# Lesson – 9

# Measures of Dispersion


## Summary

The concept of dispersion measures the degree of the variation of the variables about a central value. It denotes the extent to which the values are dispersed about the central values is called dispersion. The measures of variability give us direction to understand how the distribution scatters above and below the central tendency. In this chapter we will discuss about the various types of measures and methods of computing dispersion of different sets of statistical observations in terms of - **Range. Quartile Deviation, Mean Deviation, and Standard Deviation.**


## Meaning of Dispersion

- Dispersion is the extent to which values in a distribution differ from the average of the distribution.
- Dispersion can be measured in terms of absolute value and relative value
- If the dispersion is expressed in terms of the original units of the series, it is called absolute measure of dispersion.
- If the dispersion is expressed in terms of the original units of the series, it is called relative measure of dispersion.

**Classification of measures of dispersion -**

**Absolute measures**

- **Range**
- **Quartile Deviation or Inter Quartile Range**
- **Mean Deviation**
- **Standard Deviation**
- **Lorenz Curve**

**Relative Measures**

- **Coefficien of range**
- **Cefficient of Quartile Deviation**
- **Goefficient of Mean Deviation**
- **Coefficient of Standard Deviation**

## Range

- Range is the difference between the largest and smallest value in a series.

Range( R ) = L – S

Coefficient of Range $= \frac{L-S}{L+S}$

## Computation of Quartile Deviation and coefficient of Quartile Deviation

Difference between $Q_1$ and $Q_3$ is known as inter – quartile range

$$Quartile\ Deviation\ = \frac{Q_3 - Q_1}{2}$$

Where, $Q_1$ = First Quartile

$Q_3 = Third\ Quartile$

$$Coefficient\ of\ Q.D. = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

## Mean Deviation

Mean deviation (MD) of a series is the arithmetic average of the deviation of various items from a measure of central tendency (mean, median and mode).

**Calculation of mean deviation**

$$M.D. = \frac{\sum f\lfloor D \rfloor}{N}$$

Where, $\sum = f\lfloor D \rfloor$ = Sum of product of fre. and deviation.

N = Sum of observation

- Coefficient of M,D, = $\frac{\text{M.D.}}{\text{Median / Mean / Mode}}$

## Standard Deviation (S. D.)

- It measures the absolute dispersion or variability of a distribution. Standard deviation is the positive square root of the mean of the squared deviations of observations from their mean. It is denoted by S.D. or $\sigma_x$.

**Computation of Standard deviation - Individual Series –**

**Actual Mean Method –**

Let X variable takes on N values i.e. $X_1$, $X_2$ … $X_N$. The standard deviation of these N observations using actual mean method can be computed as follows –

- Obtain the arithmetic mean $\bar{X}$ of the given data.
- Obtain the deviation of each $i^{th}$ observation from $\bar{X}$ i.e. $(X_i - \bar{X})$. (Note that $\Sigma(X_i - \bar{X}) = 0$)
- Square each deviation i.e. $(X_i - \overline{\bar{X}})^2$.
- Obtain the sum in step 3 i.e. $\sum(X_i - \overline{\bar{X}})^2$.
- Obtain the square root of the mean of these squared deviations as follows

Standard Deviation $\sigma_x$

$$= \sqrt{\frac{\sum(X-\bar{X})^2}{N}}$$

N =Total no. of observation

**Assumed Mean Method**

- This method is applied to calculate the standard deviation when the mean of the data is in fraction.
- Thus the deviations (d) are taken from the Assumed mean (A) and standard deviation is estimated by using the following formula.

**Standard Deviation** $\sigma_x$

$$= \sqrt{\frac{\sum d^2}{N} - \left(\frac{\sum d}{N}\right)^2}$$

**Direct Method**

- The relevance of this method is particularly useful when the items are very small. To obtain standard deviations, we apply the following formula

**Standard Deviation** $(\sigma_X)$

$$= \sqrt{\frac{\sum X^2}{N} - (\bar{X})^2}$$

Where, $\bar{X} = Arithmetic\ Mean$

**Step Deviation Method**

- In this method we divide the deviations by a common class interval (c) and use the following formula for computing standard deviation.

**Standard Deviation** $\sigma_x$

$$= \sqrt{\frac{\sum d'^2}{N} - \left(\frac{\sum d'}{N}\right)^2} \times c$$

Where, $d' = \left(\frac{X-A}{c}\right)$ i.e. deviation taken from assumed mean and divided by class interval c.

## Computation of Standard Deviation in - Continuous Series

**Actual Mean Method**

Standard Deviation $\sigma_x$

$$= \sqrt{\frac{\sum fX^2}{\sum f}}$$

Where, $x = (m-\bar{X})$ i. e. deviations taken from arithmetic mean.

**Assumed Mean Method**

**Standard Deviation** $\sigma_x$

$$= \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

Where, $d = (m - A)$ i.e. deviations taken from assumed mean.

**Step Deviation Method**

- In this method we divide the deviations by a common class interval (c) and use the following formula for computing standard deviation.

  **Standard Deviation $\sigma_x$**

$$= \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} \times c$$

Where, $d' = \left(\frac{m - A}{c}\right)$ i.e. deviation taken from assumed mean and divided by class interval c.

**Computing coefficient of variation of standard deviation**

- The values of the standard deviations cannot be used as the basis of the comparison mainly because units of measurements of the two distributions may be different. The correct measure that should be used for comparison purposes is the Coefficient of Variation (C.V.) given by Karl Pearson.

$$C.V. = \frac{\sigma_X}{\bar{X}} \times 100$$

$\sigma_X = S.D.$, $\bar{X} = Mean$ of X variable

**Properties of Standard Deviation**

- The value of Standard Deviation remains same if each observation in a series is increased or decreased by a constant value i.e. Standard deviation is independent of change of origin.
- The value of Standard Deviation changes if each of observation in a series is multiplied or divided by a constant value i.e. Standard deviation is not independent of

## Evaluate Yourself

Q. Define Dispersion and also list out various measures and methods of computing dispersion.

Q. Given the following data, find out Quartile Deviation and Coefficient of Quartile Deviation.

| Wages in Rs. | No. Of Workers |
|---|---|
| 10 - 20 | 4 |
| 20 - 30 | 6 |
| 30 - 40 | 3 |
| 40 - 50 | 8 |
| 50 - 60 | 12 |
| 60 - 70 | 7 |

Q. Calculate Mean Deviation coefficient of Mean Deviation from mean and Mode of above given data.

Q. Calculate S. D. by Direct, Assume Mean Method and Step Deviation Method of data given in Q.2