

# 1

## COMPLEX NUMBERS

### 1.1 INTRODUCTION

We started our study of number systems with the set of natural numbers, then the number zero was included to form the system of whole numbers; negative numbers were defined. Thus, we extended our number system to whole numbers and integers.

To solve the problems of the type  $a \div b$  we included rational numbers and consequently for some other problems we defined irrational numbers. All the numbers taken together are termed as **real numbers**. Also, these numbers can be represented on a number line.

Now let us consider a situation where we want to solve the equation.

$$x^2 + 1 = 0$$

$$\Rightarrow x^2 = -1$$

$$\text{or } x = \pm\sqrt{-1}$$

In real numbers there is no solution to this and many other such problems like the one given below:

Consider the equation

$$x^2 - 2x + 2 = 0$$

$$\Rightarrow (x - 1)^2 + 1 = 0$$

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$$\Rightarrow (x - 1)^2 = -1$$

$$\Rightarrow x - 1 = \pm\sqrt{-1}$$

$$\Rightarrow x = 1 \pm\sqrt{-1}$$

Again we see that in the set of real numbers it has no solution.

So, there is a need to extend the system of real numbers wherein the above equations  $x^2 + 1 = 0$  and  $x^2 - 2x + 2 = 0$  and the like have solutions. This extended system of numbers is called a set of **Complex Numbers**.

## 1.2 OBJECTIVES

After going through this lesson, you will be able to

- describe the need for extending the set of real numbers to the set of complex numbers.
- define complex numbers.
- give examples of complex numbers.
- identify the real and imaginary parts of a given complex number.
- form a complex number whose real and imaginary parts are given.
- understand that all real number belong to complex number.
- define the conjugate of a complex number.
- find the conjugate of a complex number.
- state properties of the conjugate of a complex number.
- define the modulus of a complex number.
- find the modulus of a complex number.
- state properties of the modulus of a complex number.

## 1.3 PREVIOUS KNOWLEDGE

- (a) Properties of real numbers.
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- (b) Solution of linear and quadratic equations.  
 (c) Representation of a real number on the number line.

## 1.4 COMPLEX NUMBERS

In 1748, a great mathematician, L.Euler named a number ' $i$ ' as **Iota** whose square is  $-1$ . i.e.,  $i = \sqrt{-1}$  This Iota or ' $i$ ' is defined as the imaginary unit. With the introduction of the new symbol ' $i$ ' we can interpret the square root of a negative number as a product of a real number with  $i$ .

Thus  $\sqrt{-4} = \sqrt{4(-1)}$  and  $\sqrt{-4} = \sqrt{(-1)(4)} = \sqrt{(i)^2 (2)^2} = i 2$ ,

conventionally written as  $2i$

So we have  $\sqrt{-4} = 2i$

$$\sqrt{-7} = \sqrt{7}i$$

$$\sqrt{-9} = 3i$$

$\sqrt{-4}$ ,  $\sqrt{-7}$ ,  $\sqrt{-9}$  are all examples of what are known as **imaginary numbers**.

Quadratic Equation  $x^2 + 16 = 0$  can be solved as under:

$$x^2 = -16$$

or,  $x = \pm\sqrt{-16}$

or,  $x = \pm 4i$

Also,  $x^2 - 6x + 13 = 0$

or,  $(x - 3)^2 + 4 = 0$

or,  $(x - 3)^2 = -4$

or,  $(x - 3) = \sqrt{-4}$

or,  $(x - 3) = \pm 2i$

or,  $x = 3 \pm 2i$

We get numbers of the form  $x + yi$  where  $x$  and  $y$  are real numbers and  $i = \sqrt{-1}$

Any number which can be written in the form  $a + bi$  where  $a, b$  are real numbers and  $i = \sqrt{-1}$ , is called a **complex number**.

A complex number is, generally, denoted by the letter  $z$ .  
i e.,  $z = a + bi$

### 1.5 PARTS OF A COMPLEX NUMBER

The complex number  $z = a + bi$  has two parts  $a$  and  $bi$ .

' $a$ ' is called the real part of  $z$  and ' $bi$ ' is the imaginary part of  $z$  but for brevity ' $b$ ' is called the imaginary part of  $z$ .

#### Some special cases

**Case 1:** If  $a = 0$  and  $b \neq 0$  then  
 $z = 0 + bi$  or  $bi$  .....(1)

This is a purely imaginary number because the real part is 0.

Following are all purely imaginary numbers:

$$4i, -7i, \frac{1}{2}i, \pi i \text{ and } i$$

**Case 2** If  $a \neq 0$  and  $b = 0$  then  
 $z = a + 0i$  .....(2)  
 $= a$

This is a purely real number because the imaginary part is 0.

Following are all purely real numbers :

$$3, 5, \frac{1}{4}, 2.5,$$

**Case 3:** If  $a = 0$  and  $b = 0$  then  
 $z = 0 + 0i$  or 0

$z$  is 0 because the real and imaginary parts are 0.

So, we represent the complex number  $C$  as

