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Sequences and Series

Sequence

- A sequence is a collection of numbers specified in a definite order.

  $a_1, a_2, a_3, a_4, ... a_n$ is a sequence

  n = 1,2,3 - - - - is also a sequence

- **Finite Sequence :**

  A sequence having Finite number of terms 2, 4, 6, 8, 10 is a Finite sequence.

- **Infinite Sequence:**
  A sequence containing infinite number of terms is known as Infinite sequence.

- **Arithmetic Progression (A.P)**
  A sequence with Finite terms, in which the difference between two consecutive terms is the constant quantity, is called as Arithmetic progression (A.P).

  The nth term and Arithmetic progression as:
  $a_n = a + (n - 1)d$
  Where a is the first term d is the common difference.

- **Properties of Arithmetic Progression**

  (i) If the same non-zero number is added to each term of an arithmetic progression (AP) the resulting sequence is again an AP.
  (ii) If each term of an A.P. is multiplied by the same non-zero number, the resulting sequence is again an A.P.

Sum of First n terms of an AP

(i) $Sn = \frac{n}{2}(a + L)$

Where a is the first term and
L is the last term

(ii) $Sn = \frac{n}{2}\{2a + (n - 1)d\}$
where a is the first term
d is the common difference

- **Arithmetic Mean (A.M)**

  When three numbers a, A and b are in Arithmetic Progression A.P, then $A.M = \frac{a+b}{2}$

- **Geometric Progression (G.P)**
  A sequence of terms in which, the ratio of consecutive terms are constant same quantity, is called as Geometric Progression (G.P)

  General term of Geometric Progression

  $an = a\ r^{n-1}$

  Where a is the first term

  and r is the common ratio.

- **Properties of Geometric Progression**

  (i) In a Geometric progression, if a same non-zero quantity is multiplied with all the terms, then the resulting sequence is also a GP.

  If a, b, c, d - - - - are in GP and $k \neq 0$, then ak, bk, ck, dk - - - - is also a GP

(ii) If all the terms of a Geometric Progression (GP) are raised to the same constant power, then the resulting series is also a GP.

Let a, b, c, d - - - - are in GP
Then a^k, b^k, c^k, d^k are in GP for $(k \neq 0)$

- Sum of n terms of Finite GP

$$Sn = \frac{a\,(r^n - 1)}{r-1} \quad \text{For} \quad |r| >|$$

$$Sn = \frac{a\,(1-r^n)}{1-r} \quad \text{For} \quad |r| <|$$

- Sum of n terms of an Infinite GP

$$Sn = \frac{a}{1-r} \quad \text{When} \quad |r| <|$$

- ## Geometric Mean (GM)

If a, G, b are in Geometric progression, then G is called geometric mean

$$G = \sqrt{ab}$$

- Relationship between A.M and G.M
Let a and b be the two numbers, and A and G be the AM and GM respectively then

$$A > G \rightarrow AM > GM$$

Check yourself

Q1 What will be the n^{th} term of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\ldots\ldots\ldots$?

(A) $\frac{x}{1+2n}$

(B) $\frac{n}{2n}$

(C) $\frac{1}{n}$

(D) $\frac{n}{n+1}$

Q2 The 10^{th} term of the Arithmetic Progression $-2, 0, 2, 4\ldots\ldots\ldots$ is:
(A) 16
(B) 18
(C) 24
(D) 20

Q3 Which term of the Arithmetic progression $5, 11, 17\ldots\ldots$ is 119?
(A) 18
(B) 20
(C) 19
(D) 21

Q4 How many positive integers are between 100 to 200, those are divisible?
(A) 45
(B) 50
(C) 49
(D) 51

Q5 The 35^{th} term of on Arithmetic Progression is 69. Then the sum of its 69^{th} terms are equal to:
(A) 4537
(B) 4329
(C) 2345
(D) 4761

Q6 The m^{th} term of an Arithmetic Progression is n and the nth term is m. Then $(m+n)^{th}$ term of the A.P is equal to:
(A) 1
(B) 0
(C) -1
(D) 2

Q7 The 21^{st} term of an Arithmetic

progression is 124 and the term of the first is 4. Then the common difference of the sequence is:
(A) – 4
(B) 8
(C) 6
(D) 10

Q8 The sum of all natural numbers from 1 to 100 is equal to:
(A) 5050
(B) 5000
(C) 5500
(D) 5100

Q9 The sum of all 20^{th} term of the A.P 5,10,15,20……… is equal to:
(A) 1200
(B) 1250
(C) 1000
(D) 1050

Q10 The first Arithmetic mean (A.B) between the interests 8 and 12 is equal to:
(A) 20
(B) 16
(C) 10
(D) 48

Q11 In the Geometric progression (G.P) 5, –10, 20, –40 ….., which term is 320 ?
(A) 7^{th}
(B) 8^{th}
(C) 9^{th}
(D) 10^{th}

Q12 The 4^{th} and 9^{th} term of a Geometric progression (G.P) are 8 and 256 respectively. What will be the 3^{rd} term of the G.P ?
(A) 5
(B) 3
(C) 2
(D) 4

Q13 The sum of the Geometric progression (G.P) 1,3,9.27……… up to 10^{th} term is equal to:
(A) $\frac{1-3^9}{2}$
(B) $\frac{3^{10}-1}{2}$
(C) $3^{10}-2$
(D) $\frac{3^{10}-5}{2}$

Q14 How many term of the Geometric progression (G.P) 8, 16, 32, 64………. have their sum is 8184?
(A) 15
(B) 18
(C) 12
(D) 10

Q15 The sum of the infinite Geometric progression (G.P) $\frac{1}{3}, \frac{-2}{9}, \frac{4}{27}, \frac{-8}{81}$........ is equal to:
(A) $\frac{1}{5}$
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) $\frac{2}{5}$

Answer to check yourself

1 D	2 A	3 B	4C	5 D
6 B	7 C	8 A	9 D	10 C
11 A	12 D	13 B	14 D	15 A

Stretch Yourself

1. If S_n denotes the sum of n terms of an A.P., then find $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$
2. Find the sum of integers from 1 to 100 that are divisible by 2 or 3
3. If the sum to n terms of a series is given by $\frac{n(n+1)(n+2)}{6}$ then find the n^{th} term of the series
4. Find the sum of 10 terms of the series.
    a. $0.7 + .77 + .777 + \ldots$