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Matrices

Definition

A rectangular arrangement of numbers in rows and columns, is called a Matrix. This arrangement is enclosed by small ( ) or big [ ] brackets. A matrix is represented by capital letters A, B, C etc. and its element are by small letters a, b, c, x, y etc.

Order of Matrix

A matrix which has m rows and n columns is called a matrix of order m × n.

A matrix A of order m × n is usually written in the following manner-

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & ...a_{1j} & ...a_{1n} \\ a_{21} & a_{23} & a_{23} & ...a_{2j} & ...a_{2n} \\ ..... & ..... & ..... & ..... & ..... \\ a_{i1} & a_{i2} & a_{i3} & ...a_{ij} & ...a_{in} \\ ..... & ..... & ..... & ..... & ..... \\ a_{m1} & a_{m2} & a_{m3} & ...a_{mj} & ...a_{mn} \end{bmatrix} \text{ or}$$

$$A = [a_{ij}]m \times n \text{ where } \begin{matrix} i = 1, & 2,......m \\ i = 1, & 2,......n \end{matrix}$$

Here a_{ij} denotes the element of i^{th} row and j^{th} column.

Types of Matrix

Row matrix : If in a Matrix, there is only one row, then it is called a Row Matrix.

Thus $A = [a_{ij}]_{m \times n}$ is a row matrix if m = 1.

Column Matrix :

If in a Matrix, there is only one column, then it is called a Column Matrix.

Thus $A = [a_{ij}]_{m \times n}$ is a Column Matrix if n = 1.

Square Matrix If number of rows and number of column in a Matrix are equal, then it is called a Square Matrix.

Thus $A = [a_{ij}]_{m \times n}$ is a Square Matrix if m = n

Singleton Matrix :

If in a Matrix there is only one element then it is called Singleton Matrix. Thus

$A = [a_{ij}]_{m \times n}$ is a Singleton Matrix if m = n = 1.

Null or Zero Matrix :

If in a Matrix all the elements are zero then it is called a zero Matrix and it is generally denoted by O.

Thus $A = [a_{ij}]_{m \times n}$ is a zero matrix if a_{ij} = 0 for all i and j.

Diagonal Matrix :

If all elements except the principal diagonal in a **Square Matrix** are zero, it is called a Diagonal Matrix. Thus a Square Matrix

$A = [a_{ij}]$ is a Diagonal Matrix if $a_{ij} = 0$, when $i \neq j$

Scalar Matrix :

If all the elements of the diagonal of a **diagonal matrix** are equal , it is called a scalar matrix. Thus a Square Matrix $A = [a_{ij}]$ is a Scalar Matrix is

$$a_{ij} = \begin{cases} 0 & i \neq j \\ k & i = j \end{cases} \text{ where k is a constant.}$$

Unit Matrix :

If all elements of principal diagonal in a **Diagonal Matrix** are 1, then it is called

Unit Matrix. A unit Matrix of order n is denoted by I_n.

Thus a square Matrix

$A = [a_{ij}]$ is a unit Matrix if

$$a_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

Equal Matrix :

Two Matrix A and B are said to be equal Matrix if they are of same order and their corresponding elements are equal.

Addition and subtraction of matrix

If $A\ [a_{ij}]_{m \times n}$ and $[b_{ij}]_{m \times n}$ are two matrices of the same order then their sum A + B is a matrix whose each element is the sum of corresponding element.

i.e. $A + B = [a_{ij} + b_{ij}]_{m \times n}$

Similarly their subtraction A – B is defined as

$$A - B = [a_{ij} - b_{ij}]_{m \times n}$$

Properties of Matrices addition :

If A, B and C are Matrices of same order, then-

(i) A + B = B + A (Commutative Law)

(ii) (A+ B) + C = A + (B + C)

(Associative Law)

(iii) A + O = O + A = A, where O is zero matrix which is additive identity of the matrix.

(iv) A + (– A) = 0 = (–A) + A where (–A) is obtained by changing the sign of every element of A which is additive inverse of the Matrix

Scalar multiplication of matrix

Let $A = [a_{ij}]_{m \times n}$ be a matrix and k be a number then the matrix which is obtained by multiplying every element of A by k is called scalar multiplication of A by k and it is denoted by

kA thus if $A = [a_{ij}]_{m \times n}$ then

$$kA = Ak = [ka_{ij}]_{m \times n}$$

Properties of Scalar Multiplication :

If A, B are Matrices of the same order and α, μ are any two scalars then -

(i) $\alpha (A + B) = \alpha A + \alpha B$

(ii) $\alpha (\mu A) = (\alpha \mu) A = \mu(\alpha A)$

Multiplication of matrices

If A and B be any two matrices, then their product AB will be defined only when number of column in A is equal to the number of rows in B. If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$ then their product $AB = C = [c_{ij}]$, will be matrix of order m × p, where

$$(AB)_{ij} = C_{ij} = \sum_{r=1}^{n} a_{ir} b_{rj}$$

6.1 Properties of Matrix Multiplication :

If A, B and C are three matrices such that their product is defined, then

(i) AB ≠BA(Generally notcommutative)

(ii) (AB) C = A (BC) (Associative Law)

(iii)IA = A = AI

I is identity matrix for matrix multiplication

(iv)A (B + C) = AB + AC (Distributive Law)

Transpose of a Matrix

The matrix obtained from a given matrix A by changing its rows into columns or columns into rows is called transpose of Matrix A and is denoted by A^T or A'.

If order of A is m × n, then order of A^T is n × m.

Properties of Transpose :

(i) $(A^T)^T = A$

(ii) $(A \pm B)^T = A^T \pm B^T$

(iii) $(AB)^T = B^T A^T$

(iv) $(kA)^T = k(A)^T$

Symmetric Matrix : A square matrix $A = [a_{ij}]$ is called symmetric matrix if $a_{ij} = a_{ji}$ for all i,j or $A^T = A$

Skew - Symmetric Matrix : A square matrix $A = [a_{ij}]$ is called skew - symmetric matrix if $a_{ij} = -a_{ji}$ for all i, j or $A^T = -A$

Every square matrix A can uniquelly be expressed as sum of a symmetric and skew symmetric matrix i.e.

$$A = \left[\frac{1}{2}(A + A^T)\right] + \left[\frac{1}{2}(A - A^T)\right]$$

Inverse of Matrices

If A and B are two matrices such that

$$AB = I = BA$$

then B is called the inverse of A and it is denoted by A^{-1}, thus

$$A^{-1} = B \Leftrightarrow AB = I = BA$$

$$A^{-1} = \frac{\text{adj}A}{|A|}$$

Thus A^{-1} exists $\Leftrightarrow |A| \neq 0$

Properties of Inverse Matrix :

Let A and B are two invertible matrices of the same order, then

(i) $(A^T)^{-1} = (A^{-1})^T$

(ii) $(AB)^{-1} = B^{-1} A^{-1}$

(iii) $\text{adj}\,(A^{-1}) = (\text{adj}\,A)^{-1}$

(iv) $(A^{-1})^{-1} = A$

(v) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

Check Your Progress

1 If A is a matrix of order 3 × 4, then each row of A has-

(A) 3 elements (B) 4 elements

(C) 12 elements (D) 7 elements

2 In the following, upper triangular matrix is-

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & 4 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$

3 In the following, singular matrix is-

(A) $\begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$

4 If $A = \begin{bmatrix} 5 & 2 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then $|2A - 3B|$ equals-

(A) 77 (B) –53

(C) 53 (D) –77

5 If A and B are matrices of order m × n and n × n respectively, then which of the following are defined-

(A) AB, BA (B) AB, A^2

(C) A^2, B^2 (D) AB, B^2

6 If A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $A^2 + kI = 8A$, then k equals

(A) 4 (B) 8

(C) 1/4 (D) 1/16

7 If A,B,C are matrices of order 1 × 3, 3 × 3 and 3 × 1 respectively, the order of ABC will be-

(A) 3 × 3 (B) 1 × 3

(C) 1 × 1 (D) 3 × 1

8 If A = $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, B = $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$, then-

(A) AB = 0 (B) AB = 2I

(C) BA = 0 (D) $B^2 = I$

9 If A = $\begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 4 \\ 2 & -2 \end{bmatrix}$, then $(AB)^T$ is-

(A) $\begin{bmatrix} 11 & -2 \\ 5 & -6 \end{bmatrix}$ (B) $\begin{bmatrix} 11 & 5 \\ -2 & -6 \end{bmatrix}$

(C) $\begin{bmatrix} 7 & 1 \\ 0 & -8 \end{bmatrix}$ (D) $\begin{bmatrix} 7 & 0 \\ 1 & -8 \end{bmatrix}$

10 If A and B are matrices of order m × n and n × m respectively, then the order of matrix $B^T (A^T)^T$ is -

(A) m × n (B) m × m

(C) n × n (D) Not defined

11 If A, B, C, are three matrices, then $A^T + B^T + C^T$ is -

(A) zero matrix (B) A + B + C

(C) – (A + B+ C) (D) $(A + B + C)^T$

12 If A = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$, then correct statement is -

(A) AB = BA (B) $AA^T = A^2$

(C) $AB = B^2$ (D) None of these

13 If A= $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then AA^T equals-

(A) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

(B) $\begin{bmatrix} \cos^2\theta & \sin^2\theta \\ \sin^2\theta & \cos^2\theta \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

14 Matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is a-

(A) Diagonal matrix

(B) Upper triangular matrix

(C) Skew-symmetric matrix

(D) Symmetric matrix

15 If A and B are square matrices of same order, then which of the following is skew-symmetric-

(A) $\frac{A + A^T}{2}$ (B) $\frac{A^T + B^T}{2}$

(C) $\frac{A^T - B^T}{2}$ (D) $\frac{B - B^T}{2}$

Stretch Yourself

1. Find the inverse matrix of $\begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$
2. If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 0 & 2 \end{bmatrix}$, then find the value of adj (adj A) is-
3. If A = $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and X is a matrix such that A = BX, then find the value of X

Hint to Check Your Progress

1B ,2 B, 3 D, 4 B, 5 D, 6 B, 7C,
8 A, 9C, 10 D, 11 D ,12 D ,13 C ,14 C
,15 D,