SAMPLE QUESTION PAPER
MATHEMATICS (311)

Time: 3 hrs **Maximum Marks: 100**

Note:

i. This question paper consists of 45 questions in all.
ii. All questions are compulsory.
iii. Marks are given against each question.
iv. **Section A** consists of

a. **Q.No. 1 to 20** – Multiple Choice type questions (MCQs) carrying 1 mark each. Select and write the most appropriate option out of the four options given in each of these questions.
b. **Q.No. 21 to 29** – Objective type questions. **Q.No. 21 to 24** carry 02 marks each (with 2 sub-parts of 1 mark each), **Q.No. 25 to 28** carry 04 marks each (with 4 sub-parts of 1 mark each) and **Q.No. 29** carries 06 marks (with 6 sub-parts of 1 mark each). Attempt these questions as per the instructions given for each of the questions 21 – 29.

v. **Section B** consists of

a. **Q.No. 30 to 38** – Very Short questions carrying 02 marks each.
b. **Q.No. 39 to 43** – Short Answer type questions carrying 04 marks each.
c. **Q.No. 44 to 45** – Long Answer type questions carrying 06 marks each.

(An internal choice has been provided in some of the questions in **Section B**. You have to attempt only one of the given choices in such questions.)

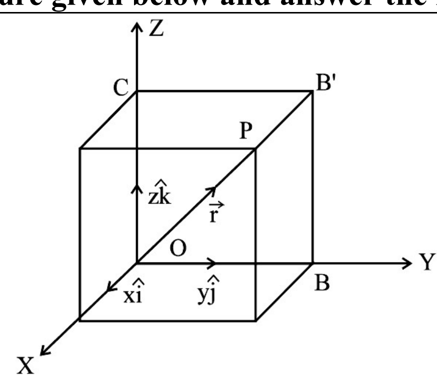
	SECTION A	
Q. No.	Questions	Marks
	<u>Q.No. 1 to 20</u> are the objective questions (MCQs) of 1 mark each:	
1.	The coordinates of the midpoint of A (4, -1) and B(7, 2) are: a) $\left(\frac{3}{2}, \frac{1}{2}\right)$ b) $\left(\frac{3}{2}, \frac{3}{2}\right)$ c) $\left(\frac{11}{2}, \frac{3}{2}\right)$ d) $\left(\frac{11}{2}, \frac{1}{2}\right)$	1
2.	The intercepts made by the line $3x - 2y + 12 = 0$ on the coordinate axis are : a) (4 and -6) b) (-4 and 6) c) (-4 and -6) d) (4 and 6)	1
3.	The centre and radius of the circle $4x^2 + 4y^2 - 2x + 3y - 6 = 0$ are : a) $\left(\frac{1}{4}, \frac{3}{8}\right)$ and $\frac{\sqrt{109}}{8}$ b) $\left(-\frac{1}{4}, -\frac{3}{8}\right)$ and $\frac{\sqrt{109}}{8}$	1

	c) $\left(\frac{1}{4},-\frac{3}{8}\right)$ and $\frac{\sqrt{109}}{8}$ d) $\left(-\frac{1}{4},\frac{3}{8}\right)$ and $\frac{\sqrt{109}}{8}$	
4.	The angles between the lines 2x+3y = 4 and 3x-2y =7 is a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$	1
5.	The coordinates of the centroid of the triangle whose vertices are: $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3) : a) $\left(\frac{x_1+y_1+x_3}{3},\frac{y_1+x_2+x_3}{3}\right)$ b) $\left(\frac{x_1+x_2+x_3}{2},\frac{y_1+y_2+y_3}{2}\right)$ c) $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$ d) $\left(\frac{x_1+x_2+x_3}{2},\frac{y_1+y_2+y_3}{3}\right)$	1
6.	If A = $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$, then B = $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$, then the value of AB is: a) $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$ b) $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$ c) $\begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$ d) $\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	1
7.	If A = $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ then the value of $A+A^T$ is: a) $\begin{pmatrix}2 & 5\\5 & 8\end{pmatrix}$ b) $\begin{pmatrix}2 & 8\\5 & 5\end{pmatrix}$	1

	c) $\begin{pmatrix} 2 & 5 \\ 8 & 5 \end{pmatrix}$ d) $\begin{pmatrix} 2 & -5 \\ 5 & 8 \end{pmatrix}$	
8.	If $\begin{bmatrix} p+q & 2 \\ 5 & q \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 2 \end{bmatrix}$ then the value of p is: a) 6 b) 4 c) 8 d) 5	1
9.	If $f(x) = x^2\ and\ g(x) = 3$, then the value of $fog(x)$ is : a) 12 b) 15 c) 9 d) 18	1
10.	If $f(x) = x + 3\ \ for\ x \in R$, then the value of $f^{-1}(x)$ is : a) $f^{-1}(x) = x - 3$ b) $f^{-1}(x) = x + 3$ c) $f^{-1}(x) = \frac{1}{x+3}$ d) $f^{-1}(x) = \frac{1}{x-3}$	1
11	Which of the following function from Z to itself are bijections? a) $f(x) = x^3$ b) $f(x) = x + 2$ c) $f(x) = 2x + 1$ d) $f(x) = x^3 + 1$	1
12.	If a binary operation * is defined on the set z of integers as a*b = 3a – b, then the value of (2*3)*4 is a) 2 b) 3 c) 4 d) 5	1
13.	If $y = x^n$, then the value of $\frac{dy}{dx}$ is a) nx^{n+1} b) nx^{n-1}	1

	c) $\frac{x^{n+1}}{n+1}$ d) $\frac{x^{n-1}}{n-1}$	
14.	If $siny = xsin(a+y)$, then $\frac{dy}{dx}$ is a) $\frac{sina}{sin^2(a+y)}$ b) $\frac{sin^2(a+y)}{sina}$ c) $sinasin^2(a+y)$ d) $\frac{sina}{sin^2(a-y)}$	1
15.	Which of the following is a vector quality? a) Mass b) Force c) Time d) Length	1
16.	The distance between the points (3,5,–1) and (9,2,–4) is a) 7 units b) 8 units c) $2\sqrt{6}$ units d) $3\sqrt{6}$ units	1
17.	If OACB is a parallelogram with $\overrightarrow{OC} = \vec{a}$ and $\overrightarrow{AB} = \vec{b}$, then $\overrightarrow{OA}$ is a) $(\vec{a}+\vec{b})$ b) $(\vec{a}-\vec{b})$ c) $\frac{1}{2}(\vec{b}-\vec{a})$ d) $\frac{1}{2}(\vec{a}-\vec{b})$	1
18.	If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} + p\hat{j} + 5\hat{k}$ are coplanar. Then the value of p is a) 4 b) – 4 c) 6 d) – 6	1
19.	If the vectors $3\hat{i} + \lambda\hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + 8\hat{k}$ are perpendicular to each other then the value if λ is a) **7**	1

	b) $\frac{1}{7}$ c) – 14 d) 14	
20.	Converse of the statement; "if n is an odd number, then n^2 in also an odd number" is a) If n^2 is not an odd number, then n is also not an odd number b) If n^2 is an odd number, then n is also an odd number c) If n is not an odd number, then n^2 is also not an odd number d) If n is not an odd number, then n^2 is an odd number	1
	<u>Q.No. 21 to 24</u> are the objective questions of 2 marks each:	
21.	**Match column –I statement with the right option of column - II**	1X2
	If matrix A is order of 2X3 and matrix B is order of 3X2,	
	Column –I | **Column - II**	
	(i) order of matrix (AB) is (ii) order of matrix (BA) is | P. 2X3 Q. 2X2 R. 3X2 S. 3X3	
22.	**Fill in the blanks:**	1X2
(i)	If $y = e^x + c$, then the value of $\frac{dy}{dx}$ is ____________	
(ii)	The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^4 - 2\left(\frac{d^2y}{dx^2}\right)^3 - \left(\frac{dy}{dx}\right)^2 + 5 = 0$ is _____	
23.	**Write TRUE for correct statement and FALSE for incorrect statements:**	1X2
(i)	A relation of R on a set A defined as (a,b) ∈ R ⇒ (b,a) ∈ R for all a,b ∈A is a symmetric relation.	
(ii)	A relation *R* is said to be an equivalence relation, if it is reflexive, symmetric but not transitive.	
24.	**Write the negation of each of the following statements:**	1X2
(i)	Sum of 2 and 3 is 6	
(ii)	$a < -7$or $a > 7$	

	Q.No. 25 to 28 are the objective questions of 4 marks each:	
25.	**Fill in the blanks:**	1X4
(i)	A square matrix A is said to be_____________ if | A | = 0.	
(ii)	A square matrix can be expressed as the sum of a symmetric and a ______________ matrix.	
(iii)	If A is a square matrix, then (A′)′ = _________	
(iv)	The number of all possible matrices of order 2 × 2 with each entry 0 or 1 is _____	
26.	**Fill in the blanks:**	1X4
(i)	If $y = sec(3x^5)$, then the value of $\frac{dy}{dx}$ is _______________	
(ii)	If $y = cos^2x$, then the value of $\frac{dy}{dx}$ is __________	
(iii)	The general solutions of the differential equation $\frac{dy}{dx} - 6x = 0$ is given by y = _____	
(iv)	$\frac{d}{dx}(sin^{-1}x) =$ _____	
27.	**Write TRUE for correct statement and FALSE for incorrect statements:**	1X4
(i)	A function f is said to be decreasing on (a,b) if $f'(x) \geq 0$ for each x in (a, b)	
(ii)	If tangent to a curve $y = f(x)$ at $x = x_0$ is parallel to x-axis, then $\frac{dy}{dx}\|_{x=x_o} = 0$	
(iii)	For a function f(x): $\int_a^b f(x)dx = -\int_b^a f(x)dx$	
(iv)	$\int_{-a}^{a} f(x)dx = 0$ if f is an even function.	
28.	**Carefully study the figure given below and answer the following:**	1X4
		

(i)	What is the name commonly given to vectors like vector $\overrightarrow{OP}$?	
(ii)	What is the name given to vectors $\hat{\imath}, \hat{\jmath}$ or $\hat{k}$?	
(iii)	What is/are the term(s) used for collection of $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$?	
(iv)	In given figure if $\lvert\vec{r}\rvert = 10\ m, x = 8\ m$ and $y = 6\ m$, Find the value of z?	
	<u>Q.No. 29</u> is the objective question of 6 marks:	1X6
29.	**Read the passage and answer the questions that follow it. (i to vi)**	
	In calculus, optimization problems involve finding the maximum or minimum values of a function, typically within a specified domain. These problems are commonly encountered in various fields, such as economics, physics, and engineering. One fundamental technique for solving optimization problems is to use derivatives. To find the maximum or minimum of a function $f(x)$, we first locate its critical points by setting $f'(x)=0$ and then analyze the behavior of the function around these points using the second derivative test. If $f''(x)>0$ at a critical point, it indicates a local minimum, whereas if $f''(x)<0$, it signifies a local maximum.	
	Consider a function $f(x) = 2x^3 - 3x^2 - 12x + 8$	
(i)	How many critical points does the function have? (a) 0 (b) 1 (c) 2 (d) 3	
(ii)	Which statement about critical points of the function f(x) is correct? (a) All the critical points of f(x) are positive. (b) All the critical points of f(x) are negative. (c) Some critical points of f(x) are positive while others are negative. (d) None of above as function f(x) does not have any critical points.	
(iii)	At positive value of critical point of f(x), function f(x) has	
	(a) Local maximum (b) Local minimum (c) Neither maximum nor minimum (d) None of above as f(x) does not have positive critical points.	
(iv)	At negative value of critical point of f(x), function f(x) has	
	(a) Local maximum (b) Local minimum (c) Neither maximum nor minimum (d) None of above as f(x) does not have negative critical points.	
(v)	Maximum value of f(x) is (a) 10 (b) 15	

	(c) 20 (d) None of above as f(x) does not have maximum value.	
(vi)	Minimum value of f(x) is (a) –20 (b) – 18 (c) – 12 (d) None of above as f(x) does not have minimum value.	

	SECTION – B	
Question Number	Question	marks
30.	Find the equation of hyperbola with vertices $(\pm 3,0)$ and the foci $(\pm 5,0)$. OR Find the equation of the parabola, whose focus is the point (2, 3) and whose directrix is the line $x - 4y + 3 = 0$.	2
31.	Find the value of the determinant of matrix A = $\begin{bmatrix} -2 & 7 \\ -8 & -6 \end{bmatrix}$.	2
32.	Find the minors of the elements of matrix A = $\begin{bmatrix} -5 & 2 \\ -6 & 8 \end{bmatrix}$. OR If $A = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 & 2 \\ 5 & 1 & 9 \end{bmatrix}$, Find (A+B) and (A-B)	2
33.	Find the value of sec$\left[cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$	2
34.	Find $\lim_{x\to\pi} \frac{\sin x}{\pi - x}$ OR Evaluate $\lim_{x\to 2} \frac{x^5-32}{x^3-8}$.	2

35.	If $y = (x^3 - 3x)\cot x$, find $\frac{dy}{dx}$. OR Differentiate $\frac{e^x}{1+sinx}$	2
36.	If $A = \pi r^2$, find $\frac{dA}{dr}$ for r = 2.	2
37.	Reduce the equation of the plane 4x – 5y + 6z – 120 = 0 to the intercept form. Find its intercepts on the co-ordinate axes.	2
38.	Find the coordinates of the point which divides the line segment joining the points (2, 4, 3) and (–4, 5, –6) internally in the ratio 2 : 1.	2
39.	Find the eccentricity, coordinates of the foci and the length of the axes of the ellipse $3x^2 + 4y^2 = 12$.	4
40.	Using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ OR Without expanding the determinants, prove that $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$	4
41.	Prove that : $\tan^{-1}\frac{27}{11} - \tan^{-1}\frac{3}{5} = \cos^{-1}\frac{4}{5}$	4
42.	Find the value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ OR Evaluate: $\int_0^1 \frac{dx}{\sqrt{2-x^2}}$	4
43.	Find the equation of the plane passing through the points (–1, 2, 3) and (2, –3, 4) and which is perpendicular to the plane 3x + y – z + 5 = 0.	4

44.	In a small scale industry a manufacturer produces two types of book cases. The first type requires 3 hours on machine P and 2 hours on machine Q for completion, whereas second type requires 3 hours on machine P and 3 hours on machine Q. The machine P runs at the most 18 hours while the machine Q for at the most 14 hours per day. There is a profit of Rs. 30 on each book case of the first type and Rs. 40 on each book case of second type. How many book cases of each type to be produced for the maximum profit? OR In a village, farmer has 50 hectare of land to grow two crops A and B. The profit from crops A and B per hectare are estimated as Rs. 10,000 and Rs. 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops A and B at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total profit?	6
45.	A square metal sheet of side 48cm. has four equal squares removed from the corners and the sides are then turned up so as to form an open box. Determine the side of the square cut so that volume of the box is maximum. OR Find the cone of maximum volume that can be inscribed in a sphere of radius R.	6

Mathematics 311

Marking Scheme

SECTION A

Q. No.	Correct Answer	Marks
1.	(d)	1
2.	(b)	1
3.	(c)	1
4.	(a)	1
5.	(b)	1
6.	(c)	1
7.	(a)	1
8.	(b)	1
9.	(c)	1
10.	(a)	1
11	(b)	1
12.	(d)	1
13.	(a)	1
14.	(b)	1
15.	(b)	1
16.	(c)	1
17.	(c)	1
18.	(b)	1
19.	(d)	1
20.	(b)	1
21.	(i)-P, (ii) – S	1 X 2
22.	(i) e^x (ii) 4	1 X 2
23.	(i) T (ii) F	1 X 2
24.	(i) Sum of 2 and 3 is not 6 (ii) a ≥ -7 *and* $a \leq 7$	1 X 2
25.	(i) singular (ii)skew – symmetric (iii) A (iv) 16	1 X 4
26.	(i) $15x^4 sec(3x^5)\ tan(3x^5)$ (ii) 2cosxsinx or sin 2x (iii)$3x^2$ + C (iv)$\frac{1}{\sqrt{1-x^2}}$	1 X 4
27.	(i) F (ii) T (iii) T	1 X 4

	(iv) F	
28.	(i) Position Vector (ii) unit vector (iii) Unit Orthogonal/ mutually perpendicular Vectors / co – initial vectors. (iv) 0	1 X 4
29.	(i) (a) (ii) (c) (iii) (b) (iv) (a) (v) (b) (vi) (c)	1 X 6

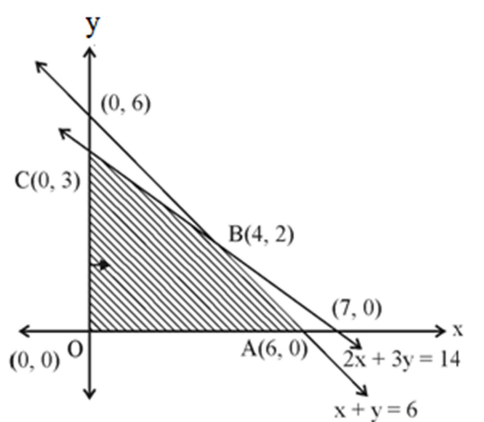
Q No.	**SECTION B** Expected Correct Solution	Step marks	Total Marks
30	Here a = 3 and ae = 5, Hence e= $\frac{5}{3}$ We know that $b^2 = a^2(e^2 - 1)$ $b^2 = 9(\frac{25}{9} - 1) = 16$ ∴ Equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$ OR Given focus is S(2,3); and the equation of the directrix is $x - 4y + 3 = 0$. By definition $SP = PM^2$ or $SP^2 = PM^2$ $(x-2)^2 + (y-3)^2 = \left\{\frac{x - 4y + 3}{\sqrt{1^2+4^2}}\right\}^2$ $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$	1/2 1/2 1/2 ½ OR ½ 1 ½	2
31	We have, A = $\begin{bmatrix} -2 & 7 \\ -8 & -6 \end{bmatrix}$ ∴ $\lvert A\rvert = (-2) \times (-6) - (-8) \times 7$ $= 12 + 56$ $= 68$	1 1	2
32	We have, $\begin{bmatrix} -5 & 2 \\ -6 & 8 \end{bmatrix}$ ∴ $M_{11} = 8$, $M_{12} = -6$ $M_{21} = 2$, $M_{22} = -5$ OR	½X4 = 2	2

	Here both A and B are 2×3 matrices $A + B = \begin{bmatrix} 3+7 & 2+3 & 4+2 \\ 0+5 & 5+1 & 3+9 \end{bmatrix} = \begin{bmatrix} 10 & 5 & 6 \\ 5 & 6 & 12 \end{bmatrix}$ and $A - B = \begin{bmatrix} 3-7 & 2-3 & 4-2 \\ 0-5 & 5-1 & 3-9 \end{bmatrix} = \begin{bmatrix} -4 & -1 & 2 \\ -5 & 4 & -6 \end{bmatrix}$	OR 1 1	
33	Let $cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \theta$ Then $\frac{\sqrt{3}}{2} = cos\ \theta$ $cos\ \theta = \cos\frac{\pi}{6}$ $\theta = \frac{\pi}{6}$ Hence $\sec[cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =]$ $= \sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$	1/2 1/2 1/2 1/2	2
34	$\lim_{x\to\pi}\frac{\sin x}{\pi - x}$ Put π–x=h. h→0 $\therefore \lim_{h\to 0}\frac{\sin(\pi - h)}{\pi - (\pi - h)}$ $= \lim_{h\to 0}\frac{\sin h}{h}$ = 1 OR $\lim_{x\to 2}\frac{x^5 - 32}{x^3 - 8} = \lim_{x\to 2}\left(\frac{x^5 - 32}{x - 2}\right) \div \left(\frac{x^3 - 8}{x - 2}\right)$ $= \lim_{x\to 2}\left(\frac{x^5 - 2^5}{x - 2}\right) \div \lim_{x\to 2}\left(\frac{x^3 - 2^3}{x - 2}\right) = 5(2)^4 \div 3(2)^2 = \frac{20}{3}$ $\left[\text{As } \lim_{x\to a}\frac{x^n - a^n}{x - a} = na^{n-1}\right]$	½ ½ ½ ½ OR 1 1	2
35	We have, $y = (x^3 - 3x)\cot x$ $\therefore \frac{dy}{dx} = (x^3 - 3x)\frac{d}{dx}(\cot x) + \cot x.\frac{d}{dx}(x^3 - 3x)$ $= (x^3 - 3x)(-\text{cosec}^2 x) + \cot x(3x^2 - 3)$ $= 3(x^2 - 1)\cot x - (x^3 - 3x)\text{cosec}^2 x$ OR $\frac{d}{dx}\left(\frac{e^x}{1 + \sin x}\right) = \frac{(1 + \sin x)\frac{d}{dx}(e^x) - e^x\frac{d}{dx}(1 + \sin x)}{(1 + \sin x)^2}$	½ 1 ½ OR 1	2

	$= \frac{(1+\sin x)e^x - e^x \cos x}{(1+\sin x)^2} = \frac{e^x(1+\sin x-\cos x)}{(1+\sin x)^2}$	1	
36	Given A = πr^2 $\therefore \frac{dA}{dr} = \pi.\frac{d}{dr}(r^{2)}$ $= \pi(2r)$ $= 2\pi r$ At r=2, $\frac{dA}{dr} = 2\pi \times 2$ $= 4\pi$	1 1	2
37	Equation of the plane is 4x – 5y + 6z – 120 = 0 or 4x – 5y + 6z = 120 This equation can be written as $\frac{x}{30} - \frac{y}{24} + \frac{z}{20} = 1$ or $\frac{x}{30} + \frac{y}{-24} + \frac{z}{20} = 1$ Intercepts on the co-ordinate axes are 30, –24 and 20 respectively.	1/2 1 1/2	2
38	Let A (2, 4, 3) and B (–4, 5, –6) be the two points. Again let P(x, y, z) divides AB in the ratio 2 : 1. $\therefore x = \frac{2 \times -4 + 1 \times 2}{2+1} = -2$ $y = \frac{2 \times 5 + 1 \times 4}{2+1} = 7$ $z = \frac{2 \times -6 + 1 \times 3}{2+1} = -3$	½ ½ ½ ½	2
39	Given equation $\frac{x^2}{4} + \frac{y^2}{3} = 1$ where $a^2 = 4$ and $b^2 = 3$ $e^2 = 1 - \frac{b^2}{a^2} = ½$ Coordinates of the foci are (1,0) and (-1,0). Length of the axes are 2a and 2b i.e. 4 and $2\sqrt{3}$.	1 1 1 1	4
40	We have A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ A = I A	½	

	$\Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$	1	
	$\Rightarrow \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \; [R_1 \to R_1 - R_2]$	1	
	$\Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A \; [R_2 \to R_2 - R_1]$	1	
	$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \; [R_1 \to R_1 - 2R_2]$		4
	Hence $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$	½	
	OR		
	$\text{L.H.S.} = \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$	OR	
	$C_1 \to C_1+C_2+C_3$		
	$= \begin{vmatrix} 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \end{vmatrix}$	1	
	Take 2 common from C_1		
	$= 2\begin{vmatrix} a+b+c & b+c & c+a \\ a+b+c & c+a & a+b \\ a+b+c & a+b & b+c \end{vmatrix}$	1	
	$C_1 \to C_1 - C_2$		
	$= 2\begin{vmatrix} a & b+c & c+a \\ b & c+a & a+b \\ c & a+b & b+c \end{vmatrix}$	1	
	$C_3 \to C_3 - C_1$		
	$= 2\begin{vmatrix} a & b+c & c \\ b & c+a & a \\ c & a+b & b \end{vmatrix}$	1	
	$C_2 \to C_2 - C_3$		
	$= 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ =R.H.S.		

41	L.H.S. = $\tan^{-1}\left(\frac{27}{11}\right)-\tan^{-1}\left(\frac{3}{5}\right)$ = $\tan^{-1}\left\{\frac{\frac{27}{11}-\frac{3}{5}}{1+\frac{27}{11}\times\frac{3}{5}}\right\}$ = $\tan^{-1}\left(\frac{102}{136}\right)$ = $\tan^{-1}\left(\frac{3}{4}\right)$ = $\cos^{-1}\left(\frac{4}{5}\right)$	$1\frac{1}{2}$ 1 ½ 1	4
42	I = $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}$ dx............(i) = $\int_0^{\pi/2}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}$ dx = $\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}$ dx...........(ii) (i) + (ii) ⇒ 2 I = $\int_0^{\pi/2}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx=\int_0^{\pi/2}1\,dx$ = $\pi/2$ ⇒ I = $\pi/4$ OR $\int\frac{dx}{\sqrt{2-x^2}}=\sin^{-1}\frac{x}{\sqrt{2}}+c$ So $\int_0^1\frac{dx}{\sqrt{2-x^2}}=\sin^{-1}\frac{x}{\sqrt{2}}\Big\vert_0^1=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)-\sin^{-1}(0)$ $=\frac{\pi}{4}-0=\frac{\pi}{4}$	1 ½ ½ 1 1 OR 1	4

		2 1	
43	Equations of any plane passing through the point (–1, 2, 3) is a (x + 1) + b (y – 2) + c (z – 3) = 0-------------(i) since the point (2, –3, 4) lies on the plane (i) so 3a – 5b + c = 0 -------------(ii) Plane (i) is perpendicular to the plane 3x + y – z + 5 = 0 hence 3a + b – c = 0 -------------(iii) By simplification we get $\frac{a}{4} = \frac{b}{6} = \frac{c}{18}$ or $\frac{a}{2} = \frac{b}{3} = \frac{c}{9}$ Required equation of the plane is 2x + 3y + 9z = 31	1/2 1/2 1 1 1	4
44	Let x be the number of book cases of the first type and y that of the second type. Maximize Z = 30x + 40y Subject to the constraints 3x + 3y ≤ 18, 2x + 3y ≤ 14 and x ≥ 0, y ≥ 0  Corner points O(0,0), A(6,0), C(0,14) Z at O (0,0) = 30(0)+40(0) = 0 Z at A (6,0) = 30(6)+40(0) = 180 Z at C (0,14/3) = 30(0) + 40(14/3) = $\frac{560}{3}$ Z at B (4,2) = 30(4) + 40(2) = 200	½ 1½ 2 ½	

	∴ The maximum profit is ` 200-------at the point B (4,2)	1 ½	6

OR

OR

Let x hectare of land be allocated to crop A and y hectare to crop B.

Obviously, $x \geq 0,\ y \geq 0$

Profit per hectare on crop A = Rs. 10000

Profit per hectare on crop B = Rs. 9000

Therefore, total profit = Rs. (10000x + 9000y)

The mathematical formulation of the problem is as follows:

Maximize $Z = 10000x + 9000y$ ½

Subject to the constraints:

$x + y \leq 50$ (which is constraint related to land) ...(i) ½

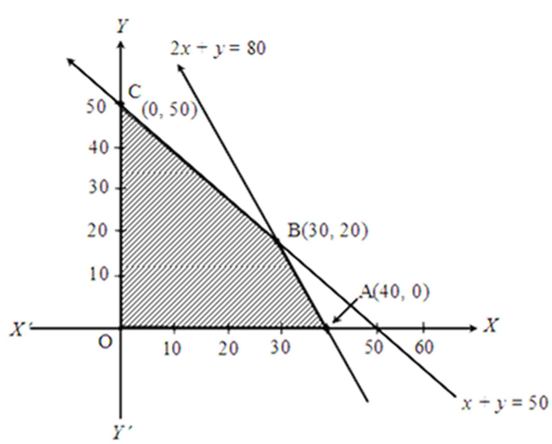
$20x + 10y \leq 800$ (which is constraint related to use of herbicide), ½

i.e., $2x + y \leq 80$ ...(ii) ½

$x \geq 0,\ y \geq 0$ (non negative constraint) ...(iii) ½

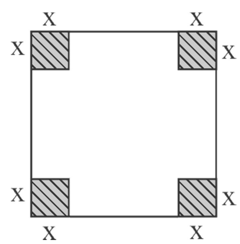
Let us draw the graph of the system of inequalities (i) to (iii). The feasible region OABC is shown (shaded) in the figure.

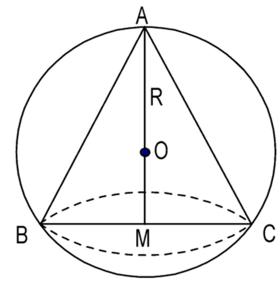
The coordinates of the corner points *O, A, B* and *C* are (0, 0), (40, 0), (30, 20) and (0, 50) respectively. Evaluation of the objective function $Z = 10000x + 9000y$ at these vertices to find which one gives the maximum profit is done as follows:



2

Corner Point	Z = 10000*x* + 9000*y*
0(0, 0)	0
A(40, 0)	400000

	B(30, 20) **480000** ← maximum C(0, 50) 450000 Hence, the farmer will get the maximum profit of Rs. 4,80,000 by allocating 30 hectares for crop *A* and 20 hectares for crop *B*.	1½	
45	Set the side of each small square be x cm.  $\therefore$ Each side of the box to be made is (48–2x) cm, and height x cm. V(volume of the box) = (48–2x) (48–2x) x $\Rightarrow$ $V=(48-2x)^2.x$ Now $\frac{dv}{dx} = (48-2x)^2.1+2(48-2x)(-2).x$ $= (48-2x)(48-6x)$ For maxima or minima, $\frac{dv}{dx}=0$ $\therefore$ $(48-2x)(48-6x)=0$ $\Rightarrow$ x=24 or x=8 $\Rightarrow$ $0<x<24$, $\therefore$ We have x=8 Now $\frac{d^2v}{dx^2}=24x-384$ $\frac{d^2v}{dx^2}$ at x=8 =192–384= –192<0 $\therefore$ V is maximum at x = 8 Hence required side of square to be cut off is 8 cm. OR Let *ABC* be the cone with radius *R* $BM = MC = x$ and height $AM = y$	1 1 1 1 1 1 OR	6

	In ΔOMB, $BM^2 + OM^2 = OB^2$ $x^2 + (y-R)^2 = R^2$ $x^2 = 2Ry - y^2$ Volume (V) of the cone $= \frac{1}{3}\pi x^2 y = \frac{\pi}{3} y\,(2Ry - y^2)$ $= \frac{\pi}{3}(2Ry^2 - y^3)$  	1	
	$\frac{dV}{dy} = \frac{\pi}{3}(4Ry - 3y^2) = \frac{\pi}{3}\,y\,(4R - 3y)$	1	
	$\frac{dV}{dy} = 0 \;\Rightarrow y = \frac{4R}{3}$, ($y$ = 0 is not possible)	1	
	$\frac{d^2V}{dy^2} = \frac{\pi}{3}(4R - 6y)$	1	
	$\frac{d^2V}{dy^2} = \frac{\pi}{3}(4R - 8R) = \frac{-4R\pi}{3}$ i.e. $\frac{d^2V}{dy^2}$ is negative when $y = \frac{4R}{3}$	1	
	$\therefore$ V is maximum when $y = \frac{4R}{3}$		
	Cone has maximum volume when height is $\frac{4R}{3}$ and radius is $\frac{2\sqrt{2}\,R}{3}$.	1	