8
INDEX NUMBERS
Meaning and Its Construction

8.1 INTRODUCTION

In the previous lesson you have already learnt about ratios, rates, percentages and arithmetic mean. Some of these concepts are used in the construction of index numbers.

In our daily life we come across remarks like 'prices are rising these days'. It means that the average prices of various commodities that we use in our daily life are rising and we have to pay more for the same goods and services. Index number is a method to find out this average change. When we say that 'prices on an average' are rising, it does not mean that prices of all commodities are rising. Some commodities might have become cheaper also. Index numbers show the average change (increase/decrease) only.

8.2 OBJECTIVES

After going through this lesson, you will be able to :

- define index number;
- define price relative;
- list the various features of an index number;
- explain the various problems faced in the construction of an index number.

8.3 MEANING AND CHARACTERISTICS OF AN INDEX NUMBER

(a) Meaning of an Index Number

An index number is a statistical device (measure) with a purpose of showing average changes in one or more related variables (like price or quantity) between two periods of time (say, between 1991 and 1996) or two places like cities (say, Delhi and Mumbai) or countries (say, India and Japan). For example, we may be interested in knowing which city of India out of Delhi, Mumbai, Calcutta and Chennai is the costliest or the cheapest in terms of price level. A tourist may be interested in knowing about the cost of living at different tourist places. Consumer price index number or cost of living index number helps in taking such decisions.

Different commodities are measured in different units. For example, wheat and rice are measured in kilograms, cloth in metres and milk in litres etc. Index number attempts some averages relating to commodities which are measured in different units.

For purpose of comparison we are interested in knowing relative changes and not absolute or total ones. These relative changes are expressed in percentage terms.

Index numbers are the indicators of the various trends in an economy. Price index numbers indicate the position of prices, whether they are rising or falling and at what rate. Similarly index numbers regarding agricultural production indicate the trend of change whether it is falling or rising and at what rate on an average over a period of time.

Index numbers may be simple or weighted depending on whether we assign equal importance to every commodity or different importance to different commodities according to the percentage of income spent on them or on the basis of some other criteria.

(b) Main Features of an Index Number

The features of an index number are as follows :

1. They are specialized type of averages :

Measures of central tendency like mean can be used to compare two or more series. But here also we face a problem of difference in units of measurement. For example, it makes no sense to say that whereas average height of students of a class is 103 cms, their average weight is 50 kilograms. This is due to the fact that whereas height is measured in terms of centimetres, the weight is measured in kilograms. The method of index number helps to overcome this difficulty. That is why they are called specialized type of averages.

2. Index numbers measure the net change in a group of related variables :

They describe the change (decrease/increase) in a group of related variables in terms of a single figure. For example, in comparing the change in prices of goods consumed by a certain group of people over two periods, say 1991 and 1996, we may construct what is known as consumer price index number. If it is calculated that compared with 1991 (= 100, index number for the year 1991 is taken as 100), the price index number in 1996 is 120, it will show that the price level has increased by (120–100) i.e 20 per cent. This 100 for 1991 and 120 for 1996 are single or summary figures of average of prices in these two years. Note again that even when an index number shows increase, it is possible that within the group some goods might have become cheaper and/or some might have become costlier in comparison to others.

3. Index numbers measure the effect of changes over a period of time or places :

Index numbers are mostly used for measuring changes over a period of time. We may find out the net change in agricultural prices from 1990 to 1997. Similarly, we can compare the agricultural production, industrial production, imports, exports, wages etc. at two different times. Index numbers can also be used to compare economic conditions of different areas (cities or countries) or different industries.

POINTS TO REMEMBER

- An index number is a statistical device (measure) to express average changes in related variables.
- Most widely used index number in practical statistics is price index number.
- A price index number measures relative changes in average prices over time.
- An index number is specialized average. It can be simple/unweighted or weighted.

INTEXT QUESTIONS 8.1

1. State whether the following statements are true or false :
   (i) Index numbers are specialised averages to measure relative changes.
   (ii) Index numbers measure net change in related variables over time.
   (iii) Simple index numbers require weights in their construction.
   (iv) Index numbers do not indicate the trend of change in the economy.

2. Fill in the blanks :
   (i) Index number is a statistical device to express............. change in related variables.
   (ii) An index number is a..................... average.
   (iii) Index number measures net change in group of .... variables.

8.4 CONSTRUCTION OF AN INDEX NUMBER

(a) Concept of Price Relative

The simplest example of an index number is a price relative. It is defined as a ratio of the price of a single commodity in a given period called current period (p_1) to its price in some past period called base or reference period (p_0) i.e.

$$\text{Price Relative (PR)} = \frac{p_1}{p_0}$$

For a better expression, price relative is multiplied by 100 i.e.

$$\text{Price Relative} = \frac{p_1}{p_0} \times 100$$

Let us take two examples to further understand the concept of price relative.

Example 1 :

The price of milk per litre was Rs.10 in 1990. In 1995 it was being sold at Rs. 20 per litre. Taking 1990 as the base year, find the percentage increase in the price of milk.

Solution :

We know that the price relative (PR) = $\frac{p_1}{p_0} \times 100$

In our example base year price (p_0) = Rs. 10.00
and current year price (p_1) = Rs. 20.00

$$PR = \frac{p_1}{p_0} \times 100 = \frac{20}{10} \times 100 = 200$$

This PR = 200 implies that price of milk in the year 1995 is 200 per cent of the price of milk in 1990. Therefore, percentage increase is 200–100 = 100%.

Example 2 :

In 1990 a loaf of bread was being sold at Rs. 2.00. But in 1998 the consumers had to pay Rs. 10.00 for the same bread. By how much the bread had become costlier in 1998?

Solution :

Here base year price = p_0 = Rs. 2.00

Current year price = p_1 = Rs. 10.00

$$\text{Price Relative} = \frac{p_1}{p_0} \times 100 = \frac{10}{2} \times 100 = 500$$

Therefore, the percentage increase in the price of bread equals 500–100 = 400%

(b) Concept of weights

Broadly speaking we have two types of index numbers : (a) Simple (or unweighted) and (b) weighted. Weights represent importance of different items in relation to each other. For example, in a poor man's monthly budget, we notice that he spends proportionately more on food, less on clothing, and still less or nothing on education and medicines. Therefore, food occupies more importance or weight in the family budget of such people. Hence, it is given more weight.

Simple index numbers are unweighted index numbers because no item is given more or less weightage in relation to other items. All items are given the same weightage.

In the case of weighted index numbers, each item is given a different weight according to the importance it occupies in one's family budget. Thus, for example, in percentage terms a poor man might spend 70 to 80 per cent of his income on food and hardly 2 to 3 per cent on recreation or entertainment. As against this, a rich man in absolute terms might spend more on food as compared to a poor man, yet in percentage terms he spends less, 10% or so of his income. Let us now discuss ways of constructing index numbers of each type i.e. weighted or unweighted index numbers.

8.5 CONSTRUCTION OF SIMPLE INDEX NUMBERS

There are two methods of constructing simple index numbers. These are : (i) Simple aggregate method and (ii) Simple average of price relatives method. Remember all items in a simple index number are assigned equal weights. These are unweighted index numbers.

(i) Simple Aggregate Method :

In this case each item is given equal weight. If we give an equal weight to each item it means the same thing, whether each item is given a weight or not. It is the simplest method of constructing an index number. We use the following three steps to find it.

a) Find the sum of current year prices of all items included in the list i.e. Σp_1

b) Find the sum of base year prices of the same items i.e. Σp_0

c) Use the formula $P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$

Example 1 :

Find the price index number, using simple aggregate method, for the following information.

Commodities	Unit	Prices (in Rupees) 1990	1995
Wheat	Quintal	400	520
Rice	Quintal	800	1200
Milk	Litres	10	20
Pulses	Kilograms	20	30
Clothing	Metres	25	40
Meat	Kilograms	40	60

Solution :

Represent prices of 1995 (current year) as p_1 and that of 1990 (base year) as p_0 and reconstruct the table as under.

Commodities	p_0	p_1
Wheat	400	520
Rice	800	1200
Milk	10	20
Pulses	20	30
Clothing	25	40
Meat	40	60
	$\Sigma p_0 = 1295$	$\Sigma p_1 = 1870$

Therefore, the price index of 1995 with base year 1990 is :

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

$$= 1870/1295 \times 100 = 144.4$$

Thus the increase in prices (or price level) is 144.4 – 100 or 44.4 %.

Example 2 :

The prices of three commodities A, B and C increased from Rs. 50, Rs. 7 and Rs. 3 in 1995 to Rs. 52, Rs. 8 and Rs. 5 respectively in 1996. Using the simple aggregate method, find by how much on an average the prices have increased ?

Solution :

Commodities	Price in 1995 (p_0)	Price in 1996 (p_1)
A	50	52
B	7	8
C	3	5
	$\Sigma p_0 = 60$	$\Sigma p_1 = 65$

Therefore, the price index of 1996 with base 1995 is :

$$P_{95,96} = \frac{\Sigma p_1}{\Sigma p_0} \times 100 = \frac{65}{60} \times 100 = \frac{325}{3} = 108.33$$

This shows that the price index for 1996, with base year 1995, is 108.33. So that the price increase in one year is 108.33 – 100 = 8.33 %.

(ii) Simple Average of Price Relatives Method

As explained earlier a price relative is nothing but the ratio of current year prices to those in base year i.e. p_1/p_0. Here the average of the price relatives is obtained by using any of the measures of central tendency. For example, if we use arithmetic mean for averaging, the formula for the index number P_{01} is

$$P_{01} = \frac{\sum\left(\frac{p_1}{p_0}\times 100\right)}{N}$$

Where, N stands for the number of commodities included in the index number.

Steps to calculate index number by Simple Average of Price Relatives Method

a) Find percentage price relative for each commodity

i.e. = $\frac{p_1}{p_0} \times 100$

b) Find the sum of these percentage price relatives

i.e. = $\sum\left(\frac{p_1}{p_0}\times 100\right)$

c) Divide $\sum\left(\frac{p_1}{p_0}\times 100\right)$ by the number of commodities included in the list.

Let us illustrate this method by suitable examples. We take the information given in examples 1 and 2 mentioned above and used to explain simple aggregate method.

Solution of example 1 by Simple Average of Price Relatives Method

S.No.	Commodities	Unit	Price (in Rs.)		Price Relative (PR)
			1990 p_0	1995 p_1	$\frac{p_1}{p_0}\times 100$
1.	Wheat	Quintal	400	520	$\frac{520}{400}\times 100 = 130.00$
2.	Rice	Quintal	800	1200	$\frac{1200}{800}\times 100 = 150.00$

3.	Milk	Litres	10	20	$\frac{20}{10}\times100 = 200.00$
4.	Pulses	Kilograms	20	30	$\frac{30}{20}\times100 = 150.00$
5.	Clothing	Metres	25	40	$\frac{40}{25}\times100 = 160.00$
6.	Meat	Kilograms	40	60	$\frac{60}{40}\times100 = 150.00$
N=6					$\sum \frac{p_1}{p_0}\times100 = 940.00$

Therefore, the price index number of 1995 (with base year 1990) is :

$$P_{01} = \frac{\sum\left(\frac{p_1}{p_0}\times100\right)}{N} = \frac{940}{6} = 156.66$$

Solution of example 2 by Simple Average of Price Relatives Method

S.No.	Commodities	Price (in Rs.)		Price Relative (PR)
		1995 p_0	1996 p_1	$\frac{p_1}{p_0}\times100$
1.	A	50	52	$\frac{52}{50}\times100 = 104.00$
2.	B	7	8	$\frac{8}{7}\times100 = 114.29$
3.	C	3	5	$\frac{5}{3}\times100 = 166.66$
N = 3				$\sum\left(\frac{p_1}{p_0}\times100\right) = 384.95$

Therefore, the price index number of 1996 (with base year 1995) is :

$$P_{01} = \frac{\sum\left(\frac{p_1}{p_0}\times100\right)}{N} = \frac{384.95}{3} = 128.32$$

POINTS TO REMEMBER

- Price relative is defined as the ratio of current year's price to some reference year's price.
- Weights represent relative importance of different items on which an index number is based.
- There are two types of index numbers - simple or unweighted and weighted index numbers.
- Simple index numbers are index numbers with equal weights. Weighted index numbers are with unequal weights.
- There are two methods of constructing simple index numbers (a) Simple aggregate method and (b) Simple average of price relatives method.
- The formula for simple/unweighted index number by
  (i) Simple Aggregate Method is

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

  (ii) Simple Average of Price Relatives method the formula is

$$P_{01} = \frac{\sum\left(\frac{p_1}{p_0} \times 100\right)}{N} \times 100$$

INTEXT QUESTIONS 8.2

1. Fill in the blanks :

   (i) In the index number represented by $P_{1990,1995}$ the base year is__________ and the current year is__________.
   (ii) Simple index number has__________weights.
   (iii) The price relative means ratio of current year price to __________.
   (iv) If the price index number increases from 100 to 300, it shows__________increase in prices.

2. State whether the following statements are true or false :

   (i) A study of family budget shows that a rich family spends proportionately more on food than a poor family does.
   (ii) Simple index numbers are weighted index numbers.
   (iii) We get the same index number whether we use simple aggregate method or simple average of price relative method.

8.6 CONSTRUCTION OF WEIGHTED INDEX NUMBERS

In weighted index number each item is given weight according to the importance it occupies in the list. There are two groups of methods to calculate index number of this category : (a) Weighted aggregate methods and (b) Weighted average of price relative methods. In this section we will study only two types of weighted aggregate method. These methods are popularly known as (a) Laspeyres' Method and (b) Paasche's Method. Both the methods are used to calculate weighted index numbers. The main difference between the two is that Laspeyres' uses base year quantities of commodities as their relative weights, while Paasche's uses current year quantities of commodities as their relative weights for preparing a price index.

(a) Laspeyres' Method :

It uses base year quantities (q_0) as the weights. Accordingly, the formula is,

$$P = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Steps to Calculate Weighted Index Number by Laspeyres' Method

1. Multiply current year price (p_1) with base year quantity (q_0) to get p_1q_0 for each item/ commodity and service.
2. Multiply base year price (p_0) with base year quantity (q_0) to get p_0q_0 for each item/ commodity and service.
3. Add all p_1q_0 and p_0q_0 separately to get Σp_1q_0 and Σp_0q_0 respectively.
4. Divide Σp_1q_0 by Σp_0q_0 and multiply by 100 to obtain Laspeyres' price index number.

Example :

Find Laspeyres' price index number from the following data.

Commodity	Price (in Rs.)		Quantity bought in units	
	1995	1996	1995	1996
A	50	52	10	12
B	7	9	3	4
C	3	5	7	6

Solution :

Represent prices and quantities in current year by p_1 and q_1 respectively and quantities in base year by p_0 and q_0 respectively.

Commodity	p_0	q_0	p_1	p_1q_0	p_0q_0
A	50	10	52	520	500
B	7	3	9	27	21
C	3	7	5	35	21
				$\Sigma p_1q_0 = 582$	$\Sigma p_0q_0=542$

Therefore, the Laspeyres' price index number is :

$$P = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{582}{542} \times 100 = 107.4$$

(b) Paasche's Method

It uses current year quantities (q_1) as the weights. Accordingly the formula is,

$$P = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

Example :

Taking data from the previous example (in Laspeyres' method), we get

Commodity	p_0	q_1	p_1	p_0q_1	p_1q_1
A	50	12	52	600	624
B	7	4	9	28	36
C	3	6	5	18	30
				$\Sigma p_0q_1 = 646$	$\Sigma p_1q_1=690$

Therefore, the Paasche's price index number is :

$$P = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 = \frac{690}{646} \times 100 = 106.8$$

You will notice that the value of Laspeyres' index number (107.4) is different from the value of Paasche's index number (106.8) for the same data. This difference has arisen because of differences in weights.

POINTS TO REMEMBER

- There are two methods of calculating weighted aggregate index numbers: (i) Laspeyres' method and (ii) Paasche's method.
- Laspeyres' uses base year quantities (q_0) as weights and Paasche's uses current year quantities (q_1) as weights.
- The formula in Laspeyres' method is

$$P = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

- The formula in Paasche's method is

$$P = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

INTEXT QUESTIONS 8.3

1. Fill in the blanks with appropriate words out of those given in the brackets :

a) In Laspeyres' method of finding index number we use ________________ quantities as weights (current year, base year).

b) In Paasche's method of finding index number we use ________________ quantities as weights (base year, current year).

c) $P = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$ is ________________ weighted index number.
(Laspeyres, Paasche's)

d) $P = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$ is ________________ weighted index number.
(Laspeyres, Paasche's)

2. State whether the following statements are true or false :

a) In the weighted index number category, the major problem is that of the decision regarding weights.

b) Laspeyres' uses current year quantities as weights.

c) Paasche's uses base year quantities as weights.

ACTIVITY

1. Prepare a 'family budget' of your family for any month under the following heads :

(a) Food items, (b) Clothing, (c) Rent, (d) Transportation and entertainment, (e) Medicines and (f) Education.

Decide the weights on the basis of their consumption.

2. Prepare a list of different formulae you have studied in this lesson to construct a price index number. Which formula will you use to construct a price index number from the following information? Also calculate the same after giving reasons.

Commodities	Unit of Measurement	Price per unit (in Rs.)	
		1990	1995
A	Kilogram	2.00	2.25
B	Metre	4.40	21.50
C	Litre	1.00	6.50
D	Kilogram	1.25	10.00
E	Quintal	75.00	325.00

3. Select 5 households in your locality and record separately expenditure on each of the items wheat, rice, cloth, house rent, transportation, medicines, entertainment and education. For each household find percentage of expenditure on each item. What conclusions can you draw from this study?

4. Construct a suitable index number of prices from the information given below :

Items	Expenditure in 1991 (in Rs.)	Price per Unit (in Rs.)		Unit
		1991	1996	
Wheat	30.00	1.55	3.20	Kilogram
Rice	40.00	4.40	8.50	Kilogram
Cloth	53.75	10.75	21.00	Metre
Pulses	22.40	3.20	6.50	Kilogram
Milk	75.00	2.50	5.50	Litre
Mustard oil	51.00	6.80	16.00	Kilogram
Potatoes	4.50	0.75	2.50	Kilogram

WHAT YOU HAVE LEARNT

- An index number is a statistical measure or device with a purpose of showing average change in one or more related variables over time and space.
- Price index numbers are more commonly used. It measures relative changes in prices over a time period.
- We can have either simple or weighted index number. Simple index number is also called unweighted index number or index number with equal weights.
- In the simple index number category we have two types :

  a) Simple Aggregate method i.e.

  $$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

  b) Simple Average of Price Relatives method i.e.

  $$P_{01} = \frac{\sum \left(\frac{p_1}{p_0} \times 100 \right)}{N} \times 100$$

- Among the weighted indices, in the Laspeyres' method, base year quantities (q_0) are taken as weights. Accordingly the formula is

  $$P = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

- Paasche's method uses current year quantities (q_1) as weights. Accordingly the formula is

  $$P = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

- It is held that Laspeyres' index number gives a different result from that of Paasche's index number.

TERMINAL EXERCISE

1. Explain the meaning of an index number and its main characteristics.
2. What is 'price relative'? Show with the help of an example how it is calculated?
3. Distinguish between simple and weighted index numbers.
4. Explain the method of construction of unweighted index number by Simple Aggregate Method.
5. Explain the method of construction of unweighted index number by Simple Aggregate of Price Relatives Method.
6. Point out the main difference between Laspeyres' and Paasche's methods of weighted index numbers. Explain Laspeyres' method.
7. Explain Paasche's method of weighted index number. Why is the result of this index different from Laspeyres' index?

ANSWERS

Intext Questions 8.1

1. (i) True (ii) True (iii) False (iv) False
2. (i) average (ii) specialised (iii) related

Intext Questions 8.2

1. (i) 1990, 1995 (ii) equal (iii) base year price (iv) 200%
2. (i) False (ii) False (iii) False

Intext Questions 8.3

1. (a) base year (b) current year (c) Laspeyres' (d) Paasche's
2. (a) True (b) False (c) False

Terminal Exercise

1. Read section 8.3
2. Read section 8.4 (a)
3. Read section 8.4 (b)
4. Read section 8.5 (i)
5. Read section 8.5 (ii)
6. Read section 8.6 (a)
7. Read section 8.6 (b)

Guidelines to Activities

1. Get this information from your monthly ration bills etc. Or consult your father or mother or any other family member who manages the household.
2. Since no weights are given, any (simple or unweighted) method out of the two will do.

   Simple aggregate method = 440

   Simple average of price relatives method = 527
3. Conclusions are in accordance with Engle's Law of Family Budget. Rich relatively spend less (percentage) of income on food than the poor etc.
4. We should construct a weighted index number. Here weights are not directly given. They can be obtained for each item by the formula :

   $$\text{Quantity} = \frac{\text{Expenditure}}{\text{Price per unit}}$$

   Since expenditure is given for 1991, the base year, therefore, we can find quantities of the base year

   $$= \frac{\text{Expenditure in the base year}}{\text{Price in the base year}}$$. Use Laspeyres' method

   Further expenditure of base year is p_0q_0. (Ans. 208)