312en08

8

ELASTIC PROPERTIES OF SOLIDS

In the previous lessons you have studied the effect of force on a body to produce displacement. The force applied on an object may also change its shape or size. For example, when a suitable force is applied on a spring, you will find that its shape as well as size changes. But when you remove the force, it will regain original position. Now apply a force on some objects like wet modelling clay or molten wax. Do they regain their original position after the force has been removed? They do not regain their original shape and size. Thus some objects regain their original shape and size whereas others do not. Such a behaviour of objects depends on a property of matter called **elasticity**.

The elastic property of materials is of vital importance in our daily life. It is used to help us determine the strength of cables to support the weight of bodies such as in cable cars, cranes, lifts etc. We use this property to find the strength of beams for construction of buildings and bridges. In this unit you will learn about nature of changes and the manner in which these can be described.

OBJECTIVES

After studying this lesson, you should be able to :

- *distinguish between three states of matter on the basis of molecular theory;*
- *distinguish between elastic and plastic bodies;*
- *distinguish between stress and pressure;*
- *study stress-strain curve for an elastic solid;*
- *define Young's modulus, bulk modulus, modulus of rigidity and Poisson's ratio; and*
- *derive an expression for the elastic potential energy of a spring.*

8.1 MOLECULAR THEORY OF MATTER : INTER-MOLECULAR FORCES

We know that matter is made up of atoms and molecules. The forces which act between them are responsible for the structure of matter. The interaction forces between molecules are known as **inter-molecular forces**.

The variation of inter molecular forces with inter molecular separation is shown in Fig. 8.1.

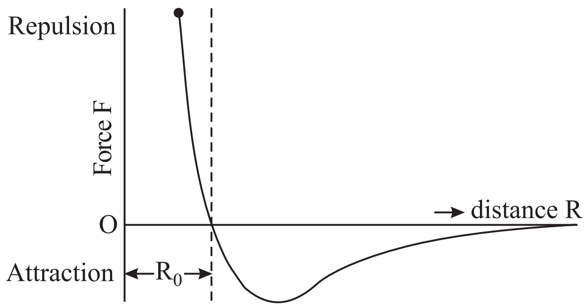


Fig. 8.1 : Graph between inter-molecular force and Inter molecular separation.

When the separation is large, the force between two molecules is attractive and weak. As the separation decreases, the net force of attraction increases up to a particular value and beyond this, the force becomes repulsive. At a distance $R = R_0$ the net force between the molecules is zero. This separation is called **equilibrium separation**. Thus, if inter-molecular separation $R > R_0$ there will be an attractive force between molecules. When $R < R_0$, a repulsive force will act between them.

In **solids**, molecules are very close to each other at their equilibrium separation ($\sim 10^{-10}$ m). Due to high intermolecular forces, they are almost fixed at their positions. You may now appreciate why a solid has a definite shape.

In **liquids**, the average separation between the molecules is somewhat larger ($\sim 10^{-8}$ m). The attractive force is weak and the molecules are comparatively free to move inside the whole mass of the liquid. You can understand now why a liquid does not have fixed shape. It takes the shape of the vessel in which it is filled.

In **gases**, the intermolecular separation is significantly larger and the molecular force is very weak (almost negligible). Molecules of a gas are almost free to move inside a container. That is why gases do not have fixed shape and size.

Ancient Indian view about Atom

Kanada was the first expounder of the atomic concept in the world. He lived around 6th century B.C. He resided at Prabhasa (near Allahabad).

According to him, everything in the universe is made up of **Parmanu** or Atom. They are eternal and indestructible. Atoms combine to form different molecules. If two atoms combine to form a molecule, it is called **duyanuka** and a triatomic molecule is called triyanuka. He was the author of "Vaisesika Sutra".

The size of atom was also estimated. In the biography of Buddha (Lalitavistara), the estimate of atomic size is recorded to be of the order 10^{-10} m, which is very close to the modern estimate of atomic size.

8.2 ELASTICITY

You would have noticed that when an external force is applied on an object, its shape or size (or both) change, i.e. deformation takes place. The extent of deformation depends on the material and shape of the body and the external force. When the deforming forces are withdrawn, the body tries to regain its original shape and size.

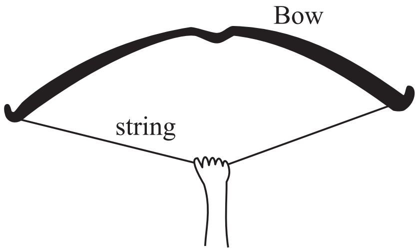


Fig 8.2 : Force applied on the string of a bow changes it shape

You may compare this with a spring loaded with a mass or a force applied on the string of a bow or pressing of a rubber ball. If you apply a force on the string of the bow to pull it (Fig 8.2), you will observe that its shape changes. But on releasing the string, the bow regains its original shape and size.

The property of matter to regain its original shape and size after removal of the deforming forces is called **elasticity**.

8.2.1 Elastic and Plastic Bodies

A body which regains its original state completely on removal of the deforming force is called **perfectly elastic**. On the other hand, if it completely retains its modified form even on removing the deforming force, i.e. shows no tendency to recover the deformation, it is said to be **perfectly plastic**. However, in practice the behaviour of all bodies is in between these two limits. There exists no perfectly elastic or perfectly plastic body in nature. The nearest approach to a perfectly elastic body is quartz fiber and to the perfectly plastic is ordinary putty. Here it can be added that the object which opposes the deformation more is more elastic. No doubt elastic deformations are very important in science and technology, but plastic deformations are also important in mechanical processes. You might have seen the processes such as stamping, bending and hammering of metal pieces. These are possible only due to plastic deformations.

The phenomenon of elasticity can be explained in terms of inter-molecular forces.

8.2.2 Molecular Theory of Elasticity

You are aware that a solid is composed of a large number of atoms arranged in a definite order. Each atom is acted upon by forces due to neighbouring atoms.

Due to inter-atomic forces, solid takes such a shape that each atom remains in a stable equilibrium. When the body is deformed, the atoms are displaced from their original positions and the inter-atomic distances change. If in deformation, the separation increases beyond their equilibrium separation (i.e., $R > R_0$), strong attractive forces are developed. However, if inter–atomic separation decreases (i.e. $R < R_0$), strong repulsive forces develop. These forces, called **restoring forces,** drive atoms to their original positions. The behaviour of atoms in a solid can be compared to a system in which balls are connected with springs.

Now, let us learn how forces are applied to deform a body.

8.2.3 Stress

When an external force or system of forces is applied on a body, it undergoes a change in the shape or size according to nature of the forces. We have explained that in the process of deformation, internal restoring force is developed due to molecular displacements from their positions of equilibrium. The internal restoring force opposes the deforming force. **The internal restoring force acting per unit area of cross-section of a deformed body is called *stress*.**

In equilibrium, the restoring force is equal in magnitude and opposite in direction to the external deforming force. Hence, stress is measured by the external force per unit area of cross-section when equilibrium is attained. If the magnitude of deforming force is F and it acts on area A, we can write

$$\text{Stress} = \frac{\text{restoring force}}{\text{area}} = \frac{\text{deforming force } (F)}{\text{area } (A)}$$

or
$$\text{Stress} = \frac{F}{A} \tag{8.1}$$

The unit of stress is Nm^{-2}. The stress may be longitudinal, normal or shearing. Let us study them one by one.

(i) **Longitudinal Stress :** If the deforming forces are along the length of the body, we call the stress produced as **longitudinal stress,** as shown in its two forms in Fig 8.3 (a) and Fig 8.3 (b).

Fig. 8.3 (a) : Tensile stress; (b) Compressive stress

(ii) **Normal Stress :** If the deforming forces are applied uniformly and normally all over the surface of the body so that the change in its volume occurs without change in shape (Fig. 8.4), we call the stress produced as normal stress. You may produce normal stress by applying force uniformly over the entire surface of the body. Deforming force per unit area normal to the surface is called pressure while restoring force developed inside the body per unit area normal to the surface is known as **stress**.

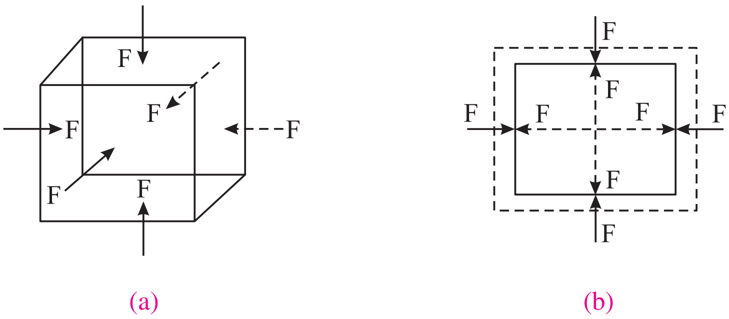


Fig. 8.4 : Normal stress

(iii) **Shearing Stress :** If the deforming forces act tangentially or parallel to the surface (Fig 8.5a) so that shape of the body changes without change in volume, the stress is called **shearing stress**. An example of shearing stress is shown in Fig 8.5 (b) in which a book is pushed side ways. Its opposite face is held fixed by the force of friction.

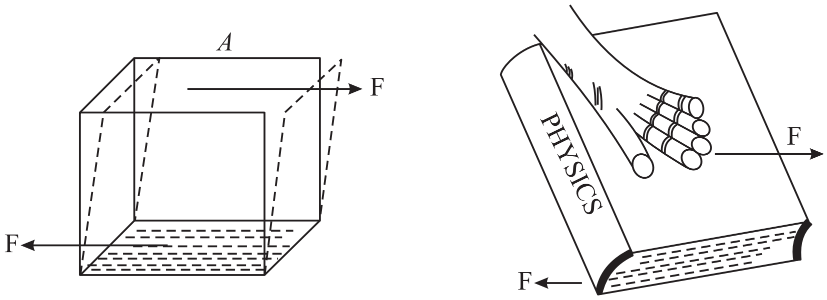


Fig. 8.5: (a) Shearing stress; (b) Pushing a book side ways

8.2.4 Strain

Deforming forces produce changes in the dimensions of the body. In general, the **strain is defined as the change in dimension (e.g. length, shape or volume) per unit dimension of the body.** As the strain is ratio of two similar quantities, it is a dimensionless quantity.

Depending on the kind of stress applied, strains are of three types : (i) linear strain,(ii) volume (bulk) strain, and (iii) shearing strain.

(i) **Linear Strain :** If on application of a longitudinal deforming force, the length ℓ of a body changes by $\Delta\ell$ (Fig. 8.6), then

Fig. 8.6: Linear strain

$$\text{linear strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta\ell}{\ell}$$

(ii) **Volume Strain :** If on application of a uniform pressure Δp, the volume V of the body changes by ΔV (Fig 8.7) without change of shape of the body, then

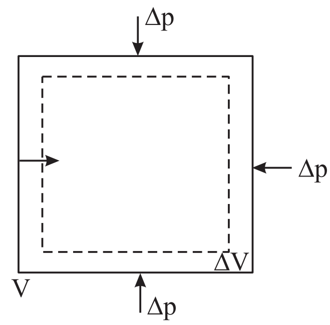
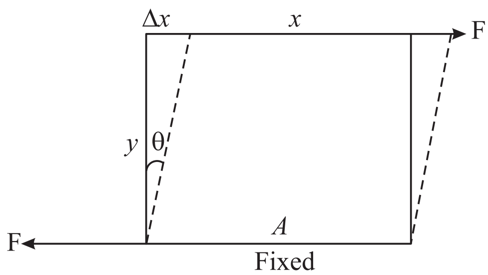


Fig. 8.7: Volume strain

$$\text{Volume strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

(i) **Shearing strain:** When the deforming forces are tangential (Fig 8.8), the shearing strain is given by the angle θ through which a line perpendicular to the fixed plane is turned due to deformation. (The angle θ is usually very small.) Then we can write



$$\theta = \frac{\Delta x}{y}$$

Fig. 8.8 : Shearing strain

8.2.5 Stress-strain Curve for a Metallic Wire

Refer to Fig. 8.9 which shows variation of stress with strain when a metallic wire of uniform cross-section is subjected to an increasing load. Let us study the regions and points

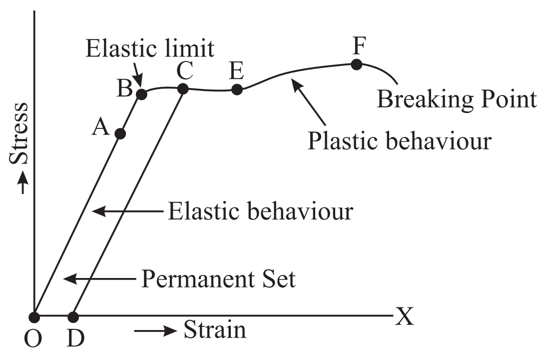


Fig. 8.9: Stress-strain curve for a steel wire

on this curve that are of particular importance.

(i) **Region of Proportionality:** OA is a straight line which indicates that in this region, stress is linearly proportional to strain and the body behaves like a perfectly elastic body.

(ii) **Elastic Limit** : If we increase the strain a little beyond A, the stress is not linearly proportional to strain. However, the wire still remains elastic, i.e. after removing the deforming force (load), it regains its original state. The maximum value of strain for which a body(wire) shows elastic property is called **elastic limit.** Beyond the elastic limit, a body behaves like a plastic body.

(iii) **Point** C : When the wire is stretched beyond the limit B, the strain increases more rapidly and the body becomes plastic. It means that even if the deforming load is removed, the wire will not recover its original length. The material follows dotted line CD on the graph on gradual reduction of load. The left over strain on zero load strain is known as a **permanent set**. After point E on the curve, no extension is recoverable.

(iv) **Breaking point** F : Beyond point E, strain increases very rapidly and near point F, the length of the wire increases continuously even without increasing of load. The wire breaks at point F. This is called the **breaking point** or **fracture point** and the corresponding stress is known as **breaking stress**.

The stress corresponding to breaking point F is called **breaking stress** or **tensile strength**. Within the elastic limit, the maximum stress which an object can be subjected to is called **working stress** and the ratio between working stress and breaking stress is called **factor of safety.** In U.K, it is taken 10, in USA it is 5. We have adopted UK norms. If large deformation takes place between the elastic limit and the breaking point, the material is called **ductile**. If it breaks soon after the elastic limit is crossed, it is called **brittle e.g.** glass.

8.2.6 Stress-Strain Curve for Rubber

When we stretch a rubber cord to a few times its natural length, it returns to its original length after removal of the forces. That is, the elastic region is large and there is no well defined plastic flow region. Substances having large strain are called **elastomers**. This property arises from their molecular arrangements. The stress-strain curve for rubber is distinctly different from that of a metallic wire. There are two important things to note from Fig. 8.10. Firstly, you can observe that there is no region of proportionality. Secondly, when the deforming force is gradually reduced, the original curve is not retraced, although the sample finally acquires its natural length. The work done by the material in returning to its original shape is less than the work done by the deforming force. This difference of energy is absorbed by the material and appears as heat. (You can feel it by touching the rubber band with your lips.) This phenomenon is called **elastic hysteresis**.

Elastic hysteresis has an important application in **shock absorbers**. A part of energy transferred by the deforming force is retained in a shock absorber and only a small part of it is transmitted to the body to which the shock absorber is attached.

8.2.7 Steel is more Elastic than Rubber

A body is said to be more elastic if on applying a large deforming force on it, the strain produced in the body is small. If you take two identical rubber and steel wires and apply equal deforming forces on both of them, you will see that the extension produced in the steel wire is smaller than the extension produced in the rubber wire. But to produce same strain in the two wires, significantly higher stress is required in the steel wire than in rubber wire. Large amount of stress needed for deformation of steel indicates that magnitude of internal restoring force produced in steel is higher than that in rubber. Thus, steel is more elastic than rubber.

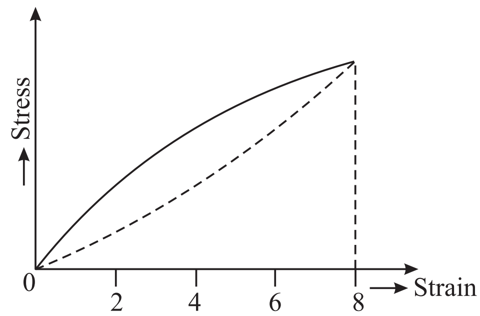


Fig. 8.10: Stress-strain curve for rubber

Example 8.1 : A load of 100 kg is suspended by a wire of length 1.0 m and cross sectional area 0.10 cm^2. The wire is stretched by 0.20 cm. Calculate the (i) tensile stress, and (ii) strain in the wire. Given, $g = 9.80$ ms^{-2}.

Solution :

$$\text{(i) Tensile stress} = \frac{F}{A} = \frac{Mg}{A}$$

$$= \frac{(100\,\text{kg})\,(9.80\,\text{ms}^{-2})}{0.10 \times 10^{-4}\,\text{m}^2}$$

$$= 9.8 \text{ x } 10^7\,\text{Nm}^{-2}$$

$$\text{(ii) Tensile strain} = \frac{\Delta\ell}{\ell} = \frac{0.20 \times 10^{-2}\,\text{m}}{1.0\,\text{m}}$$

$$= 0.20 \times 10^{-2}$$

Example 8.2 : Calculate the maximum length of a steel wire that can be suspended without breaking under its own weight, if its breaking stress = 4.0 x 10^8 Nm^{-2}, density = 7.9×10^3 kg m^{-3} and g = 9.80 ms^{-2}

Solution : The weight of the wire $W = A\ell\rho g$, where, A is area of cross section of the wire, ℓ is the maximum length and ρ is the density of the wire. Therefore, the breaking stress developed in the wire due to its own weight $\frac{W}{A} = \rho l g$. We are told that

breaking stress is 4.0 x 10^8 Nm^{-2}. Hence

$$\ell = \frac{4.0 \times 10^8\,\text{Nm}^{-2}}{(7.9 \times 10^3\,\text{kg m}^{-3})\,(9.8\,\text{ms}^{-2})}$$

$$= 0.05 \times 10^5\,\text{m}$$

$$= 5 \times 10^3\,\text{m} = 5\,\text{km}.$$

Now it is time to take a break and check your understanding

INTEXT QUESTIONS 8.1

1. What will be the nature of inter-atomic forces when deforming force applied on an object (i) increases, (ii) decreases the inter-atomic separation?
2. If we clamp a rod rigidly at one end and a force is applied normally to its cross section at the other end, name the type of stress and strain?
3. The ratio of stress to strain remains constant for small deformation of a metal wire. For large deformations what will be the changes in this ratio?
4. Under what conditions, a stress is known as breaking stress ?
5. If mass of 4 kg is attached to the end of a vertical wire of length 4 m with a diameter 0.64 mm, the extension is 0.60 mm. Calculate the tensile stress and strain?

8.3 HOOKE'S LAW

In 1678, Robert Hooke obtained the stress-strain curve experimentally for a number of solid substances and established a law of elasticity known as Hooke's law. According to this law: **Within elastic limit, stress is directly proportional to corresponding strain.**

i.e. stress α strain

or

$$\frac{\text{stress}}{\text{strain}} = \text{constant } (E) \qquad (8.2)$$

This constant of proportionality E is a measure of elasticity of the substance and is called **modulus of elasticity**. As strain is a dimensionless quantity, the modulus of elasticity has the same dimensions (or units) as stress. Its value is independent of the stress and strain but depends on the nature of the material. To see this, you may like to do the following activity.

ACTIVITY 8.1

Arrange a steel spring with its top fixed with a rigid support on a wall and a metre scale along its side, as shown in the Fig. 8.11.

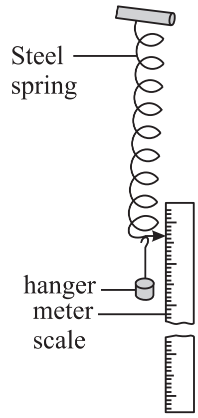


Fig. 8.11: Hooke's law apparatus

Add 100 g load at a time on the bottom of the hanger in steps. It means that while putting each 100 g load, you are increasing the stretching force by 1N. Measure the extension. Take the reading upto 500 g and note the extension each time.

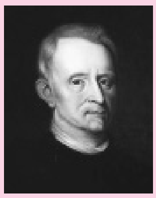
Plot a graph between load and extension. What is the shape of the graph? Does it obey Hooke's law?

The graph should be a straight line indicating that the ratio (load/ extension) is constant.

Repeat this activity with rubber and other materials.

You should know that the materials which obey Hooke's law are used in spring balances or as force measurer, as shown in the Fig. 8.11. You would have seen that when some object is placed on the pan, the length of the spring increases. This increase in length shown by the pointer on the scale can be treated as a measure of the increase in force (i.e., load applied).

Robert Hooke
(1635 – 1703)



Robert Hooke, experimental genius of seventeenth century, was a contemporary of Sir Isaac Newton. He had varied interests and contributed in the fields of physics, astronomy, chemistry, biology, geology, paleontology, architecture and naval technology. Among other accomplishments he has to his credit the invention of a universal joint, an early proto type of the respirator, the iris diaphragm, anchor escapement and balancing spring for clocks. As chief surveyor, he helped rebuild London after the great fire of 1666. He formulated Hooke's law of eleasticity and correct theory of combustion. He is also credited to invent or improve meteorological instruments such as barometer, anemometer and hygrometer.

8.3.1 Moduli of Elasticity

In previous sections, you have learnt that there are three kinds of strain. It is therefore clear that there should be three modulli of elasticity corresponding to these strains. These are **Young's modulus, Bulk Modulus and Modulus of rigidity** corresponding to linear strain, volume strain and shearing strain, respectively. We now study these one by one.

(i) **Young's Modulus:** The ratio of the longitudinal stress to the longitudinal strain is called Young's modulus for the material of the body.

Suppose that when a wire of length L and area of cross-section A is stretched by a force of magnitude F, the change in its length is equal to ΔL. Then

$$\text{Longitudinal stress} = \frac{F}{A}$$

and

$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

Hence, $$\text{Young's modulus Y} = \frac{F/A}{\Delta L/L} = \frac{F \times L}{A \times \Delta L}$$

If the wire of radius r is suspended vertically with a rigid support and a mass M hangs at its lower end, then $A = \pi r^2$ and $F = M\,g$.

$$\therefore \qquad Y = \frac{M\,g\,L}{\pi\,r^2 \Delta\,L} \qquad (8.3)$$

The SI unit of Y in is N m^{-2}. The values of Young's modulus for a few typical substances are given in Table. 8.1. Note that steel is most elastic.

Table 8.1. Young's modulus of some typical materials

Name of substance	Y (10^9Nm^{-2})
Aluminium	70
Copper	120
Iron	190
Steel	200
Glass	65
Bone	9
Polystyrene	3

(ii) **Bulk Modulus:** The ratio of normal stress to the volume strain is called bulk modulus of the material of the body.

If due to increase in pressure P, volume V of the body decreases by ΔV without change in shape, then

$$\text{Normal stress } = \Delta P$$

$$\text{Volume strain } = \Delta V/V$$

$$\text{Bulk modulus } B = \frac{\Delta P}{\Delta V/V} = V\frac{\Delta P}{\Delta V} \qquad (8.4)$$

The reciprocal of bulk modulus of a substance is called compressibility :

$$k = \frac{1}{B} = \frac{1}{V}\frac{\Delta V}{\Delta P} \qquad (8.5)$$

Gases being most compressible are least elastic while solids are most elastic or least compressible i.e. $B_{solid} > B_{liquid} > B_{gas}$

(iii) **Modulus of Rigidity or Shear Modulus:** The ratio of the shearing stress to shearing strain is called modulus of rigidity of the material of the body.

If a tangential force F acts on an area A and θ is the shearing strain, the modulus of rigidity

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} \qquad (8.6)$$

You should know that both solid and fluids have bulk modulus. However, fluids do not have Young's modulus and shear modulus because a liquid can not sustain a tensile or shearing stress.

Example 8.3 : Calculate the force required to increase the length of a wire of steel of cross sectional area 0.1 cm^2 by 50%. Given Y = 2 × 10^{11} N m^{-2}.

Solution : Increase in the length of wire = 50%. If ΔL is the increase and L is the normal length of wire then $\frac{\Delta L}{L} = \frac{1}{2}$

$$\therefore \qquad \text{Y} = \frac{F \times L}{A \times \Delta L}$$

or

$$F = \frac{\text{Y} \times A \times \Delta L}{L} = \frac{(2\times10^{11}\,\text{Nm}^{-2})\,(0.1\times10^{-4}\,\text{m}^2)\times1}{2}$$

$$= 0.1 \times 10^7 \text{ N} = 10^6 \text{ N}$$

Example 8.4 : When a solid rubber ball is taken from the surface to the bottom of a lake, the reduction in its volume is 0.0012 %. The depth of lake is 360 m, the density of lake water is 10^3 kgm^{-3} and acceleration due to gravity at the place is 10 m s^{-2}. Calculate the bulk modulus of rubber.

Solution :

Increase of pressure on the ball

$$P = h\rho g = 360\text{m} \times 10^3\,\text{kgm}^{-3} \times 10\ \text{ms}^{-3}$$
$$= 3.6 \times 10^6\ \text{Nm}^{-2}$$

$$\text{Volume strain} = \frac{\Delta V}{V} = \frac{0.0012}{100} = 1.2 \times 10^{-5}$$

$$\text{Bulk Modulus } B = \frac{PV}{\Delta V} = \frac{3.6\times10^6}{1.2\times10^{-5}} = 3.0 \times 10^{11}\ \text{Nm}^{-2}$$

8.3.2 Poisson's Ratio

You may have noticed that when a rubber tube is stretched along its length, there is a contraction in its diameter (Fig.8.12). (This is also true for a wire but may not be easily visible.) While the length increases in the direction of forces, a contraction occurs in the perpendicular direction. The strain perpendicular to the applied force is called **lateral strain**. Poisson pointed out that within elastic limit, lateral strain is directly proportional to longitudinal strain i.e. the ratio of lateral strain to longitudinal strain is constant for a material body and is known as **Poisson's ratio**. It is denoted by a Greek letter σ (sigma). If α and β are the longitudinal strain and lateral strain respectively, then Poisson's ratio

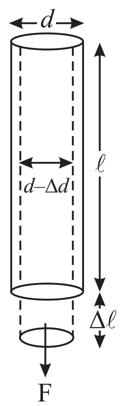


Fig. 8.12 : A stretched rubber tube.

$$\sigma = \beta/\alpha.$$

If a wire (rod or tube) of length ℓ and diameter d is elongated by applying a stretching force by an amount $\Delta\ell$ and its diameter decreases by Δd, then longitudinal strain

$$\alpha = \frac{\Delta\ell}{\ell}$$

lateral strain
$$\beta = \frac{\Delta d}{d}$$

and Possion's ratio
$$\sigma = \frac{\Delta d/d}{\Delta\ell/\ell} = \frac{\ell}{d}\ \frac{\Delta d}{\Delta\ell} \qquad (8.7)$$

Since Poisson's ratio is a ratio of two strains, it is a pure number.

The value of Poisson's ratio depends only on the nature of material and for most of the substances, it lies between 0.2 and 0.4. When a body under tension suffers no change in volume, i.e. the body is perfectly incompressible, the value of Poisson's ratio is maximum i.e. 0.5. Theoretically, the limiting values of Poisson's ratio are –1 and 0.5.

ACTIVITY 8.2

Take two identical wires. Make one wire to execute torsional vibrations for some time. After some time, set the other wire also in similar vibrations. Observe the rate of decay of vibrations of the two wires.

You will note that the vibrations decay much faster in the wire which was vibrating for longer time. The wire gets tired or fatigued and finds it difficult to continue vibrating. This phenomenon is known as **elastic fatigue**.

Some other facts about elasticity :

1. If we add some suitable impurity to a metal, its elastic properties are modified. For example, if carbon is added to iron or potassium is added to gold, their elasticity increases.
2. The increase in temperature decreases elasticity of materials. For example, carbon, which is highly elastic at ordinary temperature, becomes plastic when heated by a current through it. Similarly, plastic becomes highly elastic when cooled in liquid air.
3. The value of modulus of elasticity is independent of the magnitude of stress and strain. It depends only on the nature of the material of the body.

Example 8.5: A Metal cube of side 20 cm is subjected to a shearing stress of 10^4 Nm^{-2}. Calculate the modulus of rigidity, if top of the cube is displaced by 0.01 cm. with respect to bottom.

Solution : Shearing stress= 10^4 Nm^{-2}, $\Delta x = 0.01$ cm, and $y = 20$ cm.

$$\therefore \qquad \text{Shearing strain} = \frac{\Delta x}{y} = \frac{0.01\,\text{cm}}{20\,\text{cm}}$$

Hence, $= 0.005$

$$\text{Modulus of rigidity} \quad \eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{10^4\,\text{Nm}^{-2}}{.0005}$$

$$= 2 \times 10^7\ \text{N m}^{-2}$$

Example 8.6 : A 10 kg mass is attached to one end of a copper wire of length 5 m long and 1 mm in diameter. Calculate the extension and lateral strain, if Poisson's ratio is 0.25. Given Young's modulus of the wire = 11×10^{10} N m^{-2}.

Solution : Here L= 5 m, $r = 0.05 \times 10^{-3}$ m, $y = 11 \times 10^{10}$ Nm^{-2} $F = 10 \times 9.8$ N, and $\sigma = 0.25$.

Extension produced in the wire

$$\Delta \ell = \frac{F.\ell}{\pi r^2 Y} = \frac{(10\,\text{kg}) \times (9.8\text{ms}^{-2}) \times (5\text{m})}{3.14\,(0.5 \times 10^{-3}\,\text{m})^2 \times (11 \times 10^{10}\,\text{Nm}^{-2})}$$

$$= \frac{490}{8.63 \times 10^4}\ \text{m}$$

$$= 5.6 \times 10^{-3}\ \text{m}$$

$$\text{longitudinal streain} = \alpha = \frac{\Delta\ell}{\ell}$$

$$= \frac{5.6 \times 10^{-3}\,\text{m}}{5\text{m}}$$

$$= 1.12 \times 10^{-2}$$

$$\text{Poission's ratio } (\sigma) = \frac{\text{lateral strain}(\beta)}{\text{longitudinal strain}(\alpha)}$$

$$\therefore \qquad \text{lateral strain } \beta = \sigma \times \alpha$$

$$= 0.125 \times 1.12 \times 10^{-2}$$

$$= 0.14 \times 10^{-3}.$$

Now take a break to check your progress.

8.3.3 Elastic Energy

When a spring is either compressed or extended, it undergoes a change in its configuration and is capable of performing work.

Elastic energy is a kind of potential energy and it is the energy which is associated with the state of compression or extension of an elastic object like a spring. The force involved here is the spring force. If we compress or extend a spring, we change the relative locations of the coil of the spring. In case of a rubber like tube we change the relative locations of its different layers. A restoring force resists the change and result in work done by us due to which increases the elastic potential energy of the spring or such like objects increases.

Suppose the spring constant of a spring is k. If the spring is stretched through a distance x at any instant (Fig. 8.3.3), then the force applied is given by,

$$F = kx$$

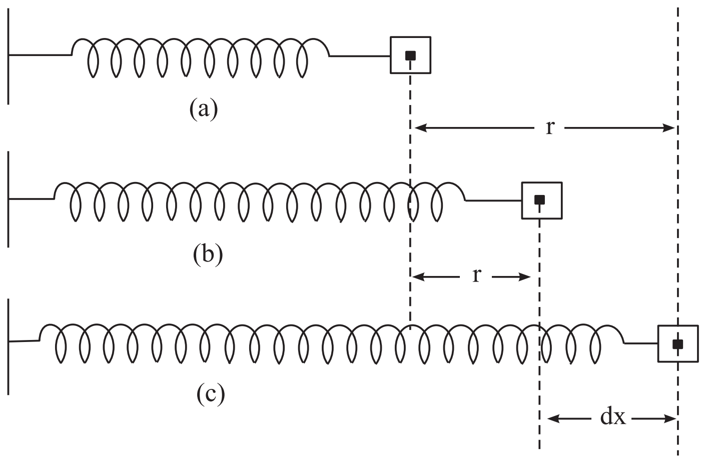


Fig. 8.13

If the spring is further stretched by a small distance dx as shown in the Fig. 8.13 then the small work done

$$dW = kx.dx$$

Therefore, the total work done in stretching the spring through a total distance r from its equilibrium position (Fig. 8.3.3) is given by

$$W = \int_0^r kxdx = k\left[\frac{x^2}{2}\right]_0^r = \frac{1}{2}kr^2$$

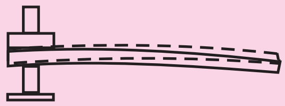
Hence the elastic potential energy $U = \frac{1}{2}kr^2$.

INTEXT QUESTIONS 8.2

1. Is the unit of longitudinal stress same as that of Young's modulus of elasticity? Give reason for your answer.
2. Solids are more elastic than liquids and gases. Justify
3. The length of a wire is cut to half. What will be the effect on the increase in its length under a given load?

4. Two wires are made of the same metal. The length of the first wire is half that of the second and its diameter is double that of the second wire. If equal loads are applied on both wires, find the ratio of increase in their lengths?
5. A wire increases by 10^{-3} of its length when a stress of 1×10^{8} Nm^{-2} is applied to it. Calculate Young's modulus of material of the wire.
6. Calculate the elastic potential energy stored in a spring of spring constant 200 Nm^{-1} when it is stretched through a distance of 10 cm.

Applications of Elastic Behaviour of Materials


(i)

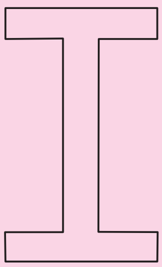
Elastic behaviour of materials plays an important role in our day to day life. Pillars and beams are important parts of our structures. A uniform beam clamped at one end and loaded at the other is called a Cantilever [Fig.(i)]. The hanging bridge of Laxman Jhula in Rishkesh and Vidyasagar Sethu in Kolkata are supported on cantilevers.

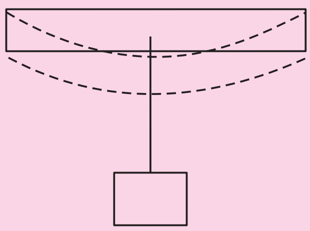
A cantilever of length *l*, breadth *b* and thickness *d* undergoes a depression δ at its free end when it is loaded by a weight of mass *M* :

$$\delta = \frac{4M\ g\,\ell^3}{\gamma\, b\, d^3}$$

It is now easy to understand as to why the cross-section of girders and rails is kept I-shaped (Fig. ii). Other factors remaining same, $\delta \propto d^{-3}$. Therefore, by increasing thickness, we can decrease depression under the same load more effectively. This considerably saves the material without sacrificing strength for a beam clamped at both ends and loaded in the middle (Fig.iii), the sag in the middle is given by

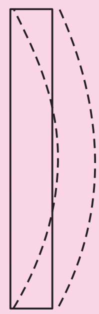
$$\delta = \frac{M\ g\,\ell^3}{4\, b\, d^3 \gamma}$$


(ii)


(iii

Thus for a given load, we will select a material with a large Young's modulus Y and again a large thickness to keep δ small. However, a deep beam may have a tendency to buckle (Fig iv). To avoid this, a large load bearing surface is provided. In the form I-shaped cross-section, both these requirements are fulfilled.

In cranes, we use a thick metal rope to lift and move heavy loads from one place to another. To lift a load of 10 metric tons with a steel rope of yield strength 300 mega pascal, it can be shown, that the minimum area of cross section required will be 10 cm or so. A single wire of this radius will practically be a rigid rod. That is why ropes are always made of a large number of turns of thin wires braided together. This provides ease in manufacturing, flexibility and strength.


(iv)

Do you know that the maximum height of a mountain on earth can be ~ 10 km or else the rocks under it will shear under its load.

WHAT YOU HAVE LEARNT

- A force which causes deformation in a body is called deforming force.
- On deformation, internal restoring force is produced in a body and enables it to regain its original shape and size after removal of deforming force.
- The property of matter to restore its original shape and size after withdrawal of deforming force is called elasticity.
- The body which gains completely its original state on the removal of the deforming forces is called perfectly elastic.
- If a body completely retains its modified form after withdrawal of deforming force, it is said to be perfectly plastic.
- The stress equals the internal restoring force per unit area. Its units is Nm^{-2}
- The strain equals the change in dimension (e.g. length, volum or shape) per unit dimension. Strain has no unit.
- In normal state, the net inter-atomic force on an atom is zero. If the separation between the atoms becomes more than the separation in normal state, the interatomic forces become attractive. However, for smaller separation, these forces become repulsive.
- The maximum value of stress up to which a body shows elastic property is called its elastic limit. A body beyond the elastic limit behaves like a plastic body.
- Hooke's law states that within elastic limit, stress developed in a body is directly proportional to strain.

- Young's modulus is the ratio of longitudinal stress to longitudinal strain.
- Bulk modulus is the ratio of normal stress to volume strain.
- Modulus of rigidity is the ratio of the shearing stress to shearing strain.
- Poisson's ratio is the ratio of lateral strain to longitudinal strain.
- The work done in stretching a spring is stored as elastic potential energy of the spring.

TERMINAL EXERCISE

1. Define the term elasticity. Give examples of elastic and plastic objects.
2. Explain the terms stress, strain and Hooke's Law.
3. Explain elastic properties of matter on the basis of inter-molecular forces.
4. Define Young's modulus, Bulk modulus and modulus of rigidity.
5. Discuss the behaviour of a metallic wire under increasing load with the help of stress-strain graph.
6. Why steel is more elastic than rubber.
7. Why poission's ratio has no units.
8. In the three states of matter i.e., solid, liquid and gas, which is more elastic and why?
9. A metallic wire 4m in length and 1mm in diameter is stretched by putting a mass 4kg. Determine the alongation produced. Given that the Young's modulus of elasticity for the material of the wire is 13.78×10^{10} N m^{-2}.
10. A sphere contracts in volume by 0.02% when taken to the bottom of sea 1km deep. Calculate the bulk modulus of the material of the sphere. You make take density of sea water as 1000 kgm^{-3} and $g = 9.8$ms^{-2}.
11. How much force is required to have an increase of 0.2% in the length of a metallic wire of radius 0.2mm. Given $Y = 9 \times 10^{10}$ N m^{-2}.
12. What are shearing stress, shearing strain and modulus of rigidity?
13. The upper face of the cube of side 10cm is displaced 2mm parallel to itself when a tangential force of 5×10^5 N is applied on it, keeping lower face fixed. Find out the strain?
14. Property of elasticity is of vital importance in our lives. How does the plasticity helps us?
15. A wire of length L and radius r is clamped rigidly at one end. When the other end of wire is pulled by a force F, its length increases by x. Another wire of the same material of length $2L$ and radius $2r$, when pulled by a force $2F$, what will be the increase in its length.

ANSWERS TO INTEXT QUESTIONS

8.1

1. If $R > R_0$, the nature of force is attractive and if (ii) $R < R_0$ it is repulsive.
2. Longitudinal stress and linear strain.
3. The ratio will decrease.
4. The stress corresponding to breaking point is known as breaking stress.
5. 0.12×10^{10} N m^{-2}.

8.2

1. Both have same units since strain has no unit?
2. As compressibility of liquids and gases is more than solids, the bulk modulus is reciprocal of compressibility. Therefore solids are more elastic than liquid and gases.
3. Half.
4. 1 : 8
5. 1×10^{11} N m^{-2}.
6. 1 J

Answers To Terminal Problems

9. 0.15 m.

10. 4.9×10^{-10} N m^{-2}

11. 22.7 N

13. 2×10^{-2}

15. *x*.