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Principle of Mathematical Induction

- **Statement : -**

The sentence, which is either true or false is called as statement

(i) 1 am 20 years old

(ii) If $x = 2$, then $x^2 = 4$

} Statement

(iii) When you leave from home?

(iv) How wonderful the garden!

} Not statement

- **The Principle of Mathematical Induction**

Let p(n) be a statement involving a natural number n, if

(i) It is true for $n = 1$, i.e P(1) is true; and

(ii) Assuming P(K) to be true, it can be proved that P(K+1) is true; Then by Principle of Mathematical induction p(n) must be true for every natural number n.

- **The Mathematical Statement**

(1) $p(n): 1 + 2 + 3 - - - - + n = \frac{n(n+1)}{2}$

(2) $p(n): 2^n > n$

(3) $p(n): 1^2 + 2^2 + 3^2 + - - - - - > n^2 = \frac{n(n+1)(2n+1)}{6}$

(4) $p(n): 1 + 4 + 7 + - - - + (3n - 2) = \frac{n(3x-1)}{26}$

(5) $p(n): \frac{1}{1x2} + \frac{1}{2x3} + - - - + \frac{1}{n(n+1)} = \frac{n}{(n+1)}$,

Where n EN, all statements are proved by Mathematical Induction.

- The word induction means, formulating a general principle (or rate) based on several particular instances.

Example : Using principle of mathematical induction prove that $\left(\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}\right)$ is a natural number for all natural number n

Solution Let $P_n : \left(\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}\right)$ is a natural number

P(1) $=\left(\frac{1}{5}+\frac{1}{3}+\frac{7}{15}\right)=1$ which is a natural number

P(1) is true.

Let P(k) :$\left(\frac{k^5}{5}+\frac{k^3}{3}+\frac{7k}{15}\right)$ is a natural number be true

Now $\left(\frac{(k+1)^5}{5}+\frac{(k+1)^3}{3}+\frac{7(k+1)}{15}\right)$

$=\frac{1}{5}[k^5+5k^4+10k^2+5k+1]+$
$\frac{1}{3}[k^3+3k^2+3k+1]+\left(\frac{7}{15}k+\frac{7}{15}\right)$

$$\left(\frac{k^5}{5}+\frac{k^3}{3}+\frac{7k}{15}\right) \textit{ is a natural number}$$

Also $k^4+2k^3+3k^2+2k$ is a natural number

P(k+1) is true , whenever P(k) is true

P(n) is true is true for all natural number.

Stretch Yourself

Prove the following by principle of mathematical induction

1. 1+2+3+4……..+P = $\frac{p(p+1)}{2}$
2. $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots\ldots\ldots\ldots\frac{1}{X(X+1)}=\frac{X(X+1)(2X+1)}{6}$
3. a+(a+d) +(a+2d)+……..a+(n-1)d $=\frac{n}{2}[2a+(n-1)d]$
4. $1\times 3+2\times 4+3\times 5+\cdots\ldots..n\times(n+2)=\frac{1}{6}n(n+1)(2n+7)$
5. $2+5+8+11+\cdots.(3n-1)=\frac{1}{2}n(3n+1)$
6. $5^{3n}-1$ is divisible by 124 for all $n\in N$
7. $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\ldots.\frac{1}{n^2}<2-\frac{1}{n}$ for all $n\geq 2, n\in N$
8. $7^{2n}+2^{3n-3}\times 3^{n-1}$ *is divisible by* 25 *for all* $n\in N$

9. $1^2 + 3^2 + 5^2 + \cdots (2n-1)^2 = \frac{1}{3}n(4n^2-1)$
10. $4^{2n} + 15n - 1$ is divisible by 9 for all $n \in N$