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DIFFERENTIATION

Derivative of A Function

The limiting process indicated by $\lim_{\delta X \to 0} \frac{\delta y}{\delta x} = \lim_{\delta X \to 0} \frac{F(X+\delta X)-F(X)}{\delta X}$ is a mathematical operation. This mathematical process is known as differentiation and it yields a result called a derivative.

(2) A function whose derivative exists at a point is said to be derivable at that point.

(3) It may be verified that if f (x) is derivable at a point x = a, then, it must be continuous at that point. However, the converse is not necessarily true.

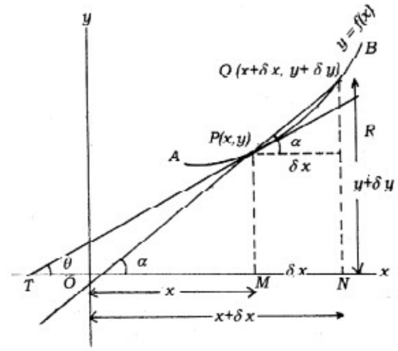
4) The symbols Δx and h are also used in place of δx

(5) If y = f (x), then $\frac{dY}{dX}$ is also denoted by y_1 or y'

Velocity as Limit

Velocity = $\lim_{\delta t \to 0} \frac{f(t+\delta t)-f(t)}{\delta t} = \frac{ds}{dt}$

Geometrical Interpretation of dy/dx



Derivative of Constant Function

The derivative of a constant is zero.

$$\frac{dx^n}{dx} = nx^{n-1}$$

Derivatives of Sum And Difference of Functions

I. $h'(x) = f'(x) + g'(x)$
(SUM Rule)

II. $h'(x) = f'(x) - g'(x)$
(Difference Rule)

III. $\frac{d[f(x)g(x)]}{dx} = f(x)g'(x) + g(x)f'(x)$
(Product Rule)

IV. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$
(Quotient Rule)

V. $\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$
(Chain Rule)

Check Yourself

1. If y = $(1+x^{1/4})(1+x^{1/2})(1-x^{1/4})$, then dy/dx equals-

   (A) –1 (B) 1

   (C) x (D) $\sqrt{x}$

2. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, then $\frac{dy}{dx}$ equals -

   (A) $\frac{1}{(1+x)^2}$ (B) $-\frac{1}{(1+x)^2}$

   (C) $\frac{1}{1+x^2}$ (D) None of these

3. If $x^y y^x = 1$, then $\frac{dy}{dx}$ equals -

   (A) $\frac{x(y+x\log y)}{y(x+y\log x)}$

   (B) $-\frac{x(x+y\log y)}{y(y+x\log x)}$

   (C) $\frac{y(y+x\log y)}{x(x+y\log x)}$

   (D) $-\frac{y(y+x\log y)}{x(x+y\log x)}$

4. If $\sqrt{1-x^2}+\sqrt{1-y^2}$ = a (x – y), then the value of dy/dx is -

   (A) $\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (B) $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

   (C) $-\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (D) $-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

5. If $f(x) = \frac{2x-4}{x^2-1}$ and $f'(x) = \frac{p}{(x^2-1)^2}$, then p equals-

   (A) $x^2 - 8x - 2$ (B) $-2x^2 + 8x + 2$

   (C) $4x + 2$ (D) $-2x^2 + 8x - 2$

6. If $y = \frac{x}{(x+5)}$, then $\frac{dx}{dy}$ equals-

   (A) $\frac{5}{(1-y)^2}$ (B) $\frac{5}{(1+y)^2}$

   (C) $\frac{1}{(1-y)^2}$ (D) None of these

7. If $y=\sqrt{\frac{1-x}{1+x}}$, then $\frac{dy}{dx}$ equals-

   (A) $\frac{y}{1-x^2}$ (B) $\frac{y}{x^2-1}$

   (C) $\frac{y}{1+x^2}$ (D) $\frac{y}{y^2-1}$

8. If $f(x) = \frac{2x^2-c}{x-2}$ and $f'(1) = 0$, then the value of c is-

   (A) 2 (B) 4

(C) 6 (D) 8

9. If $y = \frac{x+c}{1+x^2}$, then the value of xy where $\frac{dy}{dx} = 0$ is-

(A) 1/2 (B) 3/4

(C) 5/4 (D) None of these

10. If x = t +1/t , y = t –1/t , then $\frac{d^2y}{dx^2}$ equals –

(A) $-4t(t^2-1)^{-2}$

(B) $-4t^3(t^2-1)^{-3}$

(C) $(t^2+1)(t^2-1)^{-1}$

(D) $-4t^2(t^2-1)^{-2}$

Stretch Yourself

1. If $y^2x + x^2y + 3xy = 2$, then find $\frac{dy}{dx}$
2. If $x^3 - y^3 + 3xy^2 - 3x^2y + 1 = 0$, then find $\frac{dy}{dx}$ at (0, 1)
3. If $y = \frac{x\sqrt{2x+1}}{2x-1}$, then find dy/dx
4. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then find $\frac{dy}{dx}$
5. If $y = \left(1+\frac{1}{x}\right)^x$, Find $\frac{dy}{dx}$

Hint to Check Yourself

1 A	2 B	3 D	4 B	5 D
6 A	7 B	8 C	9 A	10 B