220}{3}$
29. (ii)
30. (i) $\mathrm{t}_{\mathrm{n}}=4 \mathrm{n}+1$
(ii) $\mathrm{t}_{\mathrm{n}}=-4 \mathrm{n}-3$
31. 3,2
32. 15,2
33. $30^{\circ}, 60^{\circ}, 90^{\circ}$
34. (i) $26^{\text {th }}$ term (ii) $25^{\text {th }}$ term
35. $a+b, b$
36. Yes, first term $=2 \mathrm{a}$, common difference $=2 \mathrm{~d}$
10.3
37. (i) 22 terms
(ii) 13 terms
38. 10,000
39. $6 \mathrm{r}-1$
40. 123525

MODULE-1

# Secondary Course Mathematics 

## Practice Work-Algebra

Instructions:

1. Answer all the questions on a separate sheet of paper.
2. Give the following informations on your answer sheet

Name
Enrolment number
Subject
Topic of practice work
Address
3. Get your practice work checked by the subject teacher at your study centre so that you get positive feedback about your performance.

Do not send practice work to National Institute of Open Schooling

1. The value of $a$ if $(x-a)$ is a factor of $\mathrm{x}^{6}-\mathrm{ax}^{5}+\mathrm{x}^{4}-\mathrm{ax}^{3}+3 \mathrm{x}-\mathrm{a}+2$, is
(A) $\mathrm{a}=1$
(B) $a=-1$
(C) $\mathrm{a}=2$
(D) $a=-2$
2. The reciprocal of $\frac{1}{(-3 / 5)^{-2}}$ is
(A) $\left(-\frac{3}{5}\right)^{2}$
(B) $\left(\frac{-5}{3}\right)^{2}$
(C) $(-5 / 3)^{-2}$
(D) $\left(\frac{3}{5}\right)^{-2}$
3. In an A.P., the sum of three numbers is 15 and their product is 45 . Then the three numbers are
(A) $1,3,15$
(B) $2,4,9$
(C) $1,5,9$
(D) $0,5,9$
4. If $y=\frac{x-1}{x+1}$, then $2 y-\frac{1}{2 y}$ is equal to
(A) $\frac{3 x^{2}-10 x-3}{2\left(x^{2}-1\right)}$
(B) $\frac{3 x^{2}-10 x+1}{x^{2}-1}$
(C) $\frac{3 x^{2}+10 x+3}{2\left(x^{2}-1\right)}$
(D) $\frac{3 x^{2}-10 x+3}{2\left(x^{2}-1\right)}$
5. The lowest form of the expression $\frac{4 x^{2}-25}{2 x^{2}+11 x=15}$ is
(A) $\frac{2 x-5}{x+3}$
(B) $\frac{2 x+5}{x+3}$
(C) $\frac{2 x-5}{x-3}$
(D) $\frac{2 x-5}{x-3}$
6. Find $x$, so that $\left(\frac{7}{8}\right)^{-3} \times\left(\frac{8}{7}\right)^{-11}=\left(\frac{7}{8}\right)^{x}$ :
7. Find three irrational numbers between $\sqrt{3}$ and $\sqrt{8}$. 2
8. The HCF of two polynomials is $(x-2)$ and their $L C M$ is $x^{4}+2 x^{3}-8 x-16$. If one of the polynomials is $\mathrm{x}^{3}-8$, find the other polynomial.
9. The sum of a number and its reciprocal is $\frac{50}{7}$, find the number.
10. The length of a rectangle is 5 cm less than twice its breadth. If the perimeter is 110 cm , find the area of the rectangle.
11. Show that the sum of an AP whose first term is a , the second term is b and the last term is c , is equal to $\frac{(a+c)(b+c-2 a)}{2(b-a)}$.
12. Had Ajay scored 10 more marks in his test out of 30 marks, 9 times these marks would have been the square of his actual marks. How many marks did he get in the test?

## MODULE 2

## Commercial Mathematics

It is a common saying by elders keep your expenditure, less than your income. The latent meaning of this is to save something for difficult times. You must have seen birds and animals saving eatables for rainy season, in their nests or caves. Taking the lead from this, the students have been told about the importance and need of savings in this module

Many Indian mathematicians have worked on the topic of commercial Mathematics. Yodoksu (370 B.C.) worked on fractions and ratio and proportion. In the reigns of Ashoka and Chandragupta, there is a description of levying taxes. There is a description of many mathematicians working on practice and proportion (like Aryabhatt, Mahavira, Brahmgupta, Sridharacharya). In 900 A.D., Bakhshali Manuscript was discovered which had a number of problems on Commercial mathematics.

To keep your savings safe is another tough task. Banks and other financial institutions keep the money of their customers and on the expiry of the period pay extra money, called interest, in addition to the money deposited. This encourages citizens to save and keep the money safe. This is why calculation of interest on deposits in banks is included for teaching.

The Government provides a number of facilities to the citizens. For that they levy certain taxes on citizens. One of these taxes is sales tax to which the learners are introduced in this "module. Financial transactions about buying and selling are generally done for profit. Due to greater supply of goods or sub-standard goods they are to be sold on loss. The learners are, therefore, introduced to percentage and profit and loss. Sometimes we have to buy articles on instalments because of nonavailability of adequate funds. Due to this the students are taught to calculate interest when they buy articles on instalment plan. Sometimes when we are not able to return loaned money on time, the financer starts charging interest on interest also, which is called compound interest. Due to this the study of compound interest has been included in this module. The formulae of compound interest is also used in finding increase or decrease in prices of things. This is also taught under "Appreciation and Depreciation" of value.

