13

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CARTESIAN SYSTEM OF RECTANGULAR CO-ORDINATES

You must have searched for your seat in a cinema hall, a stadium, or a train. For example, seat H-4 means the fourth seat in the H^{th} row. In other words, H and 4 are the coordinates of your seat. Thus, the geometrical concept of location is represented by numbers and alphabets (an algebraic concept).

Also a road map gives us the location of various houses (again numbered in a particular sequence), roads and parks in a colony, thus representing algebraic concepts by geometrical figures like straight lines, circles and polygons.

The study of that branch of Mathematics which deals with the interrelationship between geometrical and algebraic concepts is called Coordinate Geometry or Cartesian Geometry in honour of the famous French mathematician **Rene Descartes.**

In this lesson we shall study the basics of coordinate geometry and relationship between concept of straight line in geometry and its algebraic representation.

OBJECTIVES

After studying this lesson, you will be able to:

- define Cartesian System of Coordinates including the origin, coordinate axes, quadrants, etc;
- derive distance formula and section formula;
- derive the formula for area of a triangle with given vertices;
- verify the collinearity of three given points;
- state the meaning of the terms : inclination and slope of a line;
- find the formula for the slope of a line through two given points;
- state the condition for parallelism and perpendicularity of lines with given slopes;
- find the intercepts made by a line on coordinate axes;
- find the angle between two lines when their slopes are given;
- find the coordinates of a point when origin is shifted to some other point;
- find transformed equation of curve when oregin is shifted to another point.

EXPECTED BACKGROUND KNOWLEDGE

- Number system .
- Plotting of points in a coordinate plane.
- Drawing graphs of linear equations .
- Solving systems of linear equations .

13.1 RECTANGULAR COORDINATE AXES

Recall that in previous classes, you have learnt to fix the position of a point in a plane by drawing two mutually perpendicular lines. The fixed point O,where these lines intersect each other is called the **origin** O as shown in Fig. 13.1 These mutually perpencular lines are called the **coordinate axes**. The horizontal line XOX' is the **x-axis** or **axis** of x and the vertical line YOY' is the y- **axis** or **axis** of y.

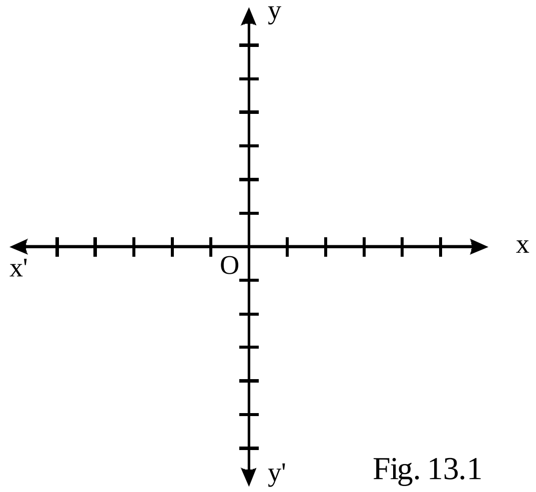


Fig. 13.1

9.1.1 CARTESIAN COORDINATES OF A POINT

To find the coordinates of a point we proceed as follows. Take X'OX and YOY' as coordinate axes. Let P be any point in this plane. From point P draw $PA \perp XOX'$ and $PB \perp YOY'$. Then the distance OA = x measured along x-axis and the distance OB = y measured along y-axis determine the position of the point P with reference to these axes. The distance OA measured along the axis of x is called the **abscissa** or x-coordinate and the distance OB (=PA) measured along $y-$axis is called the **ordinate** or y-coordinate of the point P. The abscissa and the ordinate taken together are called the **coordinates** of the point P. Thus, the coordinates of the point P are (x and y) which represent the position of the point P point in a plane. These two numbers are to form an **ordered pair** beacuse the order in which we write these numbers is important.

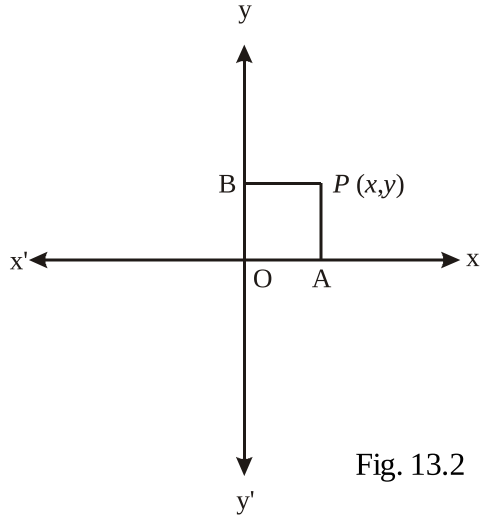


Fig. 13.2

In Fig. 13.3 you may note that the position of the ordered pair (3,2) is different from that of (2,3). Thus, we can say that (x,y) and (y,x) are two different ordered pairs representing two different points in a plane.

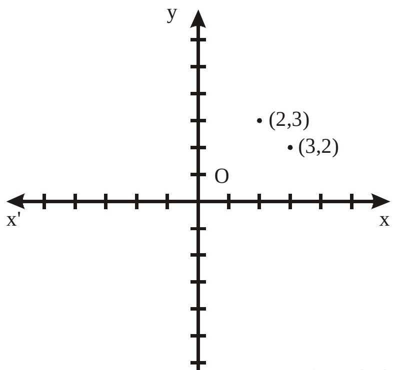


Fig.13.3

13.1.2 QUARDRANTS

We know that coordinate axes XOX' and YOY' divide the region of the plane into four regions. These regions are called the quardrants as shown in Fig. 13.4. In accordance with the convention of signs, for a point P (x,y) in different quadrants, we have

I	quadrant :	$x > 0, y > 0$
II	quadrant :	$x < 0, y > 0$
III	quadrant :	$x < 0, y < 0$
IV	quadrant :	$x > 0, y < 0$

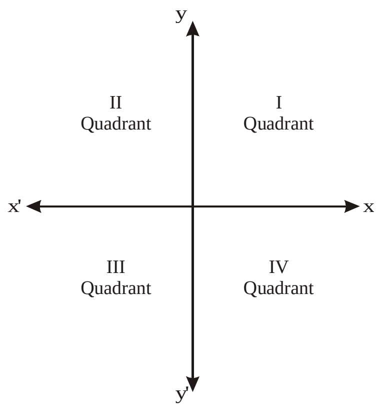


Fig.13.4

13.2 DISTANCE BETWEEN TWO POINTS

Recall that you have derived the distance formula between two points P (x_1,y_1) and Q (x_2,y_2) in the following manner:

Let us draw a line $l \parallel XX'$ through P. Let R be the point of intersection of the perpendicular from Q to the line l. Then $\Delta\, PQR$ is a right-angled triangle.

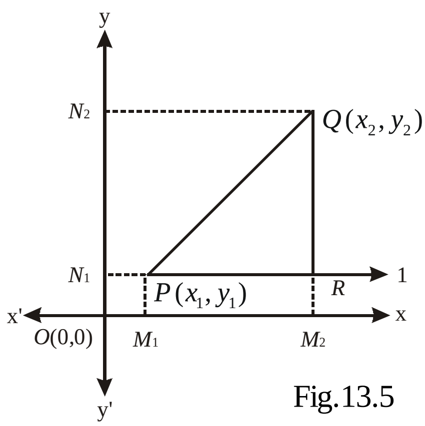


Fig.13.5

$$\text{Also } PR = M_1M_2$$

$$= OM_2 - OM_1$$

$$= x_2 - x_1$$

$$\text{and } QR = QM_2 - RM_2$$

$$= QM_2 - PM_1$$
$$= ON_2 - ON_1$$
$$= y_2 - y_1$$

Now $PQ^2 = PR^2 + QR^2$ (Pythagoras theorem)

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore \quad PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note : This formula holds for points in all quadrants.

Also the distance of a point P(x,y) from the origin O(0,0)

is OP $= \sqrt{x^2 + y^2}$.

Let us illustrate the use of these formulae with some examples.

Example 13.1 Find the distance between the following pairs of points :

(*i*) $A(14,3)$ and $B(10,6)$ (*ii*) $M(-1,2)$ and $N(0,-6)$

Solution :

(*i*) Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here $x_1 = 14$, $y_1 = 3$, $x_2 = 10$, $y_2 = 6$

$\therefore$ Distance between A and B $= \sqrt{(10-14)^2 + (6-3)^2}$

$$= \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Distance between A and B is 5 units.

(*ii*) Here $x_1 = -1$, $y_1 = 2$, $x_2 = 0$ and $y_2 = -6$

Distance between A and B $= \sqrt{(0-(-1))^2 + (-6-2)^2} = \sqrt{1+(-8)^2}$

$$= \sqrt{1+64} = \sqrt{65}$$

Distance between M and N $= \sqrt{65}$ units

Example 13.2 Show that the points P(–1, –1), Q(2, 3) and R (–2, 6) are the vertices of a right-angled triangle.

Solution: $PQ^2 = (2 + 1)^2 + (3 + 1)^2 = 3^2 + 4^2 = 9 + 16 = 25$

$QR^2 = (-4)^2 + (3)^2 = 16 + 9 = 25$

and $RP^2 = 1^2 + (-7)^2 = 1 + 49 = 50$

$\therefore$ $PQ^2 + QR^2 = 25+25=50= RP^2$

$\Rightarrow$ $\triangle PQR$ is a right-angled triangle (by converse of Pythagoras Theorem)

Example 13.3 Show that the points A(1, 2), B(4, 5) and C(–1, 0) lie on a straight line.

Solution: Here,

$$AB = \sqrt{(4-1)^2 + (5-2)^2} \text{ units} = \sqrt{18} \text{ units} = 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(-1-4)^2 + (0-5)^2} \text{ units} = \sqrt{50} \text{ units} = 5\sqrt{2} \text{ units}$$

and $$AC = \sqrt{(-1-1)^2 + (0-2)^2} \text{ units} = \sqrt{4+4} \text{ units} = 2\sqrt{2} \text{ units}$$

Now $$AB + AC = \left(3\sqrt{2} + 2\sqrt{2}\right) units = 5\sqrt{2}\, units = BC$$

i.e. BA + AC = BC

Hence, A, B, C lie on a straight line. In other words, A,B,C are collinear.

Example 13.4 Prove that the points (2a, 4a), (2a, 6a) and $\left(2a + \sqrt{3}a, 5a\right)$ are the vertices of an equilateral triangle whose side is 2a.

Solution: Let the points be A (2a, 4a), B (2a, 6a) and C $\left(2a + \sqrt{3}a, 5a\right)$

$$AB = \sqrt{0 + (2a)^2} = 2a \text{ units}$$

$$BC = \sqrt{(\sqrt{3}a)^2 + (-a)^2} \text{ units} = \sqrt{3a^2 + a^2} = 2a \text{ units}$$

and $$AC = \sqrt{(\sqrt{3}a)^2 + (+a)^2} = 2a \text{ units}$$

$\Rightarrow$ AB + BC > AC, BC + AC > AB and

AB + AC > BC and AB = BC = AC = 2a

$\Rightarrow$ A, B, C form the vertices of an equilateral triangle of side 2a.

CHECK YOUR PROGRESS 13.1

1. Find the distance between the following pairs of points.

   (a) (5, 4) and (2, –3) (b) (a, –a) and (b, b)

2. Prove that each of the following sets of points are the vertices of a right angled-trangle.

   (a) (4, 4), (3, 5), (–1,–1) (b) (2, 1), (0, 3), (–2, 1)

3. Show that the following sets of points form the vertices of a triangle:

   (a) (3, 3), (–3, 3) and (0, 0) (b) (0, a), (a, b) and (0, 0) (if ab = 0)

4. Show that the following sets of points are collinear :

   (a) (3, –6), (2, –4) and (–4, 8) (b) (0,3), (0, –4) and (0, 6)

5. (a) Show that the points (0, –1), (–2, 3), (6, 7) and (8, 3) are the vertices of a rectangle.

   (b) Show that the points (3, –2), (6, 1), (3, 4) and (0, 1) are the vertices of a square.

13.3 SECTION FORMULA

13.3.1 INTERNAL DIVISION

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two given points on a line l and $R(x, y)$ divide PQ internally in the ratio $m_1 : m_2$.

To find : The coordinates x and y of point R.

Construction : Draw PL, QN and RM perpendiculars to XX' from P, Q and R respectively and L, M and N lie on XX'. Also draw $RT \perp QN$ and $PV \perp QN$.

Method : R divides PQ internally in the ratio $m_1 : m_2$.

$\Rightarrow$ R lies on PQ and $\dfrac{PR}{RQ} = \dfrac{m_1}{m_2}$

Also, in triangles, RPS and QRT,

$$\angle RPS = \angle QRT \qquad \text{(Corresponding angles as } PS \parallel RT\text{)}$$

and $\angle RSP = \angle QTR = 90^0$

$\therefore$ $\Delta RPS \sim \Delta QRT$ (AAA similarity)

$$\Rightarrow \quad \frac{PR}{RQ} = \frac{RS}{QT} = \frac{PS}{RT} \qquad \ldots (i)$$

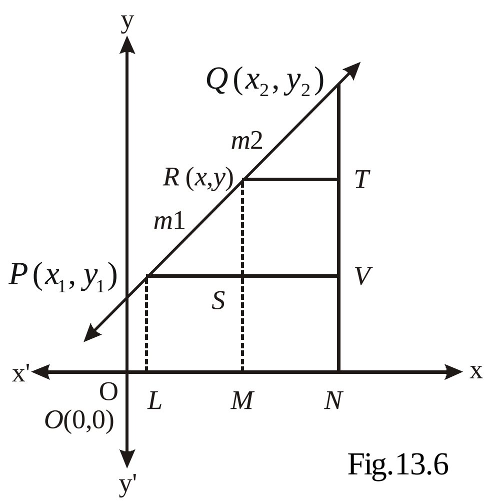


Fig.13.6

Also, $PS = LM = OM - OL = x - x_1$

$$RT = MN = ON - OM = x_2 - x$$

$$RS = RM - SM = y - y_1$$

$$QT = QN - TN = y_2 - y \, .$$

From (i), we have

$$\therefore \frac{m_1}{m_2} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$$

$$\Rightarrow m_1(x_2 - x) = m_2(x - x_1)$$

and $\quad m_1(y_2 - y) = m_2(y - y_1)$

$$\Rightarrow \quad x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \text{ and } y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Thus, the coordinates of R are:

$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

Coordinates of the mid-point of a line segment

If R is the mid point of PQ, then,

$m_1 = m_2 = 1$ (as R divides PQ in the ratio 1:1

Coordinates of the mid point are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

13.3.2 EXTERNAL DIVISION

Let R divide PQ externally in the ratio $m_1 : m_2$

To find : The coordinates of R.

Construction : Draw PL, QN and RM perpendiculars to XX' from P, Q and R respectively and $PS \perp RM$ and $QT \perp RM$.

Clearly, $\Delta RPS \sim \Delta RQT$.

$$\therefore \frac{RP}{RQ} = \frac{PS}{QT} = \frac{RS}{RT}$$

or $$\frac{m_1}{m_2} = \frac{x - x_1}{x - x_2} = \frac{y - y_1}{y - y_2}$$

$$\Rightarrow m_1(x - x_2) = m_2(x - x_1)$$

and $m_1(y - y_2) = m_2(y - y_1)$

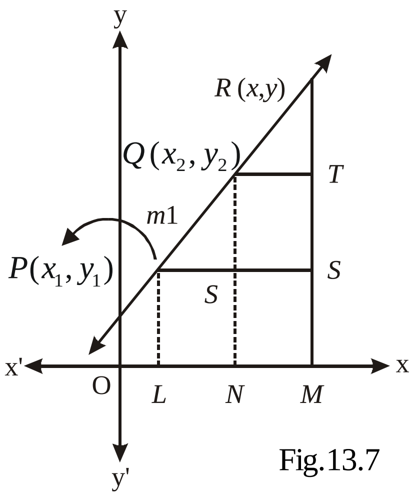


Fig.13.7

These give:

$$x = \frac{m_1x_2 - m_2x_1}{m_1 - m_2} \quad and \quad y = \frac{m_1y_2 - m_2y_1}{m_1 - m_2}$$

Hence, the coordinates of the point of external division are

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2} \right)$$

Let us now take some examples.

Example 13.5 Find the coordinates of the point which divides the line segment joining the points (4, –2) and (–3, 5) internally and externally in the ratio 2:3.

Solution:

(i) Let $P(x, y)$ be the point of internal division.

$$\therefore\ x = \frac{2(-3)+3(4)}{2+3} = \frac{6}{5} \quad \text{and} \quad y = \frac{2(5)+3(-2)}{2+3} = \frac{4}{5}$$

$\therefore$ P has coordinates $\left(\frac{6}{5}, \frac{4}{5}\right)$

If $Q(x', y')$ is the point of external division, then

$$x' = \frac{(2)(-3)-3(4)}{2-3} = 18 \quad \text{and} \quad y' = \frac{(2)(5)-3(-2)}{2-3} = -16$$

Thus, the coordinates of the point of external division are (18, –16).

Example 13.6 In what ratio does the point (3, –2) divide the line segment joining the points (1,4) and (–3, 16) ?

Solution : Let the point $P(3, -2)$ divide the line segement in the ratio $k : 1$.

Then the coordinates of P are $\left(\frac{-3k+1}{k+1}, \frac{16k+4}{k+1} \right)$

But the given coordinates of P are $(3, -2)$

$$\therefore \quad \frac{-3k+1}{k+1} = 3 \quad \Rightarrow -3k+1 = 3k+3 \qquad \Rightarrow k = -\frac{1}{3}$$

$\Rightarrow$ P divides the line segement externally in the ratio 1:3.

Example 13.7 The vertices of a quadrilateral *ABCD* are respectively (1, 4), (–2,1), (0, –1) and (3, 2). If *E, F, G, H* are respectively the midpoints of *AB, BC, CD* and *DA*, prove that the quadrilateral *EFGH* is a parallelogram.

Solution : Since *E, F, G*, and *H*, are the midpoints of the sides *AB, BC, CD* and *DA*, therefore, the coordinates of *E, F, G*, and *H* respectively are :

$$\left(\frac{1-2}{2},\frac{4+1}{2}\right),\left(\frac{-2+0}{2},\frac{1-1}{2}\right),\left(\frac{0+3}{2},\frac{-1+2}{2}\right) \text{ and } \left(\frac{1+3}{2},\frac{4+2}{2}\right)$$

$$\Rightarrow\ E\left(\frac{-1}{2},\frac{5}{2}\right),\ F(-1,0), G\left(\frac{3}{2},\frac{1}{2}\right) \text{ and } H(2,\ 3) \text{ are the required points.}$$

Also, the mid point of diagonal *EG* has coordinates

$$\left(\frac{\frac{-1}{2}+\frac{3}{2}}{2},\frac{\frac{5}{2}+\frac{1}{2}}{2}\right)=\left(\frac{1}{2},\frac{3}{2}\right)$$

Coordinates of midpoint of *FH* are $\left(\frac{-1+2}{2},\frac{0+3}{2}\right)=\left(\frac{1}{2},\frac{3}{2}\right)$

Since, the midpoints of the diagonals are the same, therefore, the diagonals bisect each other.

Hence *EFGH* is a parallelogram.

CHECK YOUR PROGRESS 13.2

1. Find the midpoint of each of the line segements whose end points are given below:

   (a) (–2, 3) and (3, 5) (b) (6,0) and (–2,10)

2. Find the coordinates of the point dividing the line segment joining

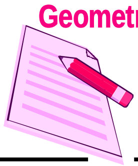
   (–5, –2) and (3, 6) internally in the ratio 3:1.

3. (a) Three vertices of a parallelogram are (0,3), (0,6) and (2,9). Find the fourth vertex.

   (b) (4, 0), (– 4, 0), (0,– 4) and (0, 4) are the vertices of a square. Show that the quadrilateral formed by joining the midpoints of the sides is also a square.

4. The line segement joining (2, 3) and (5, –1) is trisected. Find the points of trisection.

5. Show that the figure formed by joining the midpoints of the sides of a rectangle is a rhombus.



13.4 AREA OF A TRIANGLE

Let us find the area of a triangle whose

vertices are $A(x_1, y_1),\ B(x_2, y_2)$ *and* $C(x_3, y_3)$

Draw AL, BM and CN perpendiculars to XX'.

area of Δ ABC

= Area of trapzium. $BMLA$ + Area of trapzium. ALNC – Area of trapzium. BMNC

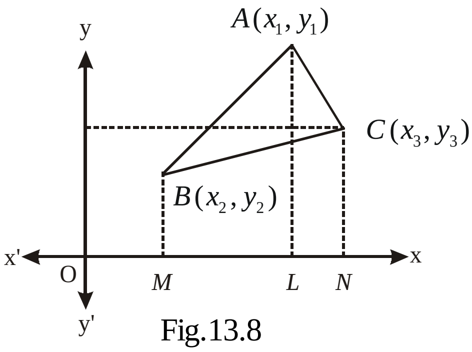


Fig.13.8

$$= \frac{1}{2}(BM + AL)ML + \frac{1}{2}(AL + CN)LN - \frac{1}{2}(BM + CN)MN$$

$$= \frac{1}{2}(y_2 + y_1)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$$

$$= \frac{1}{2}\left[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_3y_3)\right]$$

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

This can be stated in the determinant form as follows :

$$\text{Area of } \Delta\text{ ABC} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Example 13.8 Find the area of the triangle whose vertices are $A(3, 4)$, $B(6, -2)$ and $C(-4, -5)$.

Solution: The area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ 6 & -2 & 1 \\ -4 & -5 & 1 \end{vmatrix}$

$$= \frac{1}{2}\left[3(-2+5) - 4(6+4) + 1(-30-8)\right] = \frac{1}{2}\left[9 - 40 - 38\right] = \frac{-69}{2}$$

As the area is to be positive

$\therefore$ Area of $\Delta ABC = \frac{69}{2}$ square units

Example 13.9 If the vertices of a triangle are (1, *k*), (4, –3) and (–9, 7) and its area is 15 square units, find the value(s) of k.

Solution : Area of triangle $= \frac{1}{2}\begin{vmatrix} 1 & k & 1 \\ 4 & -3 & 1 \\ -9 & 7 & 1 \end{vmatrix}$

$$= \frac{1}{2}\left[-3-7-k(4+9)+1(28-27)\right] = \frac{1}{2}\left[-10-13k+1\right] = \frac{1}{2}\left[-9-13k\right]$$

Since the area of the triangle is given to be15,

$\therefore \quad \frac{-9-13k}{2} = 15$ or, $-9-13k = 30$, $-13k = 39$, or, $k = -3$

CHECK YOUR PROGRESS 13.3

1. Find the area of each of the following triangles whose vertices are given below :

   (1) (0, 5), (5, –5), and (0,0) (b) (2, 3), (–2,–3) and (–2, 3)

   (c) (a, 0), (0, – a) and (0, 0)

2. The area of a triangle ABC, whose vertices are A (2, –3), B(3, –2) and $C\left(\frac{5}{2}, k\right)$ is $\frac{3}{2}$ sq unit. Find the value of k

3. Find the area of a rectangle whose vertices are (5, 4), (5,– 4), (–5, 4) and (–5, – 4)

4. Find the area of a quadrilateral whose vertices are (5, –2), (4, –7), (1, 1) and (3, 4)

13.5 CONDITION FOR COLLINEARITY OF THREE POINTS

The three points $A(x_1, y_1)$, $B(x_2, y_2)$ *and* $C(x_3, y_3)$ are collinear if and only if the area of the triangle ABC becomes zero.

i.e. $$\frac{1}{2}\left[x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3\right] = 0$$

i.e. $$x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3 = 0$$

In short, we can write this result as

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Let us illustrate this with the help of examples:

Example 13.10 Show that the points $A(a,\ b+c),\ B(b, c+a)\ and\ C(c, a+b)$ are collinear.

Solution : Area of triangle ABC $= \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$ (Applying $C_1 \to C_1 + C_2$)

$$= \frac{1}{2}\begin{vmatrix} a+b+c & b+c & 1 \\ a+b+c & c+a & 1 \\ a+b+c & a+b & 1 \end{vmatrix} = \frac{1}{2}(a+b+c)\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix} = 0$$

Hence the points are collinear.

Example 13.11 For what value of k, are the points (1, 5) , (k, 1) and (4, 11) collinear ?

Solution : Area of the triangle formed by the given points is

$$= \frac{1}{2}\begin{vmatrix} 1 & 5 & 1 \\ k & 1 & 1 \\ 4 & 11 & 1 \end{vmatrix} = \frac{1}{2}[-10-5k+20+11k-4] = \frac{1}{2}[6k+6] = 3k+3$$

Since the given points are collinear, therefore

$3k + 3 = 0 \Rightarrow k = -1$

Hence, for $k = -1$, the given points are collinear.

CHECK YOUR PROGRESS 13.4

1. Show that the points (–1,–1), (5, 7) and (8, 11) are collinear.
2. Show that the points (3, 1), (5, 3) and (6, 4) are collinear.
3. Prove that the points $(a, 0)$, $(0, b)$ and (1, 1) are collinear if $\frac{1}{a}+\frac{1}{b}=1$.
4. If the points $(a,\ b), (a_1, b_1)\ and\ (a-a_1,\ b-b_1)$ are collinear, show that $a_1 b = ab_1$

5. Find the value of k for which the points (5, 7), (k, 5) and (0, 2) are collinear.

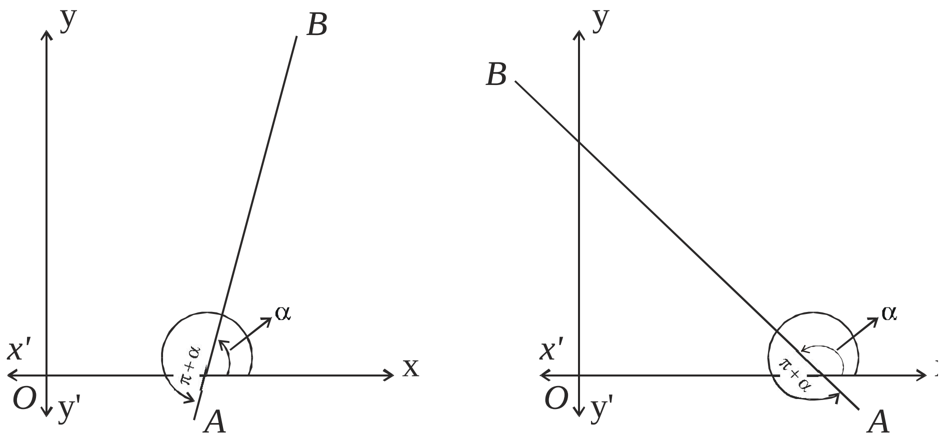
6. Find the values of k for which the point (k, 2–2k), (– k+1, 2k) and (– 4 – k, 6 – 2k) are collinear.

13.6 INCLINATION AND SLOPE OF A LINE

Look at the Fig. 13.9. The line AB makes an angle or $\pi + \alpha$ with the x-axis (measured in anticlockwise direction).

The *inclination* of the given line is represented by the measure of angle made by the line with the positive direction of x-axis (measured in anticlockwise direction)

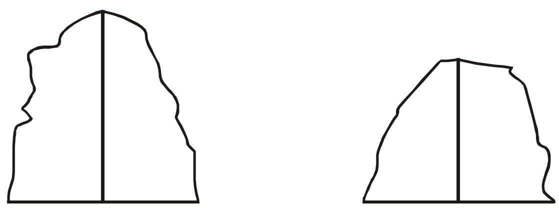
In a special case when the line is parallel to x-axis or it coincides with the *x-axis*, the inclination of the line is defined to be 0^0.



(a) (b)

Fig. 13.9

Again look at the pictures of two mountains given below. Here we notice that the mountain in Fig. 13.10 (a) is more steep compaired to mountain in Fig. 13.10 (b).



(a) Fig.13.10 (b)

How can we quantify this steepness ? Here we say that the angle of inclination of mountain (a) is more than the angle of inclination of mountain (b) with the ground.

Try to see the difference between the ratios of the maximum height from the ground to the base in each case.

Naturally, you will find that the ratio in case (a) is more as compared to the ratio in case (b). That means we are concerned with height and base and their ratio is linked with tangent of an angle, so mathematically this ratio or the tangent of the inclination is termed as *slope*. We define the slope as tangent of an angle.

The *slope* of a line is the the tangent of the angle θ (say) which the line makes with the positive direction of x-axis. Generally, it is denoted by m $(= \tan\theta)$

Note : If a line makes an angle of 90^0 or 270^0 with the x–axis, the slope of the line can not be defined.

Example 13.12 In Fig. 13.9 find the slope of lines *AB* and *BA*.

Solution : Slope of line *AB* = $\tan\alpha$

Slope of line *BA* = $\tan(\pi+\alpha) = \tan\alpha$.

Note : From this example, we can observe that "slope is independent of the direction of the line segement".

Example 13.13 Find the slope of a line which makes an angle of 30^0 with the negative direction of *x*-axis.

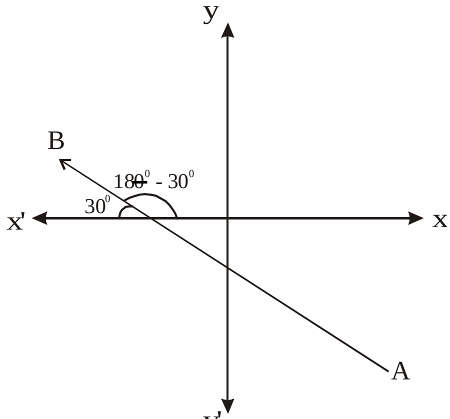


Fig. 13.11

Solution : Here $\theta = 180^0 - 30^0 = 150^0$

$\therefore$ m = slope of the line = $\tan(180^0 - 30^0)$

$= -\tan 30^0$

$= -\dfrac{1}{\sqrt{3}}$

Example 13.14 Find the slope of a line which makes an angle of 60^0 with the positive direction of *y*-axis.

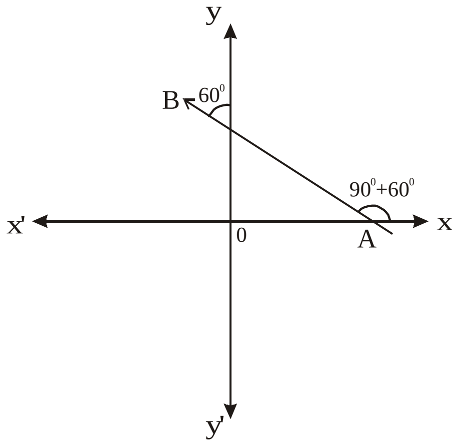


Fig. 13.12

Solution : Here $\theta = 90^0 + 60^0$

$\therefore$ m = slope of the line

$= \tan(90^0 + 60^0)$

$= -\cot 60^0$

$= -\tan 30^0 \quad = -\dfrac{1}{\sqrt{3}}$

Example 13.15 If a line is equally inclined to the axes, show that its slope is ± 1.

Solution : Let a line *AB* be equally inclined to the axes and meeting axes at points *A* and *B* as shown in the Fig. 13.13

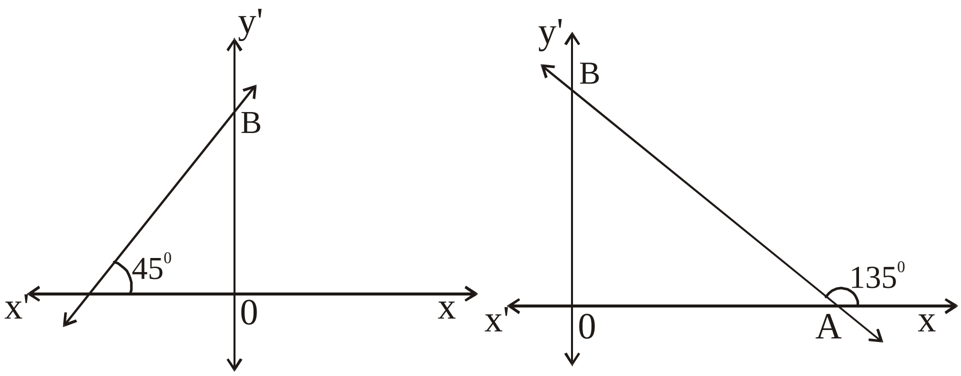


Fig. 13.13

In Fig 13.13(a), inclination of line $AB = \angle XAB = 45^0$

$\therefore$ Slope of the line $AB = \tan 45^0 = 1$

In Fig. 13.13 (b) inclination of line $AB = \angle XAB = 180^0 - 45^0 = 135^0$

$\therefore$ Slope of the line $AB = \tan 135^0 = \tan(180^0 - 45^0) = -\tan 45^0 = -1$

Thus, if a line is equally inclined to the axes, then the slope of the line will be ± 1.

CHECK YOUR PROGRESS 13.5

1. Find the Slope of a line which makes an angle of (i) 60^0, (ii) 150^0 with the positive direction of *x*-axis.

2. Find the slope of a line which makes an angle of 30^0 with the positive direction of *y*-axis.

3. Find the slope of a line which makes an angle of 60^0 with the negative direction of *x*-axis.

13.7 SLOPE OF A LINE JOINING TWO DISTINCT POINTS

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two distinct points. Draw a line through *A* and *B* and let the inclination of this line be θ. Let the point of intersection of a horizontal line through *A* and a vertical line through *B* be *M*, then the coordinates of *M* are as shown in the Fig. 13.14

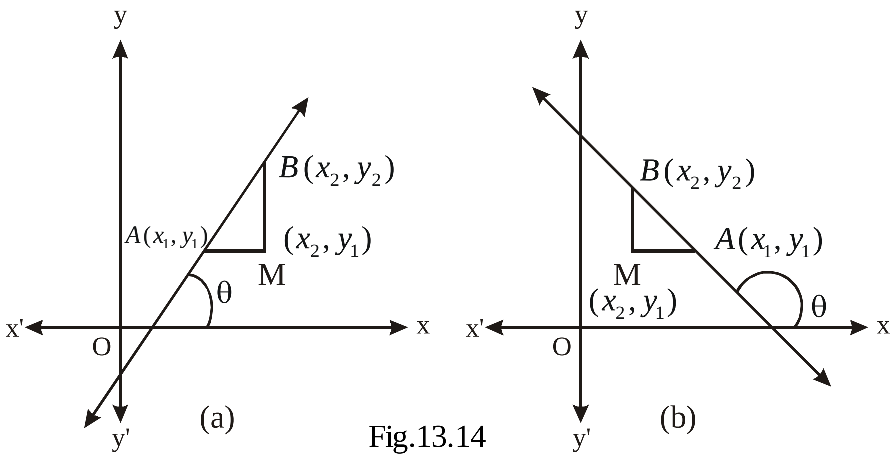


Fig.13.14

(A) In Fig 13.14 (a), angle of inclination MAB is equal to θ (acute). Consequently.

$$\tan\theta = \tan(\angle MAB) = \frac{MB}{AM} = \frac{y_2 - y_1}{x_2 - x_1}$$

(B) In Fig. 13.14 (b), angle of inclination θ is obtuse, and since θ and $\angle MAB$ are supplementary, consequently,

$$\tan\theta = -\tan(\angle MAB) = -\frac{MB}{MA} = -\frac{y_2 - y_1}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

Hence in both the cases, the slope m of a line through $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

***Note :** if $x_1 = x_2$, then m is not defined. In that case the line is parallel to y-axis.*

Is there a line whose slope is 1? Yes, when a line is inclined at 45^0 with the positive direction of x-axis.

Is there a line whose slope is $\sqrt{3}$? Yes, when a line is inclined at 60^0 with the positive direction of x-axis.

From the answers to these questions, you must have realised that given any real number m, there will be a line whose slope is m (because we can always find an angle α such that $\tan\alpha = m$).

Example 13.16 Find the slope of the line joining the points $A(6, 3)$ and $B(4, 10)$.

Solution : The slope of the line passing through the points (x_1, y_1) and $(x_2, y_2) = \dfrac{y_2 - y_1}{x_2 - x_1}$

Here, $x_1 = 6,\ y_1 = 3;\ x_2 = 4,\ y_2 = 10.$

Now substituting these values, we have slope $= \dfrac{10-3}{4-6} = -\dfrac{7}{2}$

Example 13.17 Determine *x*, so that the slope of the line passing through the points (3, 6) and (*x*, 4) is 2.

Solution :

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-6}{x-3} = \frac{-2}{x-3}$$

$$\therefore \frac{-2}{x-3} = 2 \qquad \text{.............. (Given)}$$

$$\therefore \quad 2x - 6 = -2 \quad or \quad x = 2$$

CHECK YOUR PROGRESS 13.6

1. What is the slope of the line joining the points *A*(6, 8) and *B*(4, 14) ?
2. Determine *x* so that 4 is the slope of the line through the points *A*(6,12) and *B*(*x*, 8).
3. Determine *y*, if the slope of the line joining the points $A(-8, 11)$ and $B(2, y)$ is $-\dfrac{4}{3}$.
4. *A*(2, 3) *B*(0, 4) and $C(-5, 0)$ are the vertices of a triangle *ABC*. Find the slope of the line passing through *the point B* and the mid point of *AC*
5. $A(-2, 7)$, B(1, 0), C(4, 3) and D(1, 2) are the vertices of a quadrilateral *ABCD* . Show that

   (i) slope of *AB* = slope of CD (ii) slope of BC = slope of AD

13.8 CONDITIONS FOR PARALLELISM AND PERPENDI CULARITY OF LINES.

9.8.1 Slope of Parallel Lines

Let l_1, l_2, be two (non-vertical) lines with their slopes m_1 and m_2 respectively.

Let θ_1 and θ_2 be the angles of inclination of these lines respectively.

Case I : Let the lines l_1 *and* l_2 *be parallel*

Then $\theta_1 = \theta_2 \Rightarrow \tan\theta_1 = \tan\theta_2$

$$\Rightarrow m_1 = m_2$$

Thus, if two lines are parallel then their slopes are equal.

Case II : Let the lines l_1, and l_2 have equal slopes.

i.e. $m_1 = m_2 \Rightarrow \tan\theta_1 = \tan\theta_2$

$\Rightarrow \theta_1 = \theta_2 \ (0^\circ \le \theta \le 180^\circ)$

$\Rightarrow l_1 \parallel l_2$

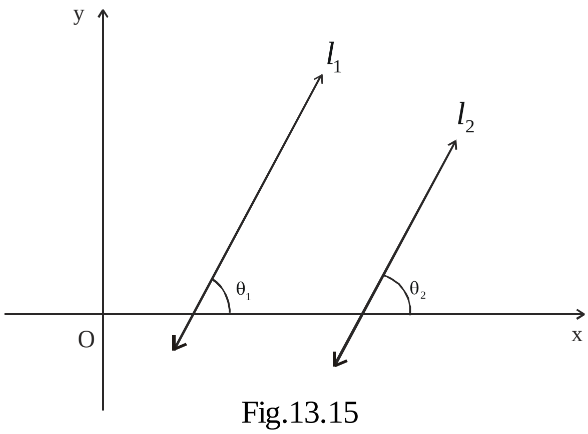


Fig.13.15

Hence, two (non-vertical) lines are parallel if and only if $m_1 = m_2$

13.8.2 SLOPES OF PERPENDICULAR LINES

Let l_1 and l_2 be two (non-vertical)lines with their slopes m_1 and m_2 respectively. Also let θ_1 and θ_2 be their inclinations respectively.

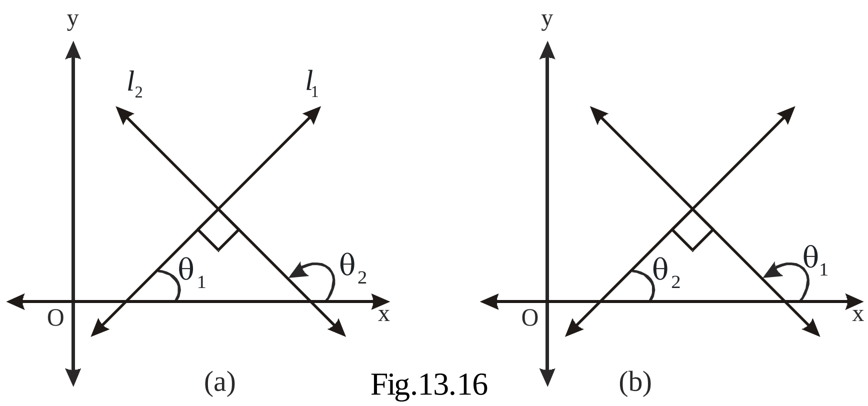


Fig.13.16

Case-I : Let $l_1 \perp l_2$

$\Rightarrow \theta_2 = 90^0 + \theta_1$ or $\theta_1 = 90^0 + \theta_2$

$\Rightarrow \tan\theta_2 = \tan(90^0 + \theta_1)$ or $\tan\theta_1 = \tan(90^0 + \theta_2)$

$\Rightarrow \tan\theta_2 = -\cot(\theta_1)$ or $\tan\theta_1 = -\cot(\theta_2)$

$\Rightarrow \tan\theta_2 = -\dfrac{1}{\tan\theta_1}$ or $\Rightarrow \tan\theta_1 = -\dfrac{1}{\tan\theta_2}$

$\Rightarrow$ In both the cases, we have

$$\tan\theta_1 \tan\theta_2 = -1$$

or $\quad m_1.m_2 = -1$

Thus, if two lines are perpendicular then the product of their slopes is equal to –1.

Case II : Let the two lines l_1 and l_2 be such that the product of their slopes is –1.

i.e. $m_1.m_2 = -1$

$$\Rightarrow \tan\theta_1 \tan\theta_2 = -1$$

$$\Rightarrow \tan\theta_1 = -\frac{1}{\tan\theta_2} = -\cot\theta_2 = \tan\left(90^0 + \theta_2\right)$$

or

$$\tan\theta_2 = \frac{-1}{\tan\theta_1} = -\cot\theta_1 = \tan\left(90 + \theta_1\right)$$

$\Rightarrow$ Either $\theta_1 = 90^0 + \theta_2$ or $\theta_2 = 90^0 + \theta_1$ $\Rightarrow$ In both cases $l_1 \perp l_2$.

Hence, two (non-vertical) lines are perpendicular if and only if $m_1.m_2 = -1$.

Example 13.18 Show that the line passing through the points A(5,6) and B(2,3) is parallel to the line passing, through the points C(9,–2) and D(6,–5).

Solution : Slope of the line AB $= \dfrac{3-6}{2-5} = \dfrac{-3}{-3} = 1$

and slope of the line CD $= \dfrac{-5+2}{6-9} = \dfrac{-3}{-3} = 1$

As the slopes are equal $\therefore$ AB $\parallel$ CD.

Example 13.19 Show that the line passing through the points A(2,–5) and B(–2,5) is perpendicular to the line passing through the points L(6,3) and M(1,1).

Solution : Here

$$m_1 = \text{slope of the line AB} = \frac{5+5}{-2-2} = \frac{10}{-4} = \frac{-5}{2}$$

$$\text{and } m_2 = \text{slope of the line LM} = \frac{1-3}{1-6} = \frac{2}{5}$$

$$\text{Now } m_1.m_2 = \frac{-5}{2} \times \frac{2}{5} = -1$$

Hence, the lines are perpendicular to each other.

Example 13.20 Using the concept of slope, show that A(4,4), B(3,5) and C (-1,-1) are the vertices of a right triangle.

Solution : Slope of line AB = $m_1 = \dfrac{5-4}{3-4} = -1$

Slope of line BC = $m_2 = \dfrac{-1-5}{-1-3} = \dfrac{3}{2}$

and slope of line AC = $m_3 = \dfrac{-1-4}{-1-4} = 1$

Now $m_1 \times m_3 = -1 \quad \Rightarrow AB \perp AC$

$\Rightarrow \triangle$ABC is a right-angled triangle.

Hence, A(4,4), B(3,5) and C(– 1, – 1) are the vertices of right triangle.

Example 13.21 What is the value of y so that the line passing through the points A(3,y) and B(2,7) is perpendicular to the line passing through the point C (– 1,4) and D (0,6)?

Solution : Slope of the line AB = $m_1 = \dfrac{7-y}{2-3} = y-7$

Slope of the line CD = $m_2 = \dfrac{6-4}{0+1} = 2$

Since the lines are perpendicular,

$\therefore m_1 \times m_2 = -1$ or $(y-7)\times 2 = -1$

or $2y-14=-1$ or $2y=13$ or $y=\dfrac{13}{2}$

CHECK YOUR PROGRESS 13.7

1. Show that the line joining the points (2,–3) and (– 4,1) is

   (i) parallel to the line joining the points (7,–1) and (0,3).

   (ii) perpendicular to the line joining the points (4,5) and (0,–2).

2. Find the slope of a line parallel to the line joining the points (– 4,1) and (2,3).

3. The line joining the points (–5,7) and (0,–2) is perpendicular to the line joining the points (1,3) and (4,x). Find x.

4. A(–2,7), B(1,0), C(4,3) and D(1,2) are the vertices of quadrilateral ABCD. Show that the sides of ABCD are parallel.

5. Using the concept of the slope of a line, show that the points A(6, – 1), B(5,0) and C(2,3) are collinear.[Hint: slopes of AB, BC and CA must be equal.]

6. Find k so that line passing through the points (k,9) and (2,7) is parallel to the line passing through the points (2,–2) and (6,4).

7. Using the concept of slope of a line, show that the points (–4,–1), (–2–4), (4,0) and (2,3) taken in the given order are the vertices of a rectangle.

8. The vertices of a triangle ABC are A(–3,3), B(–1,– 4) and C(5,–2). M and N are the midpoints of AB and AC. Show that MN is parallel to BC and MN = $\frac{1}{2}$ BC.

13.9 INTERCEPTS MADE BY A LINE ON AXES

If a line l (not passing through the Origin) meets x-axis at A and y-axis at B as shown in Fig. 13.17, then

(i) OA is called the x-intercept or the intercept made by the line on x-axis.

(ii) OB is called y-intercept or the intercept made by the line on y-axis.

(iii) OA and OB taken together in this order are called the intercepts made by the line l on the axes.

(iv) AB is called the portion of the line intercepted between the axes.

(v) The coordinates of the point A on x-axis are (a,0) and those of point B are (0,b)

To find the intercept of a line in a given plane on x-axis, we put y = 0 in the given equation of a line and the value of x so obtained is called the x intercept.

To find the intercept of a line on y-axis we put x = 0 and the value of y so obtained is called the y intercept.

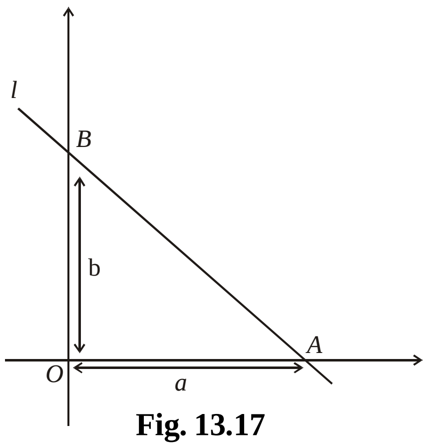


Fig. 13.17

Note: 1. A line which passes through origin makes no intercepts on axes.

2. A horizontal line has no x-intercept and vertical line has no y-intercept.

3. The intercepts on x- axis and y-axis are usually denoted by a and b respectively. But if only y-intercept is considered, then it is usually denoted by c.

Example 13.22 If a line is represented by $2x+3y=6$, find its x and y intercepts.

Solution : The given equation of the line is $2x+3y=6$ ... (i)

Putting x = 0 in (i), we get y = 2

Thus, y-intercept is 2.

Again putting $y = 0$ in (i), we get $2x = 6 \Rightarrow x = 3$

Thus, x-intercept is 3.

CHECK YOUR PROGRESS 13.8

1. Find x and y intercepts, if the equations of lines are :

   (i) $x + 3y = 6$ (ii) $7x + 3y = 2$ (iii) $\dfrac{x}{2a} + \dfrac{y}{2b} = 1$ (iv) $ax + by = c$

   (v) $\dfrac{y}{2} - 2x = 8$ (vi) $\dfrac{y}{3} - \dfrac{2x}{3} = 7$

13.10 ANGLE BETWEEN TWO LINES

Let l_1 and l_2 be two non vertical and non perpendicualr lines with slopes m_1 and m_2 respectively. Let α_1 and α_2 be the angles subtended by l_1 and l_2 respectively with the positive direction of x-axis. Then $m_1 = \tan\alpha_1$ and $m_2 = \tan\alpha_2$.

From figure 1, we have $\alpha_1 = \alpha_2 + \theta$

$$\therefore \quad \theta = \alpha_1 - \alpha_2$$

$$\Rightarrow \quad \tan\theta = \tan(\alpha_1 - \alpha_2)$$

$$\text{i.e.} \quad \tan\theta = \frac{\tan\alpha_1 - \tan\alpha_2}{1 + \tan\alpha_1 . \tan\alpha_2}$$

$$\text{i.e.} \quad \tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} \quad ...(1)$$

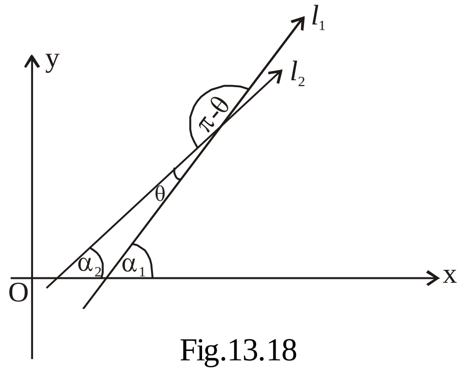


Fig.13.18

As it is clear from the figure that there are two angles θ and $\pi - \theta$ between the lines l_1 and l_2.

$$\text{We know,} \quad \tan(\pi - \theta) = -\tan\theta$$

$$\therefore \quad \tan(\pi - \theta) = -\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$$

Let $\pi - \theta = \phi$

$\therefore$ $$\tan\phi = -\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right) \quad ...(2)$$

- If $\frac{m_1 - m_2}{1 + m_1 m_2}$ is positive then $\tan\theta$ is positive and $\tan\phi$ is negative i.e. θ is acute and ϕ is obtuse.

- If $\frac{m_1 - m_2}{1 + m_1 m_2}$ is negative then $\tan\theta$ is negative and $\tan\phi$ is positive i.e. θ is obtuse and ϕ is acute.

Thus the acute angle (say θ) between lines l_1 and l_2 with slopes m_1 and m_2 respectively is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \text{ where } 1 + m_1 m_2 \neq 0.$$

The obtuse angle (say ϕ) can be found by using the formula $\phi = 180° - \theta$.

Example 13.23 Find the acute and obtuse angles between the lines whose slopes are $\frac{3}{4}$ and $\frac{-1}{7}$.

Solution : Let θ and ϕ be the acute and obtuse angle between the lines respectively.

$\therefore$ $$\tan\theta = \left|\frac{\frac{3}{4} + \frac{1}{7}}{1 + \left(\frac{3}{4}\right)\left(\frac{-1}{7}\right)}\right| = \left|\frac{21 + 4}{28 - 3}\right| = |1| = 1$$

$\Rightarrow$ $\theta = 45°$

$\therefore$ $\phi = 180° - 45° = 135°.$

Example 13.24 Find the angle (acute or obtuse) between x-axis and the line joining the points (3, –1) and (4, –2),

Solution : Slope of x-axis (say m_1) = 0

Slope of given line (say m_2) = $\frac{-2 + 1}{4 - 3} = -1$

$\therefore$ $$\tan\theta = \left|\frac{0 + 1}{1 + (0)(-1)}\right| = 1$$

$\Rightarrow$ $\theta = 45°$ as acute angle.

Example 13.25 If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

Solution : Here, $$\tan\frac{\pi}{4} = \left|\frac{\frac{1}{2}-m_2}{1+\left(\frac{1}{2}\right)(m_2)}\right|$$

$$\Rightarrow \quad \left|\frac{1-2m_2}{2+m_2}\right| = 1$$

$$\Rightarrow \quad \frac{1-2m_2}{2+m_2} = 1 \text{ or } \frac{1-2m_2}{2+m_2} = -1.$$

$$\Rightarrow \quad m_2 = -\frac{1}{3} \text{ or } m_2 = 3.$$

$\therefore$ Slope of other line is 3 or $-\frac{1}{3}$.

CHECK YOUR PROGRESS 13.9

1. Find the acute angle between the lines with slopes 5 and $\frac{2}{3}$.

2. Find the obtuse angle between the lines with slopes 2 and –3.

3. Find the acute angle between the lines l_1 and l_2 where l_1 is formed by joining the points (0, 0) and (2, 3) and l_2 by joining the points (2, –2) and (3, 5)

13.11 SHIFTING OF ORIGIN :

We know that by drawing x-axis and y-axis, any plane is divided into four quadrants and we represent any point in the plane as an ordered pair of real numbers which are the lengths of perpendicular distances of the point from the axes drawn. We also know that these axes can be chosen arbitrarily and therefore the position of these axes in the plane is not fixed. Position of the axes can be changed. When we change the position of axes, the coordinates of a point also get changed correspondingly. Consequently equations of curves also get changed.

The axes can be changed or transformed in the following ways :

(i) Translation of axes (ii) Rotation of axes (iii) Translation and rotation of axes. In the present section we shall discuss only one transformation i.e. translation of axes.

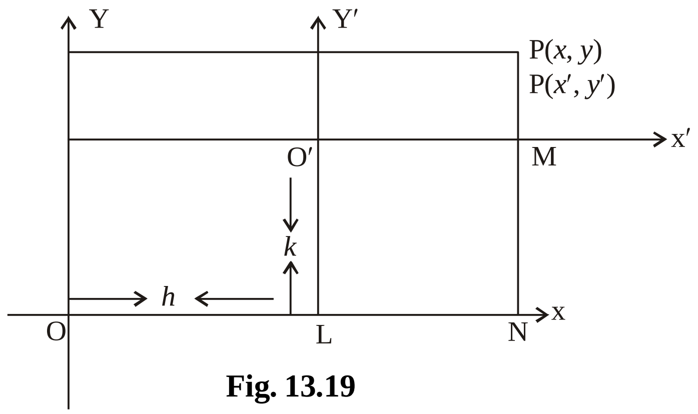


Fig. 13.19

The transformation obtained, by shifting the origin to a given point in the plane, without changing the directions of coordinate axes is called **translation of axes**.

Let us see how coordinates of a point in a plane change under a translation of axes. Let $\overrightarrow{OX}$ and $\overrightarrow{OY}$ be the given coordinate axes. Suppose the origin O is shifted to O′(*h*, *k*) by the translation of the axes $\overrightarrow{OX}$ and $\overrightarrow{OY}$. Let $\overrightarrow{O'X'}$ and $\overrightarrow{O'Y'}$ be the new axes as shown in the above figure. Then with reference to $\overrightarrow{O'X'}$ and $\overrightarrow{O'Y'}$ the point O' has coordinates (0, 0).

Let P be a point with coordinates (x, y) in the system $\overrightarrow{OX}$ and $\overrightarrow{OY}$ and with coordinates (x', y') in the system $\overrightarrow{O'X'}$ and $\overrightarrow{O'Y'}$. Then $O'L = K$ and $OL = h$.

Now $x = ON = OL + LN$

$$= OL + O'M$$

$$= h + x'.$$

and $y = PN = PM + MN = PM + O'L = y' + k$.

Hence $x = x' + h$; $y = y' + k$

or $x' = x - h$, $y' = y - k$

- If the origin is shifted to (h, k) by translation of axes then coordinates of the point $P(x, y)$ are transformed to $P(x - h, y - k)$ and the equation $F(x, y) = 0$ of the curve is transformed to $F(x' + h, y' + k) = 0$.

- Translation formula always hold, irrespective of the quadrant in which the origin of the new system happens to lie.

Example 13.26 When the origin is shifted to (–3, 2) by translation of axes find the coordinates of the point (1, 2) with respect to new axes.

Solution : Here $(h, k) = (-3, 2)$, $(x, y) = (1, 2)$, $(x', y') = ?$

$x' = x - h = 1 + 3 = 4$

MODULE-IV
Co-ordinate Geometry

Notes

$y' = y - k = 2 - 2 = 0$

Therefore $(x', y') = (4, 0)$

Example 13.27 When the origin is shifted to the point (3, 4) by the translation of axes, find the transformed equation of the line $3x + 2y - 5 = 0$.

Solution : Here $(h, k) = (3, 4)$

$\therefore \quad x = x' + 3$ and $y = y' + 4$.

Substituting the values of x and y in the equation of line

we get $3(x' + 3) + 2(y' + 4) - 5 = 0$

i.e. $3x' + 2y' + 12 = 0$.

CHECK YOUR PROGRESS 13.10

1. (i) Does the length of a line segment change due to the translation of axes? Say yes or no.

   (ii) Are there fixed points with respect to translation of axes? Say yes or no.

   (iii) When the origin is shifted to the point (4, –5) by the translation of axes, the coordinates of the point (0, 3) are ...

   (iv) When the origin is shifted to (2, 3), the coordinates of a point P changes to (4, 5), coordinates of point P in original system are ...

   (v) If due to translation of axes the point (3, 0) changes to (2, –3), then the origin is shifted to the point ...

LET US SUM UP

- Distance between any two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- Coordinates of the point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

- Coordinates of the point dividing the line segment joining the the points (x_1, y_1) and (x_2, y_2) externally are in the ratio $m_1 : m_2$ are.

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

- Coordinates of the mid point of the line segment joining the points (x_1,y_1) and (x_2, y_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- The area of a triangle with vertices (x_1, y_1) and (x_2, y_2) and (x_3, y_3) is given by

$$\frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)]$$

- Three points A, B, and C are collinear if the area of the triangle formed by them is zero.
- If θ is the angle which a line makes with the positive direction of x-axis, then the slope of the line is $m = \tan\theta$.
- Slope (m) of the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- A line with the slope m_1 is parallel to the line with slope m_2 if $m_1 = m_2$.
- A line with the slope m_1 is perpendicular to the line with slope m_2 if $m_1 \times m_2 = -1$.
- If a line l (not passing through the origin) meets x- axis at A and y- axis at B then OA is called the x- intercept and OB is called the y- intercept.
- If θ be the angle between two lines with slopes m_1 and m_2, then

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$$

where $1 + m_1m_2 \neq 0$

- If $\tan\theta$ is +ve, the angle (θ) between the lines is acute and if $\tan\theta$ is -ve then it is obtuse.
- When origin is shifted to (h,k) then transformed coordinates (x',y') (say) of a point P (x,y) are (x-h, y-k)

SUPPORTIVE WEB SITES

http://www.youtube.com/watch?v=VhNkWdLGpmA
http://www.youtube.com/watch?v=5ctsUsvIp8w
http://www.youtube.com/watch?v=1op92ojA6q0

TERMINAL EXERCISE

1. Find the distance between the pairs of points:

   (a) (2, 0) and (1, cot θ) (b) (–sin A, cosA) and (sin B, cos B)

2. Which of the following sets of points form a triangle?

   (a) (3, 2), (–3, 2) and (0, 3) (b) (3, 2), (3, –2) and (3, 0)

3. Find the midpoint of the line segment joining the points (3. –5) and (–6, 8).

4. Find the area of the triangle whose vertices are:

   (a) (1, 2), (–2, 3), (–3, –4) (b)(c, a), (c + a, a), (c – a, –a)

5. Show that the following sets of points are collinear (by showing that area formed is 0).

   (a) (–2, 5) (2, –3) and (0, 1) (b) (a, b + c), (b, c + a) and (c, a + b)

6. If (–3, 12), (7, 6) and (x, a) are collinear, find x.

7. Find the area of the quadrilateral whose vertices are (4,3) (–5,6) (0,7) and (3,–6).

8. Find the slope of the line through the points

   (a) (1,2), (4,2) (b) (4, –6), (–2, –5)

9. What is the value of y so that the line pasing through the points (3, y) and (2,7) is parallel to the line passing through the points (–1, 4) and (0, 6) ?

10. Without using Pythagoras theorem, show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right-angled triangle.

11. Using the concept of slope, determine which of the following sets of points are collnear:

    (i) (–2, 3) , (8, –5) and (5, 4), (ii) (5, 1) , (1, –1) and (11, 4),

12. If A (2, –3) and B (3, 5) are two vertices of a rectangle ABCD, find the slope of

    (i) BC (ii) CD (iii) DA.

13. A quadrilateral has vertices at the points (7, 3), (3, 0), (0, –4) and (4, –1). Using slopes, show that the mid-points of the sides of the quadrilatral form a parallelogram.

14. Find the x-intercepts of the following lines:

    $(i)\ 2x-3y=8$ $(ii)\ 3x-7y+9=0$ $(iii)\ x-\frac{y}{2}=3$

15. When the origin is shifted to the point (3, 4) by translation of axes, find the transformed equation of $2x^2+4xy+5y^2=0$.

16. If the origin is shifted to the point (3, –4), the transformed equation of a curve is $(x^1)^2+(y^1)^2=4$, find the original equation of the curve.

17. If A(–2, 3), B(3, 8) and C(4, 1) are the vertices of a ΔABC. Find $\angle$ABC of the triangle.

18. Find the acute angle between the diagonals of a quadrilateral ABCD formed by the points A(9, 2), B(17, 11), C(5, – 3) and D(–3, –2) taken in order.

19. Find the acute angle between the lines AB and BC given that A = (5, –3), B = (–3, –2) and C = (9, 12).

ANSWERS

CHECK YOUR PROGRESS 13.1

(a) $\sqrt{58}$ (b) $\sqrt{2(a^2+b^2)}$

CHECK YOUR PROGRESS 13.2

1. (a) $\left(\frac{1}{2},4\right)$ (b) (2,5) 2. (1,4) 3. (a) (2,6)

4. $\left(3,\frac{5}{3}\right),\left(4,\frac{1}{3}\right)$

CHECK YOUR PROGRESS 13.3

1. (a) $\frac{25}{2}$ sq. units (b) 12 sq. units (c) $\frac{a^2}{2}$ sq. units

2. $k=\frac{5}{3}$ 3. 80 sq. units 4. $\frac{41}{2}$ sq. units

CHECK YOUR PROGRESS 13.4

5. $k = 3$ 6. $k=\frac{1}{2},-1$

CHECK YOUR PROGRESS 13.5

1. (*i*) $\sqrt{3}$ (*ii*) $-\frac{1}{\sqrt{3}}$ 2. $-\sqrt{3}$ 3. $-\sqrt{3}$

CHECK YOUR PROGRESS 13.6

1. -3 2. 5 3. $-\frac{7}{3}$ 4. $\frac{5}{3}$

CHECK YOUR PROGRESS 13.7

2. $\frac{1}{3}$ 3. $\frac{14}{3}$. 6. $k=\frac{10}{3}$

CECK YOUR PROGRESS 13.8

1. (*i*) x-intercept = 6, y-intercept = 2

(*ii*) x-intercept = $\frac{2}{7}$, y-intercept = $\frac{2}{3}$

(*iii*) x-intercept = 2a, y-intercept = $2b$

(*iv*) x-intercept $= \dfrac{c}{a}$, y-intercept $= \dfrac{c}{b}$

(*v*) x-intercept $= -4$, y-intercept $= 16$

(*vi*) x-intercept $= \dfrac{-21}{2}$, y-intercept $= 21$

CHECK YOUR PROGRESS 13.9

1. $45°$ 2. $135°$ 3. $\tan = \dfrac{11}{23}$

CHECK YOUR PROGRESS 13.10

1. (i) No (ii) No (iii) (-4,8) (iv) (6,8) (v) (1,3)

TERMINAL EXERCISE

1. (a) cosec θ (b) $2 \sin \dfrac{A+B}{2}$

2. None of the given sets forms a triangle.

3. $\left(-\dfrac{3}{2}, \dfrac{3}{2}\right)$ 4. (a) 11 sq. unit (b) a^2 sq. unit.

6. $\dfrac{51-5a}{3}$ 7. 29 sq. unit.

8. (*a*) 0 (b) $-\dfrac{1}{6}$

9. $y = 3$ 11. Only (*ii*)

12. (*i*) $-\dfrac{1}{8}$ (*ii*) 8 (*iii*) $-\dfrac{1}{8}$

14. (*i*) 4 (*ii*) -3 (*iii*) 3

15. $x^2 + 4y^2 + 4xy + 116x + 2y + 259 = 0$

16. $x^2 + y^2 - 6x + 8y + 21 = 0$

17. $\tan^{-1}\left(\dfrac{4}{3}\right)$ 18. $\tan^{-1}\left(\dfrac{48}{145}\right)$ 19. $\tan^{-1}\left(\dfrac{62}{55}\right)$