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## CONGRUENCE OF TRIANGLES

- Two figures, which have the same shape and same size are called congruent figures and this property is called congruence.
- Two line segments are congruent when they are of equal length.
- Two squares are congruent if their sides are equal.
- Two triangles are congruent, if all the sides and all the angles of one are equal to the corresponding sides and angles of other.
For example in triangles PQR and XYZ

$P Q=X Y, P R=X Z, Q R=Y Z$
$\angle \mathrm{P}=\angle \mathrm{X}, \angle \mathrm{Q}=\angle \mathrm{Y}, \angle \mathrm{R}=\angle \mathrm{Z}$
Thus $\triangle \mathrm{PQR}$ is congruent to $\triangle \mathrm{XYZ}$ and we write $\triangle \mathrm{PQR} \cong \triangle \mathrm{XYZ}$ where $\cong$ is symbol of congruence.
- SAS Criterion of congruence: If the two sides and the included angle of one triangle are equal to the corresponding sides and included angle of the other triangle, the two triangles are congruent.
For example:

$\mathrm{AB}=\mathrm{PQ}, \mathrm{BC}=\mathrm{QR}, \angle \mathrm{ABC}=\angle \mathrm{PQR}$
Hence $\triangle \mathrm{ABC} \cong \triangle \mathrm{PQR}$
- ASA or AAS Criterion of Congruence:If any two angles and one side of a triangle are equal to corresponding angles and the side of the another triangle, then the two triangles are congruent.
For example

$\angle \mathrm{ABC}=\angle \mathrm{PQR}, \angle \mathrm{ACB}=\angle \mathrm{PRQ}$ and $\mathrm{BC}=\mathrm{QR}$ Hence $\triangle \mathrm{ABC} \cong \triangle \mathrm{PQR}$
- SSS Criterion of Congruence: If the three sides of one triangle are equal to the corresponding sides of another triangle, then the two triangles are congruent.
For example:

$\mathrm{AB}=\mathrm{PQ}, \mathrm{BC}=\mathrm{QR}, \mathrm{AC}=\mathrm{PR}$, Hence $\triangle \mathrm{ABC} \cong \triangle \mathrm{PQR}$
- RHS Criterion of Congruence: If the hypotenuse and a side of one right triangle are respectively equal to the hypotenuse and a side of another right tringle, then the two triangles are congruent.
For example

$\mathrm{AC}=\mathrm{PR}, \mathrm{AB}=\mathrm{PQ}, \angle \mathrm{ABC}=\angle \mathrm{PQR}=90^{\circ}$
Hence $\triangle \mathrm{ABC} \cong \triangle \mathrm{PQR}$
- The angles opposite to equal sides of a triangle are equal.
- The sides opposite to equal angles of a triangle are equal.
- Perpendiculars or altitudes drawn on equal sides, from opposite vertices of an isosceles triangle are equal.
- If two sides of a triangle are unequal, then the longer side has the greater angle opposite to it.
- In a triangle, the greater angle has longer side opposite to it.
- Sum of any two sides of a triangle is greater than the third side.


## CHECK YOUR PROGRESS:

1. In triangle ABC if $\angle \mathrm{C}>\angle \mathrm{B}$, then :
A. $\mathrm{BC}>\mathrm{AC}$
B. $\mathrm{AB}>\mathrm{AC}$
C. $\mathrm{AB}<\mathrm{AC}$
D. $\mathrm{BC}<\mathrm{AC}$
2. In figure if $\mathrm{AB}=\mathrm{AC}$ and $\mathrm{BD}=\mathrm{DC}$, then $\angle \mathrm{ADB}$ is :

A. $45^{\circ}$
B. $90^{\circ}$
C. $60^{\circ}$
D. None of these
3. Two sides of a triangle are of length 6 cm and 2.5 cm . The length of the third side of the triangle can not be :
A. 4.5 cm
B. 5 cm
C. 6 cm
D. 3.2 cm
4. In $\triangle \mathrm{PQR}, \mathrm{QR}=\mathrm{PQ}$ and $\angle \mathrm{Q}=40^{\circ}$, then $\angle \mathrm{P}$ is equal to -
A. $40^{\circ}$
B. $70^{\circ}$
C. $50^{\circ}$
D. $80^{\circ}$
5. In $\triangle \mathrm{ABC}$, if $\angle \mathrm{B}=\angle \mathrm{C}$ and $\mathrm{AD} \perp \mathrm{BC}$, then $\triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$ by the criterian:

A. RHS
B. ASA
C. SAS
D. SSS
6. $\triangle \mathrm{ABC}$ is a right triangle in which $\angle \mathrm{B}=90^{\circ}$ and $\mathrm{AB}=\mathrm{BC}$. Find $\angle \mathrm{A}$ and $\angle \mathrm{C}$.
7. In figure. Find $\angle \mathrm{DAC}$

8. Prove that angles opposite to equal sides of a triangle are equal.
9. Prove that each angle of an equilateral triangle is $60^{\circ}$.
10. S is any point on side QR of a $\Delta \mathrm{PQR}$. Show that $\mathrm{PQ}+\mathrm{QR}+\mathrm{RP}>2 \mathrm{PS}$


## STRETCH YOURSELF

## ANSWERS:

1. Show that in a quadrilateral ABCD , $A B+B C+C D+D A>A C+B D$.
2. A triangle $A B C$ is right angled at $A$. $A L$ is drawn perpendicular to BC. Prove that $\angle \mathrm{BAL}=\angle \mathrm{ACB}$.
3. Prove that the medians of a triangle are equal.
4. In figure $\angle \mathrm{A}=\angle \mathrm{C}$ and $\mathrm{AB}=\mathrm{AC}$. Prove that $\triangle \mathrm{ABD} \cong \triangle \mathrm{CBE}$.

5. B
6. B
7. D
8. B
9. A
10. $\angle \mathrm{A}=45^{\circ}, \angle \mathrm{B}=45^{\circ}$
11. $\angle \mathrm{DAC}=40^{\circ}$
