4

QUADRATIC EQUATIONS

4.1 INTRODUCTION

In earlier classes, we have studied about quadratic equation. Quadratic equations are equations, involving the second power of the variable. For example $3x^2 + 2x + 120$, $x^2 - 4=0$ are all quadratic equations. The general quadratic equation is given by

$$ax^2 + bx + c = 0$$

which we can solve by the formula

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

Here, $D = b^2 - 4ac$ (Discriminent)

Let us consider the quadratic equation $x^2 - 104x + 400=0$

$$\therefore \quad x =$$

$$x = \frac{104 \pm 96}{2}$$

$$\therefore \quad x = \quad = 100 \quad \text{or } x = \quad = 4$$

Now try solving $y^2 + 3y + 5 = 0$

Here, $y = \frac{-3 \pm \sqrt{-11}}{2}$

$\sqrt{-11}$ is not a real number

Having learnt in the previous chapter, we know that $\sqrt{-11} = \sqrt{11}i$ which is a complex number. So, the solution of the equation exists in complex number.

In this lesson, we will learn to solve the quadratic equations in complex number and their applications in finding square roots of complex number and other allied problems.

4.2 OBJECTIVES

After studying this lesson, you will be able to:

- solve quadratic equation with real coefficients
- find cube roots of unity
- find square root of complex numbers
- understand and apply the properties of cube roots of unity

4.3 PREVIOUS KNOWLEDGE

(a) what are quadratic equations

(b) factorization method to solve them

(c) quadratic formula for finding the roots of a quadratic equations.

4.4 METHOD TO SOLVE QUADRATIC EQUATION WHEN D < 0

Let us consider the followng quadratic equation

(a) $x^2 + 3x + 4 = 0$

$\therefore \quad x = \dfrac{-3 \pm \sqrt{9-16}}{2} =$

Since D < 0

So the roots are

,

or $\quad , \quad \dfrac{-3-\sqrt{-7}i}{2}$

So the roots are complex and conjugate

(b) Solve $x^2 - x + 2 = 0$

$$x = \frac{1 \pm \sqrt{-7}}{2}$$

$$x = \frac{1 + \sqrt{-7}}{2}$$

or $\quad x = \dfrac{1+\sqrt{-7}i}{2}$ and $x = \dfrac{1-\sqrt{-7}i}{2}$

(c) Solve $-3x^2 + \sqrt{5}x - 4 = 0$

$$x = \frac{\sqrt{5} \pm \sqrt{5-4(-3)(-4)}}{2(-3)}$$

or $\quad x = \dfrac{\sqrt{5} \pm \sqrt{5-48}}{-6}$

or $\quad x = \dfrac{\sqrt{5} \pm \sqrt{43}}{-6}$

Roots are

$$x = \frac{\sqrt{5} \pm \sqrt{43}i}{-6}, \quad \frac{\sqrt{5} - \sqrt{43}i}{-6}$$

In all the above examples, we can make following **observations**

(i) D < 0 in all cases

(ii) Roots are complex in nature

(iii) Roots are conjugate of ech other

Is it always true that complex roots occur in conjugate pair?

Let us form a quadratic equation whose one root is $2 + 3i$ and other is $4 - 5i$.

The equation will be $\{x - 2 + 3i)\}\ \{x - (4 - 5i)\} = 0$

or $x^2 - (2 + 3i)x - (4 - 5i)x + (2 + 3i)(4 - 5i) = 0$

or $x^2 + (-6 + 2i)x + 23 + 2i = 0,$

But this is a equation in the complex coefficients.

4.4.1 Equations Reducible to Quadratic Forms

Example A:

Find the value of $\sqrt{2+\sqrt{2+\sqrt{2+\ldots\ldots\ldots\ldots\infty}}}$

Solution:

Let $x = \sqrt{2+\sqrt{2+\sqrt{\ldots\ldots\ldots\ldots\infty}}}$

then $x = \sqrt{2+x}$

$\therefore$ $x = \sqrt{2+\sqrt{2+\sqrt{2+\ldots\ldots\ldots\ldots\infty}}}$

(Since the terms are infite. Remember this will give only an approximation).

or $x^2 = 2+x$

or $x^2 - x - 2 = 0$

or $x^2 - 2x + x - 2 = 0$

or $x (x - 2) + 1 (x - 2) = 0$

or $(x + 1) (x - 2) = 0$

$\Rightarrow$ $x + 1 = 0$ or $x - 2 = 0$

$\Rightarrow$ $x = -1$ or $x = 2$

Example B: Find the value of $\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\ldots\ldots\ldots\infty}}}$

Solution:

Let $x = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\ldots\ldots\ldots}}}$

Refer to the above example,

x =

again will give us approximate value

or $x(2 + x) = 1$

or $x^2 + 2x - 1 = 0$

$\Rightarrow$ x =

x = $-1 \pm$

$\therefore$ Roots are

$x = -1 + \sqrt{2}$, $x = -1 - \sqrt{2}$

So, we may note that **if the quadratic equation has two complex roots, which are not conjugate of each other, quadratic equation is equation with complex coefficients.**

Check-point 1:

1. Tick mark the appropriate answer.

(a) If $D < 0$, for a quadratic equation

  (i) Roots are real and equal
  (ii) Roots are real and unequal
  (iii) Roots are complex and conjugate of each other

(b) If $D = 0$, for a quadratic equation

  (i) Roots are not real
  (ii) Roots are always real and equal
  (iii) Roots are always real, but unequal.

{ **Ans: 1 (a) (iii), (b) (ii)** }

INTEXT QUESTIONS 4.1

1. Solve the following quadratic equations

(a) (i) $x^2 + 9x + 10 = 0$

(ii) $x^2 + 5x + 11 = 0$

(iii) $2x^2 + x + 3 = 0$

(b) (i) $x^2 - 2x + 4 = 0$

(ii) $x^2 - 5x + 8 = 0$

(iii) $2x^2 - 3x + 3 = 0$

(c) (i) $-2x^2 + \sqrt{3}\,x - 4 = 0$

(ii) $-x^2 + \sqrt{2}\,x - 1 = 0$

(iii) $-4x^2 + \sqrt{5}\,x - 3 = 0$

(d) (i) $x^2 + x + 1 = 0$

(ii) $3x^2 + \sqrt{2}\,x + 5 = 0$

(iii) $x^2 - 5x + 6 = 0$

2. Find the value of

(a) (i) $\sqrt{8+\sqrt{8+\sqrt{8+\ldots\ldots\ldots\ldots\infty}}}$

(ii) $\sqrt{7+\sqrt{7+\sqrt{7+\ldots\ldots\ldots\ldots\infty}}}$

(ii) $\sqrt{\frac{1}{5}+\sqrt{\frac{1}{5}+\sqrt{\frac{1}{5}+\ldots\ldots\ldots\ldots\infty}}}$

(b) (i) 1

7 + 1

7 + 1

7 + 1

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(iii) $2x^2 - 3x + 3 = 0$

(c) (i) $-2x^2 + \sqrt{3}x - 4 = 0$

(ii) $-x^2 + \sqrt{2}x - 1 = 0$

(iii) $-4x^2 + \sqrt{5}x - 3 = 0$

(d) (i) $x^2 + x + 1 = 0$

(ii) $3x^2 + \sqrt{2}x + 5 = 0$

(iii) $x^2 - 5x + 6 = 0$

2. Find the alue of :

(a) (i) $\sqrt{8+\sqrt{8+\sqrt{8+\ldots\ldots\ldots\infty}}}$

(ii) $\sqrt{7+\sqrt{7+\sqrt{7+\ldots\ldots\ldots\infty}}}$

(iii) $\sqrt{\frac{1}{5}+\sqrt{\frac{1}{5}+\sqrt{\frac{1}{5}+\ldots\ldots\ldots\infty}}}$

(b) (i) $\cfrac{1}{7+\cfrac{1}{7+\cfrac{1}{7+\cfrac{1}{7+\cfrac{1}{7+\ldots\ldots\infty}}}}}$

(ii) $$\cfrac{1}{5+\cfrac{1}{5+\cfrac{1}{5+\cfrac{1}{5+\ldots\ldots\ldots\ldots}}}}$$

(iii) $$\cfrac{1}{10+\cfrac{1}{10+\cfrac{1}{10+\cfrac{1}{10+\ldots\ldots\ldots\ldots}}}}$$

(c) Find the value of

$$x = 2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\ldots\ldots\ldots}}}$$

{ Hint $$x-2 = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\ldots\ldots\ldots}}}$$

or $x - 2 =$ $\frac{1}{\;+(x-2)}$ }

Word Probelms

Example C:

A pilot flies a distance of 1500 km. He saves 30 minutes by increasing the speed by 250 km/hour. Find the actual speed of the aircraft.

Solution:

Let the original speed of the aircraft be x km/hour.

Time taken in travelling 1500 km = $\frac{1500}{x}$ hour.

Increased speed of the aircraft = (x + 250) km/hour.

Time taken now = hour.

By the given condition $\frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60}$

or $\frac{1500(x+250) - 1500x}{x(x+250)} = \frac{1}{2}$

or 375000 = x^2 + 250x

or x^2 + 250x – 375000 = 0

∴ $x = \frac{-250 \pm \sqrt{62500 + 1500000}}{2}$

= –

or x =

Rejecting negative value, we have

or x = = 1000 = 500 km/hour

Example D:

A piece of cloth costs Rs.35. If the piece were 4 metre longer and each metre costs Rs 1, less, the total cost would remain unchanged. How long is the piece ?

Solution:

Let the length of piece be x metres.

Then, rate per metre = Rs.

New rate = Rs.

So $\frac{35}{x} - \frac{35}{x+4} = 1$

$\Rightarrow \quad 35(x + 4) - 35x = x(x + 4)$

$\Rightarrow \quad 35x + 140 - 35x = x^2 + 4x$

or $\quad x^2 + 4x - 140 = 0$

$\Rightarrow \quad (x - 10)(x + 14) = 0$

$\Rightarrow \quad x = 10$ or $x = -14$

But, the length of cloth can never be negative. So, the required length = 10mtrs.

INTEXT QUESTION 4.2

1. The time taken by a man to cover 150 km was 2.5 hours more than the time taken in return journey. If he returned at a speed of 10 km/hour more than the speed of going, what was the speed in each direction?
2. A train travels a distance of 800 `km with some average speed. It could have saved 40 minutes by increasing the average speed of the train by 40 km/hour Find the actual speed of the train?
3. The current of a stream runs at the rate of 2 km/hour A motorboat goes 10 km upstream and back again to the starting point in 55 minutes. Find the speed of the motorboat in still water.
4. Some students planned a picnic. The budget for food was Rs. 480. But eight of them failed to go and thus the cost of food for each member increased by Rs.10. How many students attended the picnic?
5. A number of points are marked on a plane and are connected pairwise by a line segment. If the total number of line segments is 10, how many points are marked on the plane?

4.5 CUBE ROOTS OF UNITY

Having studied the quadratic equation therefore we are now ready to tackle the situation where a given equation of degree 3 or more can be expressed as a product of linear and quadratic equation.

The simplest situation that comes for our consideration is

$x^3 - 1 = 0$ .... (a)

$\therefore$ $x^3 - 1 = (x - 1)(x^2 + x + 1) = 0$

$\Rightarrow$ $x-1 = 0$ or $x^2 + x + 1 = 0$

$\Rightarrow$ $x = 1$ or $x = \frac{-1 \pm \sqrt{3}i}{2}$

$\therefore$ Roots are

$$1, \quad -\frac{1}{2} - \frac{\sqrt{3}i}{2}, \quad -\frac{1}{2} + \frac{\sqrt{3}i}{2},$$

Let us consider another equation

$x^3 + 1 = 0$ .... (b)

$\Rightarrow$ $(x + 1)(x^2 - x + 1) = 0$

$\Rightarrow$ $x + 1 = 0$ or $x^2 - x + 1 = 0$

$\Rightarrow$ $x = -1$ or $x = \frac{1 \pm \sqrt{1-4}}{2}$

Þ $x = -1$ or $x = \frac{1 \pm \sqrt{-3}}{2}$

$\therefore$ Roots are

$$-1, \quad \frac{1 + \sqrt{3}i}{2}, \quad \frac{1 - \sqrt{3}i}{2}$$

Refer back to equation in example (a)

i.e., $x^3 = 1$

has roots $1, \ -\frac{1}{2} - \frac{\sqrt{3}i}{2}, \ -\frac{1}{2} + \frac{\sqrt{3}i}{2},$

are called **cube roots of unity**

Do you see any relationship between two non-real roots of unity obtained above?

We may note that

1. Two complex roots are conjugate of each other.

2. Let $w = -\frac{1}{2}+\frac{\sqrt{3}i}{2}$,

Squaring both sides

$$w^2 = \left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)^2$$

$$= \frac{1}{4}(1+3i^2-2\sqrt{3}i)$$

$$= \frac{1}{4}(1-3-2\sqrt{3}i)$$

$$= \frac{-2-2\sqrt{3}i}{4}$$

$$= \frac{-2\left(1+\sqrt{3}i\right)}{4} = \frac{-\left(1+\sqrt{3}i\right)}{2}$$

$\therefore$ $w^2 = -\frac{1}{2}-\frac{\sqrt{3}i}{2}$, = other complex root

$\therefore$ we denote cube roots of unity as 1, w, w^2

Note

(i) Cube roots of -1 are -1, $-w$, $-w^2$

(ii) In general roots of any cubic equation $x^3 = a^3$ would be a, aw, aw^2

Square of one complex root is the other complex root.

3. Sum of roots are

$$1 + \left(-\frac{1}{2}-\frac{\sqrt{3}i}{2}\right) + \left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)$$

$$= \quad 1-\frac{1}{2}-\frac{1}{2}$$

$$= \quad 1 - 1$$

$$= \quad 0$$

$$\therefore \quad \boxed{1 + w + w^2 = 0}$$

Sum of cube roots of unity is zero.

4. Product of roots is

$$1.w.w^2 = w^3 = 1$$

($\because$ w is root of the quadratic equation)

$$x^2+x+1 = 0$$

Check-point 2:

1. Are all the cube roots of unity square of each other ?

   True or False.

2. Tick mark the appropriate answer :

   (a) Sum of cube roots of unity is

   (i) one

   (ii) zero

   (iii) three

   (b) Product of cube roots of unity is

   (i) zero

   (ii) any real number

   (iii) one

Examples E

If 1, w, w^2 are cube roots of unity, prove that

(a) $w^6 = 1$

Solution

$$w^6 = (w^3)^2 = (1)^2 = 1$$

$$w^6 = 1$$

(b) $1 + w^2 + w^7 = 0$

Solution

Consider $1 + w^2 + w^6 . w$

$= 1 + w^2 + w$ ($w^6 = 1$)

$= 0$

(c) $(1 - w + w^2)(1 + w - w^2) = 4$

Solution:

Consider $(1 - w + w^2)(1 + w - w^2)$

$= (1 + w^2 - w)(1 + w - w^2)$ ($1 + w + w^2 = 0$)

$= (-w - w)(-w^2 - w^2)$ $\Rightarrow$ $1 + w^2 = -w$

$= (-2w)(-2w^2)$ and $1 + w = -w^2$

$= 4w^3$

$= 4$

(d) $(1 + w)^3 - (1 + w^2)^3 = 0$

Solution:

$(1 + w)^3 - (1 + w^2)^3$

$= (-w^2)^3 - (-w)^3$ ($1 + w = -w^2$

$= -w^6 + w^6$ $1 + w^2 = -w$)

$= -(w^3)^2 + 1$

$= -1 + 1$

$= 0$

(e) $(1 - w + w^2)^3 = -8$ and $(1 + w - w^2)^3 = -8$

Solution:

Consider $(1 - w + w^2)^3 = (1 + w^2 - w)^3$

$= (-w - w)^3$

$= (-2w)^3$

$$= -8w^3$$

$$= -8$$

$$(1 + w - w^2)^3 = (-w^2 - w^2)^3$$

$$= (-2w^2)^3$$

$$= -8w^6$$

$$= -8(w^3)^2$$

$$= -8$$

INTEXT QUESTIONS 4.3

1. Solve the following :

(a) (i) $x^3 = 27$

(ii) $x^3 = 64$

(b) (i) $x^3 = -27$

(ii) $x^3 = -64$

2. If 1, w, w² are cube roots of unity show that

(a) (i) $w^9 = 1$ (iii) $w^{18} = 1$

(ii) $x^4 = w$

(b) (i) $1 + w^4 + w^5 = 0$

(ii) $w + w^3 + w^7 = 0$

(iii) $w^9 + w^{10} + w^{11} = 0$

(c) (i) $(1 - w^2 + w^4)(1 + w^4 - w^5) = 4$

(ii) $(1 + w)(1 + w^2)(1 + w^4)(1 + w^8) = 1$

(iii) $(1 - w)(1 - w^2)(1 - w^4)(1 - w^5) = 9$

(d) (i) $(1 - w + w^2)^5 + (1 + w - w^2)^5 = 32$

(ii) $(1 + w)^4 + (1 + w^2)^4 = -1$

(iii) $(1 + w^3)^3 = 8$

(e) (i) $(2 + 5w + 2w^2)^6 = (2 + 2w + 5w^2)^6 = 729$

(ii) $(1 - w + w^2)^6 = (1 + w - w^2)^6 = 64$

(iii) $(1 + w)^{16} - w = (1 - w)^{16} + w = -1$

4.6 SQUARE ROOTS OF A COMPLEX NUMBER

Let us consider a quadratic equation with complex coefficients

$x^2 - 4ix + 1 + 3i = 0$

Solution $x =$

$$= \frac{4i \pm \sqrt{-16-4-13i}}{2}$$

$$= \frac{4i \pm \sqrt{-20-121}}{2}$$

In order to write answer as a complex root we need to find the square root of -20-12i.

Let us now learn to find the square roots of complex numbers.

Example F

$$\frac{i \pm \sqrt{(-4i)^2 - 4(1+3i)}}{2}$$

Find $\sqrt{3+2i}$

Solution:

Let $x + iy = \sqrt{3+2i}$, $x \in R$, $y \in R$

then, $x^2 - y^2 + 2xyi = 3 + 2i$

Equating real and imaginary parts on both sides, we get

$x^2 - y^2 = 3$ .... (i)

$2xy = 2$ .... (ii)

Consider

$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$ (why ?)

$= (x^2 - y^2)^2 + (2xy)^2$

$= (3)^2 + (2)^2$

$= 9 + 4$

$= 13$

$\therefore$ $x^2 + y^2 = \pm\sqrt{13}$

we reject $x^2 + y^2 = -\sqrt{13}$

$\because \quad x \in R, \quad y \in R \quad \Rightarrow \quad x^2 \geq 0, \; y^2 \geq 0$

$\Rightarrow \quad x^2 + y^2 \geq 0$

$\therefore \quad x^2 + y^2 =$ .... (a)

$x^2 - y^2 = 3$ .... (b)

Adding equations (a) and (b), we get

$$2x^2 = \sqrt{13} + 3$$

$$x^2 = \frac{\sqrt{13}+3}{2} \Rightarrow \quad x = \pm\sqrt{\frac{13+3}{2}}$$

Taking difference of the equations (a) & (b), we get

$$2y^2 = \sqrt{13} - 3$$

$$y^2 = \frac{\sqrt{13}-3}{2}$$

or $$y = \pm\sqrt{\frac{13-3}{2}}$$

$\therefore$ square roots of the complex number $3 + 2i$ are

$$\pm\left(\sqrt{\frac{\sqrt{13}+3}{2}} + \sqrt{\frac{\sqrt{13}-3i}{2}}\right)$$

Example G:

Find $\sqrt{5-2i}$

Solution:

Let $\sqrt{5-2i} = x + iy; \quad x \in R, \quad y \in R$

(Note that $\sqrt{5-2i}$ is not taken as $x - iy$ or $\sqrt{5-4i}$ is not taken as $-x - iy$ as x and y are real number.)

$\therefore \quad 5 - 2i = (x + iy)^2$

$5 - 2i = x^2 - y^2 + 2ixy$

Equating the real and imaginary parts on both sides, we get

$x^2 - y^2 = 5$

$2xy = -2$

Now, $x^2 + y^2 = \sqrt{(5)^2 + (-2)^2} = \sqrt{29}$

Now we have

$x^2 + y^2 = \sqrt{29}$ .... (i)

$x^2 - y^2 = 5$ .... (ii)

Add (i) and (ii), we get

$$x^2 = \frac{\sqrt{29}+5}{2}$$

or $x = \pm\sqrt{\dfrac{\sqrt{29}+5}{2}}$

Subtrarct (ii) from (i), we get

$$y = \pm\sqrt{\frac{\sqrt{29}+5}{2}}$$

But $2xy = -2$

$\Rightarrow \quad xy = -1$

Since product of x and y is negative, this implies when x is positive, y is negative and vice-versa.

So, we can write the square root of $5-2i$ as

$$\sqrt{5-2i} = \pm\sqrt{\frac{\sqrt{29}+5}{2}} \mp \sqrt{\frac{\sqrt{29}-5}{2}}\,i$$

Example H:

Find $\sqrt{-3-4i}$

Solution:

Let $\sqrt{-3-4i} = x + iy,\ x \in R,\ y \in R$

Squaring both sides, we get

$$-3 - 4i = (x^2 - y^2) + 2ixy$$

Comparing the real and imaginary part on both sides, we get

$$x^2 - y^2 = -3 \quad \text{..........} \quad \text{(i)}$$

$$2xy = -4 \quad \text{..........} \quad \text{(ii)}$$

$$\therefore \quad x^2 + y^2 = \sqrt{9+16} = \sqrt{25} = 5 \quad \text{..........} \quad \text{(iii)}$$

Solving (i) & (iii) we get

$$x = \pm 1,\ y = \pm 2$$

But $2xy = -4 \Rightarrow$ when $x = 1,\ y = -2$

$x = -1 \quad y = 2$

$$\therefore \quad \sqrt{-3-4i} = \pm 1 \mp 2i$$

or, we can say roots are $1 - 2i$ and $-1 + 2i$

INTEXT QUESTION 4.4

1. Find the square root of the following complex numbers.

(a) (i) $1 + i$
(ii) $2 + 4i$
(iii) $7 + 6i$

(b) (i) $5 - 4i$
(ii) $3 - 7i$
(iii) $9 - 8i$

(c) (i) $-5 - 12i$
(ii) $-1 - 2i$
(iii) $-4 - 8i$

(d) (i)
(ii) $13 - 20\sqrt{3}i$
(iii) i
(iv) $-4 + 5i$

WHAT YOU HAVE LEARNT

- Roots of quadratic equation $ax^2 + bx + c = 0$ are complex and conjugate of each other, when $D<0$

· Cube roots of unity are $1, w, w^2$

where $w = \frac{-1}{2} - \frac{\sqrt{3}i}{2}$

and $w^2 = \frac{-1}{2} + \frac{\sqrt{3}i}{2}$

- Sum of cube roots of unity is zero

  i.e., $1 + w + w^2 = 0$

- Product of cube roots of unity is one.

  $w^3 = 1$

- Complex roots w and w^2 are conjugate of each other
- One complex root is square of other complex root.
- To find the square root of a complex number,

  the formula required is $(x^2 + y^2)^2 = (x^2 - y^2) - (2xy)^2$

TERMINAL QUESTIONS

1. Solve the following quadratic equations:

   (a) $x^2 + 5x + 7 = 0$

   (b) $5x^2 - 8x - 2 = 0$

   (c) $-3x^2 + 4x + 1 = 0$

   (d) $3x^2 - 5x - 3 = 0$

2. Find the value of x, if

$$x = 7 + \cfrac{1}{7 + \cfrac{1}{7 + \cfrac{1}{7 + \cfrac{1}{7 + \ldots\ldots\ldots\infty}}}}$$

3. A plane left 30 minutes later than the scheduled time and in order to reach its destiuation 4500 km away, in

time, it had to increase its speed by 250 km/hour from its usual speed. Find its usual speed.

4. A number of points are marked on a plane and are connected pairwise by a line segment. If the total number of line segments is 10, how many points are marked on the plane.

5 Show that the roots of the equation

$$2(a^2 + b^2)\ x^2 + 2\ (a + b)\ x + 1 = 0$$

are imaginary, when $a \neq b$

6 Show that the roots of the equation

$$bx^2 + (b - c)\ x = c + a - b$$

are always real if those of $ax^2 + b\ (2x + 1) = 0$ are imaginary.

(Hint : since the roots of $ax^2 + b\ (2x + 1) = 0$ are imaginary, its discriminant must be negative.

So, $4b(b - a) < 0$

$\therefore \quad 4b(a - b) > 0$

Discriminant of $bx^2 + (b - c)\ x = c + a - b$ is

$(b + c)^2 + 4b\ (a - b) > 0 \quad (\because\ 4b\ (a - b) > 0)$)

7. Prove that the roots of the equation

$x^2\ (a^2 + b^2) + 2x\ (ac + bd) + (c^2 + d^2) = 0$
are imaginary. But if $ad = bc$, roots are real and equal

8. Find the square root of

(a) $-3i$ (c) $-1-7i$

(b) $2+9i$ (d) $-1+i$

9. If 1, w, w^2 are cube roots of unity, prove that

(a) $(1 - w + w^2)^5 + (1 + w - w^2)^5 = 32$

(b) $(2+ 5w + 2w^2)^6 = (2 + 2w +5w^2)^6 = 729$

(c) $(2 - w)\ (2 - w^2)\ (2 - w^{10})\ (2 - w^{11}) = 49$

(d) $(x - y)\ (xw - y)\ (xw^2 - y) = x^3 - y^3$

10. Prove that $\frac{\left(-1+\sqrt{-3}\right)^{29}}{2}+\frac{\left(-1-\sqrt{-3}\right)^{29}}{2}=-1$

Hint : $\frac{-1+\sqrt{-3}}{2} = \qquad = w$

$\frac{-1-\sqrt{-3}}{2} = w^2$

11. If w is a cube root of unity and n is a positive integer which is not a multiple of 3 then show that

$1 + wn + w^2n = 0$

12. If $x = a + b$, $y = aw + bw^2$, $z = aw^2 + bw$, prove that

(a) $x^2 + y^2 + z^2 = 6ab$

(b) $xyz = a^3 + b^3$

13. Prove that

$\frac{9\pm\sqrt{43}i}{2}$,

$(a + b)^2 + (aw + bw^2)^2 + (aw^2 + bw)^2 = 6ab$

14. Prove that

$(1 - w)(1 - w^2)(1 - w^4)(1 - w^8) = 9$

ANSWERS TO INTEXT QUESTIONS

4.1

1. (a) (i) (ii) $\frac{-5\pm\sqrt{19}i}{2}$ (iii) $\frac{-1\pm\sqrt{23}i}{4}$

(b) (i) $1\pm\sqrt{3}i$ (ii) $\frac{-5\pm\sqrt{7}i}{2}$ (iii) $\frac{3\pm\sqrt{15}i}{4}$

(c) (i) $\frac{\sqrt{3}\pm\sqrt{29}i}{4}$ (ii) $\frac{1\pm i}{\sqrt{2}}$ (iii) $\frac{\sqrt{5}\pm\sqrt{43}i}{8}$

(d) (i) $\frac{-1\pm\sqrt{3}i}{2}$ (ii) $\frac{-\sqrt{2}\pm\sqrt{58}i}{6}$ (iii) 3 or 2

2. (a) (i) $\frac{1+\sqrt{33}}{2}$ or $\frac{1-\sqrt{33}}{2}$ (ii) $\frac{1+\sqrt{29}}{2}$ or $\frac{1-\sqrt{29}}{2}$

(iii) $\frac{5+\sqrt{45}}{10}$ or $\frac{5-\sqrt{45}}{10}$

(b) (i) $\frac{-7+\sqrt{53}}{2}$ or (ii) or

(iii) $-5+$ or $-5-\sqrt{26}$

(c) $1+\sqrt{2}$ or $1-\sqrt{2}$

4.2

1. Speed of going = 20 km/hour

   Speed of coming back = 30 km/hour

2. 200 km/hour

4. 24

4.3

1. (a) (i) $x = 3$, (ii) $x = 4$

   (b) (i) $x = 3$, (ii) $x = -4$

4.4

1. (a) (i) $\pm\sqrt{\frac{\sqrt{2}+1}{2}} \pm \sqrt{\frac{\sqrt{2}-1i}{2}}$

   (ii) $\pm\sqrt{\sqrt{5}+1} \pm \sqrt{\sqrt{5}-1}$

   (iii) $\pm\sqrt{\frac{\sqrt{85}+7}{2}} \pm \sqrt{\frac{\sqrt{85}-7i}{2}}$

   (b) (i) $\pm\sqrt{\frac{\sqrt{41}+5}{2}} \mp$

(ii) $\pm\sqrt{\dfrac{\sqrt{58}+3}{2}}\ \mp$

(iii) $\pm\sqrt{\dfrac{\sqrt{145}+9}{2}}\ \mp$

(c) (i) $2-3i,\ -2+3i$, (ii) $\pm\sqrt{\dfrac{\sqrt{5}-1}{2}}\ \mp$

(iii) $\pm\sqrt{2\left(\sqrt{5}-1\right)}\ \mp$

(d) (i) $\pm\ \sqrt{3}\pm\sqrt{2}i,$ (ii) $5-2\sqrt{3}i,\ \ -5+2\sqrt{3}i$

(iii) $\pm\sqrt{\dfrac{1}{2}}\pm\sqrt{\dfrac{1i}{2}}$ $\pm$ (iv) $\pm\sqrt{\dfrac{\sqrt{41}-4}{2}}\pm\sqrt{\dfrac{\sqrt{41}+4}{2}}$

ANSWERS TO TERMINAL QUESTIONS

1. (a) $\dfrac{-5\pm\sqrt{3}i}{2}$ (b) $\dfrac{4\pm\sqrt{26}}{5}$ (c) $\dfrac{2\pm\sqrt{7}}{3}$ (d) $\dfrac{5\pm\sqrt{61}}{6}$

2. $\dfrac{97\pm\sqrt{197}}{14}$

3. (a) $\pm\sqrt{\dfrac{3}{2}}\mp\sqrt{\dfrac{3i}{2}}$ (b) $\mp$ $\pm\sqrt{\dfrac{\sqrt{85}-2i}{2}}$

(c) $\pm\sqrt{\dfrac{5\sqrt{2}-1}{2}}\ \mp$ (d) $\pm\sqrt{\dfrac{1}{2}}\pm\sqrt{\dfrac{3i}{2}}$