Lesson-8

Measures of Central Tendency

Summary

In this chapter, we will find out the single value which represents the characteristics of statistical distribution. The study of measures of central tendency provides sufficient basis for comparative of different data sets and contribute s to draw conclusion about the characteristics of entire statistical observations of given data sets. For this purpose , we learn how to calculate mean , median , quartiles and mode. These measures also help to solve our day to day business prolems.

Meaning of Central Tendency

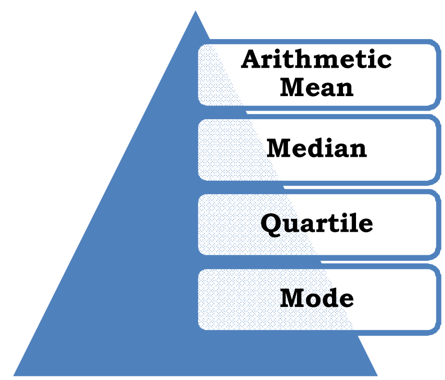
Definition -The measure of central tendency is defined as statistical measure that identifies a single value as the identity of an entire distribution. It aims to provide an accurate description of entire data.

The measure of central tendency summarizes the data in a single value in such a way that this single value represents the entire data. The word average is commonly used in day-to-day conversation.

Types of Average or Measures of central Tendency

The following are important types of average-

Measures of Central Tendency



Arithmetic Mean

Computation of Arithmetic Mean in **Individual Series - Direct Method**

$$\bar{X} = A + \frac{\sum X}{N}$$

Where, $\bar{X}$ = Arithmetic Mean

$\sum X$ = Sum of X_1, X_2, X_3………X_N

N = Sum of observations

Arithmetic Mean

Computation of Arithmetic Mean in **Individual Series - Short-Cut Method**

$$\bar{X} = A + \frac{\sum d}{N}$$

Where, $\bar{X}$ = Arithmetic Mean

A = Assumed Mean

$\sum d = \sum(X - A)$ = Sum of deviations

N = Sum of observations

Arithmetic Mean

Computation of Arithmetic Mean in **Discrete Series - Direct Method**

$$\bar{X} = \frac{\sum fX}{\sum f}$$

Where, $\bar{X}$ = Arithmetic Mean

$\sum fX$ = Sum of Product of observations and frequencies

$\sum f$ = Sum of frequencies

Arithmetic Mean

Computation of Arithmetic Mean in **Discrete Series - Short-Cut Method**

$$\bar{X} = A + \frac{\sum fX}{\sum f}$$

Where, $\bar{X}$ = Arithmetic Mean

A = Assumed Mean

$\sum fX$ = Sum of Product of observations and frequencies

$\sum f$ = Sum of frequencies

Arithmetic Mean

Computation of Arithmetic Mean in **Discrete Series - Step - Deviation Method**

$$\bar{X} = A + \frac{\sum fd'}{\sum f} \times c$$

Where, $\bar{X}$ = Arithmetic Mean

A = Assumed Mean

$\sum fd'$ = Sum of Product of frequency and step-deviation and observation

$$d' = \frac{X-A}{c}$$

$\sum f$ = Sum of frequency

c = Cumulative frequen

Arithmetic Mean

Computation of Arithmetic Mean in **Continuous series-Direct Method**

$$\bar{X} = \frac{\sum fm}{\sum f}$$

Where, $\bar{X}$ = Arithmetic Mean

$\sum fm$ = Sum of product of mid-point and frequency

$\sum f$ = Sum of frequencies

Arithmetic Mean

Computation of Arithmetic Mean in **Continuous series-Assumed Mean Method**

$$\boldsymbol{\bar{X} = A + \frac{\sum fd}{\sum f}}$$

$\bar{X}$ = Arithmetic Mean

A = Assumed Mean

$d = m - A$ Deviation from midpoints

$\sum f$ = Sum of frequencies

Arithmetic Mean

Computation of Arithmetic Mean in **Continuous series-Step Deviation Method**

$$\boldsymbol{\bar{X} = A + \frac{\sum fd'}{\sum f}}$$

$\boldsymbol{\bar{X}}$ = Arithmetic Mean

A = Assumed Mean

$d' = \frac{m-A}{c}$ Deviation from mid- point

$\sum f$ = Sum of frequencies

Combined Mean

If a series of N observations consists of two components having N1 and N2 observations (N1+ N2 = N), and means X1 and X2 respectively then the Combined mean X of N observations is given by

Combined $\bar{X} = \frac{N_1X_1+N_2X_2}{N_1+N_2}$

Properties of Arithmetic Mean

1. The sum of the deviations, of all the values of X, from their arithmetic mean, is zero.

2. The product of the arithmetic mean and the number of items gives the total of all items.

3. The sum of the squares of the deviations of the items taken from arithmetic mean is minimum,

4. If a constant is added or subtracted to all the variables, mean increases or decreases by that constant.

5. If all the variables are multiplied or divided by a constant, mean also gets multiplied or divided by the constant

Weighted Arithmetic Mean

$$\boldsymbol{\bar{X}\,\mathbf{w} = \frac{\sum wX}{\sum w}}$$

Where, $\boldsymbol{\bar{X}}$ **w =** Weighted Arithmetic Mean

$\boldsymbol{\sum wX}$ **=** Product of weights & observations

$\boldsymbol{\sum w}$ **=** Sum of Weight

Median Computation of Median in **Individual Series**

- Arrange data in ascending or descending order
- Median =Size of $\frac{(N+1)th}{2}$ item
- This will help to identify position of $\frac{(N+1)th\ item}{2}$ in the Series and that item will be value of median

Median

Computation of Median in **Discrete Series**

The steps involved in the calculation of median are as follows –

Step 1: Arrange the data in ascending or descending order of magnitude.

Step 2: Find out the cumulative frequency (c.f.)

Step 3: Median = Size of $\frac{(N+1)th}{2}$ item

Step 4 Now look at the cumulative frequency column and find that total which is either equal to N 1 2 + or next higher to that and determine the value of the variable corresponding to it. That gives the value of median.

Computation of Medan in Continuous Series

The steps involved in the calculation of median are as follows

Step 1: Calculate Cumulative Frequencies.

Step 2: Ascertain $\left\{\frac{N}{2}\right\} th$ item.

Step 3: Find out the cumulative frequency which includes $\left\{\frac{N}{2}\right\} th$ item and corresponding class frequency. The corresponding class of this cumulative frequency is called the median class.

Step 4: Calculate Median as Follows

$$\text{Median} = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i$$

Where, l_1 = Lower limit of the median class

c.f. = cumulative frequency of the preceding class

f = frequency of the median class

i = class interval of m1e1dian class

Quartiles

The calculation of quartiles is done exactly in the same manner as it is in case of the calculation of median.

Computation of Q_1 of Q_3 in - **individual and Discrete Series**

First Quartile -

$$Q_1 = Size\ of\ \frac{(N+1)}{4}\ th\ item$$

Third Quartile -

$$Q_3 = Size\ of\ \frac{3(N+1)}{4}\ th\ item$$

Computation of Q_1 of Q_3 in –

Continuous Series

$$Q_1 = Size\ of\ \frac{N}{4}\ th\ item$$

$$Q_1 = l_1 + \frac{(\frac{N}{4} - c.f.)}{f} \times i$$

$$Q_3 = l_1 + \frac{3(\frac{N}{4} - c.f.)}{f} \times i$$

Mode

Mode (M0) is the value around which maximum concentration of items occurs.

Computation of Mode in - **Ungrouped Data/Individual series.**

The mode of this series can be obtained by mere inspection. The number which occurs most often is the mode.

Computation of mode in case of discrete series/and continuous series

(a)Simple inspection method: By simple inspection, the modal value is the value of the variable against which the frequency is the largest.

(b) Grouping and Analysis Table method: This method is generally used when the difference between the maximum frequency and the frequency preceding it or succeeding it is very small.

Process of Computation: In order to find mode, a grouping table and an analysis table are to be prepared in the following manner –

A grouping table consists of 6 columns.

- Arrange the values in ascending order and write down their corresponding frequencies in the column-1.
- In column-2 the frequencies are grouped into two's and added.
- In column-3 the frequencies are grouped into two's, leaving the first frequency and added.
- In column-4 the frequencies are grouped into three's, and added.
- In column-5 the frequencies are grouped into three's, leaving the first frequency and added.

- In column-5 the frequencies are grouped into three's, leaving the first frequency and added.
- In column-6 the frequencies are grouped into three's, leaving the first and second frequencies and added.
- Now in each these columns mark the highest total with a circle.

Formula

$$\text{Mode} = l_1 + \frac{f_1 - f_2}{2f_1 - f_0 - f_2} \times i$$

l_1 = *Lower limit of Modal* Class

f_1 = *Frequency of* Modal Class

f_0 = *Frequency of preceeding* Modal Class

f_2 = *Frequency of sugceeding* Modal Class

i = Class interval of the modal class

Evaluate Yourself

Q. Mention formula for to calculate Arithmetic, Median and Mode in continuous series.

Q. Calculate A.M. and Median from following data-

Wage (Rs.)	20-25	25-30	30-35	35-40	40-45	45-50
Labour	1	3	8	12	7	5

Q. List out the properties of arithmetic mean.

Q. If the quartiles for the following distribution are Q1= 23.125 and Q3=43.5, find the median:

Daily Wages	No of Workers
0-10	5
10-20	-
20-30	20
30-40	30
40-50	-
50-60	10