2

EXPONENTS AND RADICALS

- **Expoential Notion:** The notation for writing the product of a number by itself several times e.g. $a \times a \times a \times a = a^4$
- **Base and Exponent:** $(a)^n = a \times a \times a....$ n times, a = base, n = exponent
- **Reading of an exponent:** $5 \times 5 \times 5 \times .....$ 20 times = 5^{20} is read as '20th powers of 5' or 5 raised to the power 20.
- **Prime factorisation:** Any natural number other then1, can be expressed as a product of power of prime numbers.
- **Laws of exponents:** $a^m \times a^n = a^{m+n}$ a $a \neq 0$

  $a^m \div n^a = a^{m-n}$ (if $m > n$),

  $a^m \div a^n = \frac{1}{a^{n-m}}$ (if $m < n$)

  $(a^m)^n = a^{mn}$, $a \neq 0$

  $a^0 = 1$, $a \neq 0$

  $(a \times b)^m = a^m \times b^m$, $a \neq 0, b \neq 0$

  $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $a \neq 0, b \neq 0$
- **Negative integers as exponents :** a is non-zero rational number and m is any positive integer, then the reciprocal of a^{-m} is written as a^m or $\frac{1}{a^m}$ and read as 'a raised to the power (–m) or $\frac{1}{a^m} = a^{-m}$
- **Radicals or surds:** $\sqrt[n]{x}$ is a surd if and only if it is an irrational number and it is a root of the positive rational number. $\sqrt{}$ is called a radical sign. The index n is called the order of the surd and x is called the radicand.
- **Pure and mixed surd :** A surd with rational factor 1 only other factor being irrational is called pure surd e.g. $\sqrt[5]{16}, \sqrt[3]{50}$.

  A surd having rational factor other then 1 alongwith the irrational factor is called a mixed surd e.g. $5\sqrt[2]{3}, 4\sqrt[3]{7}$
- **Laws of surds:** If x, y are the positive rational numbers and m, n and p are positive integers then $\left(\sqrt[n]{x}\right)^n = x$

  $\sqrt[n]{x}.\sqrt[n]{y} = \sqrt[n]{xy}$ or $x^{\frac{1}{n}}.y^{\frac{1}{n}} = (xy)^{\frac{1}{n}}$

  $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$ or $\frac{x^{1/n}}{y^{1/n}} = \left(\frac{x}{y}\right)^{\frac{1}{n}}$

  $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x} = \sqrt[n]{\sqrt[m]{x}}$ or

  $\left(x^{\frac{1}{n}}\right)^{\frac{1}{m}} = x^{\frac{1}{mn}} = \left(x^{\frac{1}{m}}\right)^{\frac{1}{n}}$

  $\sqrt[n]{x^m} = x^{\frac{m}{n}}$ or $(x^m)^{\frac{1}{n}} = x^{\frac{m}{n}}$

  $\sqrt[n]{x^p} = \sqrt[mn]{x^{mp}}$ or $x^{\frac{pm}{mn}} = (x^{pm})^{\frac{1}{mn}}$
- **Similar or like surds:** Two sruds are said to be similar if they have same irrational factor e.g $3\sqrt{5}$ and $7\sqrt{5}$ are like or similar surds.
- **Simplest or lowest form of a surd :** A surd is said to be in simplest form, if it has Smallest possible index of the sign, no fraction under radical sign, no factor of the form a^n where a is a positive integer under the radical sign of index n.
- **Properties of surds:** Similar surds can be subtracted and added

  Order of surds can be changed by multiplying index of the surds and index of the radinard by the same positive number surds of the same order can be multiplided and divided

- **Comparision fo Surds:** Change the given surds to surds of the same order, then compare their radicands alongwith co-efficients.
- **Rationalising factor of a surd:** If the product of the two surds is rational, each is called the rationalising factor of the other $x+\sqrt{y}$ is called the rationalising factor of $x-\sqrt{y}$ and vice-versa

CHECK YOUR PROGRESS:

1. $\left(-\frac{2}{3}\right)^3 \times \left(-\frac{2}{3}\right)^5$ equals to:

   (A) $\left(-\frac{2}{3}\right)^{15}$ (B) $\left(\frac{2}{3}\right)^{-15}$ (C) $\left(-\frac{2}{3}\right)^{8}$ (D) $\left(-\frac{2}{3}\right)^{2}$

2. The order of the surd $3\sqrt[5]{47}$ is:

   (A) 5 (B) 3 (C) 47 (D) $\frac{1}{5}$

3. The rationalising factor of $\sqrt[3]{25}$ is :

   (A) 5 (B) $\sqrt{5}$ (C) $\sqrt[3]{5}$ (D) $\sqrt[3]{25}$

4. $\sqrt{8}$ is a:

   (A) Pure Surd (B) Mixed Sured (C) Not a Surd (D) Rational Number

5. $\left(-\frac{3}{4}\right)^0$ is equal to :

   (A) -1 (B) 1 (C) $\frac{-3}{4}$ (D) $\frac{-4}{3}$

6. Express the following as a product of prime factors in exponential form:

   (A) 194400 (B) 864360

7. Express the following as a mixed surd in simplest form

   (A) $\sqrt[4]{1215}$ (B) $\sqrt[3]{1024}$

8. Express the following as a pure surd:

   (A) $5\sqrt{2}$ (B) $4\sqrt[3]{5}$ (C) $2\sqrt[5]{2}$

9. Simplify each of the following:

   (i) $3\sqrt{80}-\frac{3}{2}\sqrt{\frac{1}{5}}+3\sqrt{120}$ (ii) $2\sqrt{50}\times\sqrt{32}\times2\sqrt{72}$ (iii) $\frac{15\sqrt[3]{13}}{6\sqrt[6]{5}}$

10. (i) Arrange in ascending order: $\sqrt[3]{2}, \sqrt{3}$ and $\sqrt[6]{5}$

(ii) Arrange in descending order : $\sqrt[3]{2}, \sqrt[4]{3}, \sqrt[3]{4}$

11. Simplify the following by rationalising the denominator

(i) $\frac{28}{\sqrt{7}+\sqrt{3}}$ (ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2}$ (iii) $\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$

STRETCH YOURSELF :

1. Find the value of x, if $\left(\frac{5}{7}\right)^{5+x} \times \left(\frac{25}{49}\right)^{x} = \left(\frac{7}{5}\right)^{2}$.

2. Simplify: $\left(\frac{-5}{6}\right)^{2} \div \left(\frac{-3}{5}\right)^{2}$.

3. if $x = 7 + 4\sqrt{3}$, Find the value of $x + \frac{1}{x}$.

4. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$, Find the values of a and b.

ANSWERS

CHECK YOUR PROGRESS:

1. C 2. A 3. C
4. B 5. B
6. (i) $2^5\ 3^5\ 5^2$ (ii) $2^5\ 3^2\ 5^1\ 7^4$
7. (i) $3\sqrt[4]{15}$ (ii) $8\sqrt[3]{2}$
8. (i) $\sqrt{50}$ (ii) $\sqrt[3]{320}$ (iii) $\sqrt[5]{64}$
9. (i) $\frac{88}{5}\sqrt{5}$ (ii) $960\sqrt{2}$ (iii) $\frac{1}{2}\sqrt[6]{845}$
10. (i) $\sqrt[3]{2}$, $\sqrt[6]{5}$, $\sqrt{3}$ (ii) $\sqrt[3]{4}$, $\sqrt[4]{3}$, $\sqrt[3]{2}$
11. (i) $7\left(\sqrt{7}-\sqrt{3}\right)$ (ii) $\frac{11-4\sqrt{7}}{3}$ (iii) $\frac{2\sqrt{3}+3\sqrt{2}+\sqrt{30}}{12}$

STRETCH YOURSELF

1. $x = 7$
2. $\frac{625}{324}$
3. 14
4. a = 11, b = –6