7

Some Special Sequences

Series:

- A series is associated with Sequence. A series is a sum of terms with definite order.
- An expression of the form $u_1 + u_2 + \cdots .. u_n$ is called series, where u_1, u_2 ….. is a sequence of numbers.
  Denoted by $\sum_{r=1}^{n} u_r$

  If n is finite then the series is finite series, otherwise the series is infinite.

- Sum of the powers of the first n-natural numbers

  $$Sn = \frac{n\,(n+1)}{2}$$

- Sum of squares of the first n-natural numbers

  $$Sn = 1^2 + 2^2 + 3^2 + - - - + n^2$$

  $$Sn = \frac{n\,(n+1)(2n+1)}{6}$$

  $$\sum n^2 = \frac{n\,(n+1)(2n+1)}{6}$$

- The sum of the Cubes of the first n-natural numbers

  $$Sn = 1^3 + 2^3 + 3^3 + - - - \mp n^3$$

  $$\sum n^3 = \left[\frac{n\,(n+1)}{2}\right]$$

- The sum of the series the n tn term of the series (tn), Sn = ∑ tn

Check Your Self

Find the sum of the following series to n terms

1. $\frac{1^2}{1} + \frac{1^2+2^2}{1+3} + \frac{1^2+2^2+3^2}{1+3+5} + \cdots.$
2. $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \cdots.$
3. 1.2.4+2.3.7+3.4.10+ ……
4. 1+ (1+2) + (1+2+3)+ (1+2+3+4)……
5. 1.2.5+2.3.6+3.4.7+…….
6. 5+7+13+31+85+….
7. 2+4+7+11+16+….
8. $2^2 + 4^2 + 6^2 + \cdots ....$
9. $1.2^2 + 2.3^2 + 3.4^2 + \cdots ..$
10. $2 + 10 + 30 + 68 + 130 + ...$

Hint to check yourself

1. $\frac{n}{24}(2n^2 + 9n + 12)$
2. $\frac{n}{12}(n + 1^2)(n + 2)$
3. $\frac{n}{12}(n + 1)(9n^2 + 25n + 14)$
4. $\frac{n}{6}(n + 1)(n + 2)$
5. $\frac{n}{12}(n + 1)(3n^2 + 23n + 34)$

6. $\frac{1}{2}(3^n + 8n - 1)$
7. $\frac{n}{6}(n^2 + 3n + 8)$
8. $\frac{2n}{3}(n + 1)(2n + 1)$

9. $\frac{n}{2}(n + 1)(n + 2)(3n + 5)$
10. $\frac{n}{4}(n + 1)(n^2 + n + 2)$