

## INDEX NUMBERS

Of the important statistical devices and techniques, Index Numbers have today become one of the most widely used for judging the pulse of economy, although in the beginning they were originally constructed to gauge the effect of changes in prices. Today we use index numbers for cost of living, industrial production, agricultural production, imports and exports, etc.

Index numbers are the indicators which measure percentage changes in a variable (or a group of variables) over a specified time. By saying that the index of export for the year 2001 is 125 , taking base year as 2000, it means that there is an increase of $25 \%$ in the country's export as compared to the corresponding figure for the year 2000.

## OBJECTIVES

After studying this lesson, you will be able to :

- define index numbers and explain their uses;
- identify and use the following methods for construction of index numbers :
(i) aggregate method (ii) simple average of relative method; and
- explain the advantages of different methods of construction.


## EXPECTED BACKGROUND KNOWLEDGE

- Knowledge of commercial mathematics
- Measures of Central Tendency


### 38.1 INDEX NUMBERS-DEFINITION

Some prominent definitions, given by statisticians, are given below:

## According to the Spiegel :

"An index number is a statistical measure, designed to measure changes in a variable, or a group of related variables with respect to time, geographical location or other characteristics such as

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income, profession, etc."

## According to Patternson :

" In its simplest form, an index number is the ratio of two index numbers expressed as a percent . An index is a statistical measure, a measure designed to show changes in one variable or a group of related variables over time, with respect to geographical location or other characteristics".

## According to Tuttle :

"Index number is a single ratio (or a percentage) which measures the combined change of several variables between two different times, places or situations".

We can thus say that index numbers are economic barometers to judge the inflation (increase in prices) or deflationary (decrease in prices ) tendencies of the economy. They help the government in adjusting its policies in case of inflationary situations.

### 38.2 CHARACTERISTICS OF INDEX NUMBERS

Following are some of the important characteristics of index numbers :

- Index numbers are expressed in terms of percentages to show the extent of relative change
Index numbers measure relative changes. They measure the relative change in the value of a variable or a group of related variables over a period of time or between places.
Index numbers measures changes which are not directly measurable.
The cost of living, the price level or the business activity in a country are not directly measurable but it is possible to study relative changes in these activities by measuring the changes in the values of variables/factors which effect these activities.


### 38.3 PROBLEMS IN THE CONSTRUCTION OF INDEX NUMBERS

The decision regarding the following problems/aspect have to be taken before starting the actual construction of any type of index numbers.
(i) Purpose of Index numbers under construction
(ii) Selection of items
(iii) Choice of an appropriate average
(iv) Assignment of weights (importance)
(v) Choice of base period

Let us discuss these one-by-one

### 38.3.1 Purpose of Index Numbers

An index number, which is designed keeping, specific objective in mind, is a very powerful tool. For example, an index whose purpose is to measure consumer price index, should not include wholesale rates of items and the index number meant for slum-colonies should not consider luxury items like A.C., Cars refrigerators, etc.

### 38.3.2 Selection of Items

After the objective of construction of index numbers is defined, only those items which are

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related to and are relevant with the purpose should be included.

### 38.3.3 Choice of Average

As index numbers are themselves specialised averages, it has to be decided first as to which average should be used for their construction. The arithmetic mean, being easy to use and calculate, is preferred over other averages (median, mode or geometric mean). In this lesson, we will be using only arithmetic mean for construction of index numbers.

### 38.3.4 Assignment of weights

Proper importance has to be given to the items used for construction of index numbers. It is universally agreed that wheat is the most important cereal as against other cereals, and hence should be given due importance.

### 38.3.5 Choice of Base year

The index number for a particular future year is compared against a year in the near past, which is called base year. It may be kept in mind that the base year should be a normal year and economically stable year.

### 38.4 USES OF INDEX NUMBERS

(i) Index numbers are economic barometers. They measure the level of business and economic activities and are therefore helpful in gauging the economic status of the country.
(ii) Index numbers measure the relative change in a variable or a group of related variable(s) under study.
(iii) Consumer price indices are useful in measuring the purchasing power of money, thereby used in compensating the employes in the form of increase of allowances.

### 38.5 TYPES OF INDEX NUMBERS

Index numbers are names after the activity they measure. Their types are as under :
Price Index : Measure changes in price over a specified period of time. It is basically the ratio of the price of a certain number of commodities at the present year as against base year.

Quantity Index : As the name suggest, these indices pertain to measuring changes in volumes of commodities like goods produced or goods consumed, etc.

Value Index : These pertain to compare changes in the monetary value of imports, exports, production or consumption of commodities.

### 38.6 CONSTRUCTION OF INDEX NUMBERS

Suppose one is interested in comparing the sum total of expenditure on a fixed number of commodities in the year 2003 as against the year 1998. Let us consider the following example.

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|  |  |  | Inde |
| :---: | :---: | :---: | :---: |
| OPTIONAL - II | Commodity |  | (in Rupees) |
| Commerce, Economics |  | 1998 | 2003 |
| - | Wheat | 200 | 400 |
| - | Petrol | 25 | 36 |
| Notes | Pulses | $12 \frac{1}{2}$ | 24 |
|  | Sugar | 10 | 18 |
|  | Cooking Oil | 80 | 80 |
|  | Cloth | 40 | 50 |

Since all the commodities are in different units and their prices are not enlarged proportionally, we just cannot get an average for comparison. For that reason, we express the rates of all commodities in 1998 as 100 each and proportionally increase for the corresponding commodities for 2003.

| Commodity | 1990 |  | 2003 |  |
| :--- | :--- | :--- | :--- | :--- |
|  | price | Index | Price | Index |
| Wheat | 200 | 100 | 400 | $\frac{400 \times 100}{200}=200$ |
| Petrol | 25 | 100 | 36 | $\frac{100}{25} \times 36=144$ |
| Pulses | $12 \frac{1}{2}$ | 100 | 24 | $\frac{100}{12.5} \times 24=192$ |
| Sugar | 10 | 100 | 18 | $\frac{100}{10} \times 18=180$ |
| Cooking Oil | 80 | 100 | 80 | $\frac{100}{80} \times 80=100$ |
| Cloth | 40 | 100 | 50 | $\frac{100}{40} \times 50=125$ |
| Average |  | 100 | Average | $941 \div 6=156.83$ |

We find that the average number (Index) for 2003 is 156.83 as against 100 for the year 1998. We can say that the prices have gone up by $56.83 \%$ in the year 2003 as against 1998. This method is used for finding price index numbers.

### 38.7 METHODS OF CONSTRUCTING INDEX NUMBERS

Construction of index numbers can be divided into two types :
(a) Unweighted indices

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(b) Weighted indices

In this lesson, we will discuss only the unweighted indices:
The following are the methods of constructing unweighted index numbers :
(i) Simple Aggregative method
(ii) Simple average of price relative method

### 38.7.1 Simple Aggregative Method

This is a simple method for constructing index numbers. In this, the total of current year prices for various commodities is divided by the corresponding base year price total and multiplying the result by 100 .
$\therefore$ Simple Aggregative Price Index $\mathrm{P}_{01}=\frac{\Sigma \mathrm{p}_{1}}{\Sigma \mathrm{p}_{0}} \times 100$
Where $P_{01}=$ Current price Index number
$\Sigma \mathrm{p}_{1}=$ the total of commodity prices in the current year
$\Sigma \mathrm{p}_{0}=$ the total of same commodity prices in the base year.
Let us take an example to illustrate :
Example 38. 1 Construct the price index number for 2003, taking the year 2000 as base year

| Commodity | Price in the year | Price in the year |
| :---: | :---: | :---: |
|  | $\mathbf{2 0 0 0}$ | $\mathbf{2 0 0 3}$ |
| A | 60 | 80 |
| B | 50 | 60 |
| C | 70 | 100 |
| D | 120 | 160 |
| E | 100 | 150 |

Solution : Calculation of simple Aggregative index number for 2003 (against the year 2000)

| Commodity | Price in 2000 <br> (in Rs) $p_{0}$ | Price in 2003 <br> (in Rs.) $p_{1}$ |
| :---: | :---: | :---: |
| A | 60 | 80 |
| B | 50 | 60 |
| C | 70 | 100 |
| D | 120 | 160 |
| E | 100 | 150 |
| Total | $\sum \mathrm{p}_{0}=400$ | $\sum \mathrm{p}_{1}=550$ |

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$$
\begin{array}{ll}
\text { Here } & \sum \mathrm{p}_{0}=400, \sum \mathrm{p}_{1}=550 \\
\therefore & \mathrm{P}_{\mathrm{ol}}=\frac{\Sigma \mathrm{p}_{1}}{\Sigma \mathrm{p}_{0}} \times 100=\frac{550}{400} \times 100 \\
& =\frac{275}{2}=137.5
\end{array}
$$

i.e. the price index for the year 2003, taking 2000 as base year, is 137.5 , showing that there is an increase of $37.5 \%$ in the prices in 2003 as against 2000.

Example 38.2 Compute the index number for the years 2001, 2002, 2003 and 2004, taking
2000 as base year, from the following data :

| Year | $\mathbf{2 0 0 0}$ | $\mathbf{2 0 0 1}$ | $\mathbf{2 0 0 2}$ | $\mathbf{2 0 0 3}$ | $\mathbf{2 0 0 4}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Price | 120 | 144 | 168 | 204 | 216 |

Solution : Price relatives for different years are

| 2000 | $\frac{120}{120} \times 100=100$ |
| :---: | :--- |
| 2001 | $\frac{144}{120} \times 100=120$ |
| 2002 | $\frac{168}{120} \times 100=140$ |
| 2003 | $\frac{204}{120} \times 100=170$ |
| 2004 | $\frac{216}{120} \times 100=180$ |

$\therefore$ Price index for different years are :

| Year | 2000 | 2001 | 2002 | 2003 | 2004 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Price-Index | 100 | 120 | 140 | 170 | 180 |

Example 38.3 Prepare simple aggregative price index number from the following data :

| Commodity | Rate Unit | Price (1995) | Price (2004) |
| :--- | :---: | :---: | :---: |
| Wheat | per 10 kg | 100 | 140 |
| Rice | per 10 kg | 200 | 250 |
| Pulses | per 10 kg | 250 | 350 |
| Sugar | per kg | 14 | 20 |
| Oil | per litre | 40 | 50 |

## Index Numbers

Solution : Calculation of simple aggregative index number.

| Commodity | Rate Unit | Price (1995) | Price (2004) |
| :--- | :---: | :---: | :---: |
| Wheat | per 10 kg | 100 | 140 |
| Rice | per 10 kg | 200 | 250 |
| Pulses | per 10 kg | 250 | 350 |
| Sugar | per kg | 14 | 20 |
| Oil | per litre | 40 | 50 |
|  |  | $\mathbf{6 0 4}$ | $\mathbf{8 1 0}$ |

Simple Aggregative index number

$$
=\frac{810}{604} \times 100=134.1
$$

CHECK YOUR PROGRESS 38.1

1. Write the characteristics and uses of index numbers.
2. Enumerate the problems /aspects in the construction of index numbers.
3. Find the simple aggregative index number for each of the following :
(i) For the year 2000 with 1980 as base year

| Commodity | Price in 1980 | Price in 2000 |
| :---: | :---: | :---: |
| A | 200 | 250 |
| B | 110 | 150 |
| C | 20 | 30 |
| D | 210 | 250 |
| E | 25 | 25 |

(ii) For the years 1999, 2000, 2001, 2002, 2003 taking 1998 as base year

| Year | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 |
| :---: | :--- | :---: | :---: | :--- | :--- | :--- |
| Price | 20 | 25 | 28 | 30 | 35 | 40 |

(iii) For the years 2001 and 2002 taking 1999 as base year.

| Commodity |  | A | B | C | D | E |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| price in | 1999 | 10 | 25 | 40 | 30 | 25 |
|  | 12 | 30 | 50 | 30 | 25 | 110 |
|  | 2001 | 15 | 30 | 60 | 40 | 30 |
|  | 2002 |  |  |  |  |  |

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Notes

### 38.7.2 Simple Average of Price Relatives Method

In this method, the price relatives for all commodities is calculated and then their average is taken to calculate the index number.
Thus, $\mathrm{P}_{01}=\frac{\Sigma \frac{\mathrm{p}_{1}}{\mathrm{p}_{0}} \times 100}{\mathrm{~N}}$, if A.M. is used as average where $\mathrm{P}_{01}$ is the price index, N is the number of items, $\mathrm{p}_{0}$ is the price in the base year and $\mathrm{p}_{1}$ of corresponding commodity in present year (for which index is to be calculated)

Let us take an example.
Example 38.4 Construct by simple average of price relative method the price index of 2004, taking 1999 as base year from the following data :

| Commodity | A | B | C | D | E | F |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Price (in 1999) | 60 | 50 | 60 | 50 | 25 | 20 |
| Price (in 2004) | 80 | 60 | 72 | 75 | $37 \frac{1}{2}$ | 30 |

Solution :

| Commodity | Price (in 1999)$\text { (in Rs.) }\left[\mathrm{p}_{0}\right]$ | Price (in 2004) | Price Relatives |
| :---: | :---: | :---: | :---: |
|  |  | (in Rs.) [ $\mathrm{p}_{1}$ ] | $\left(\frac{\mathrm{p}_{1}}{\mathrm{P}_{0}} \times 100\right)$ |
| A | 60 | 80 | 133.33 |
| B | 50 | 60 | 120.00 |
| C | 60 | 72 | 120.00 |
| D | 50 | 75 | 150.00 |
| E | 25 | $37 \frac{1}{2}$ | 150.00 |
| F | 20 | 30 | 150.00 |
|  |  |  | 823.33 |

$$
\mathrm{P}_{01}=\frac{\sum \frac{\mathrm{p}_{1}}{\mathrm{p}_{0}} \times 100}{\mathrm{~N}}=\frac{823.33}{6}=137.22
$$

Price index for 2004, taking 1999 for base year $=137.22$
Example 38.5 Using simple average of Price Relative Method find the price index for 2001, taking 1996 as base year from the following data :

## Index Numbers

| Commodity | Wheat | Rice | Sugar | Ghee | Tea |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Price (in 1996) per unit | 12 | 20 | 12 | 40 | 80 |
| Price (in 2001) per unit | 16 | 25 | 16 | 60 | 96 |

Solution :
\(\left.$$
\begin{array}{|l|c|c|c|}\hline \text { Commodity } & \text { Price (in 1996) } & \text { Price (in 2001) } & \begin{array}{c}\text { Price Relatives } \\
\text { (in Rs.) }\left[\mathrm{p}_{0}\right]\end{array}
$$ <br>

(in Rs.)\left[\mathrm{p}_{1}\right] \& \left(\frac{\mathrm{p}_{1}}{\mathrm{P}_{0}} \times 100\right)\end{array}\right]\)| Wheat |
| :--- |
| Rice |
| Sugar |
| Ghee |

$$
\begin{aligned}
\therefore \quad \mathrm{P}_{01} & =\frac{\Sigma \frac{\mathrm{p}_{1}}{\mathrm{p}_{0}} \times 100}{\mathrm{~N}} \\
& =\frac{661.66}{5}=132.33
\end{aligned}
$$

$\therefore$ Price Index for 2001, taking 1996 as base year, $=132.33$

## (D) CHECK YOUR PROGRESS 38.2

Using Simple Average of Relatives Method, find price index for each of the following :
(i) For 2004, taking 2000 as base year

| Commodity | A | B | C | D | E |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Price in 2000 | 15 | 16 | 60 | 40 | 20 |
| Price in 2004 | 20 | 20 | 80 | 50 | 25 |

(ii) For 2001, taking 1999 as base year


| Commodity | Wheat | Rice | Sugar | Ghee | Tea |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Price (per unit) in 1999 | 10 | 20 | 60 | 40 | 16 |
| Price (per unit) in 2001 | 12 | 22 | 80 | 50 | 20 |

## LET USSUM UP

- An index number is a statistical measure, designed to measure changes in a variable(s) with time/geographical location/other criteria
- Index Numbers are of three types :
(i) Price-Index Numbers
(ii) Quantity Index Numbers
(iii) Value-index Numbers
- Method of construction of Index numbers
(i) Simple Aggregative method $\mathrm{P}_{01}=\Sigma \frac{\mathrm{p}_{1}}{\mathrm{p}_{0}} \times 100$
where $\quad \mathrm{P}_{01}$ is the price index
$\mathrm{p}_{0}$ is the price of a commodity in base year
$\mathrm{p}_{1}$ is the price of the commodity in present year
(ii) Simple Average of Price Relatives Method

$$
\mathrm{P}_{01}=\frac{\Sigma \frac{\mathrm{p}_{1}}{\mathrm{p}_{0}} \times 100}{\mathrm{~N}}
$$

Where N is the number of commodities and all others as in (i) above.


SUPPORTIVE WEB SITES
http :// www.wikipedia.org
http :// mathworld.wolfram.com

## TERMINAL EXERCISE

1. Use Simple Aggregative Method, find the price index for each of the following :

## Index Numbers

(i) For the year 2000, taking 1990 as base year

| Commodity | A | B | C | D | E |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Price (in Rs.) in 1990 | 10 | 14 | 18 | 20 | 100 |
| Price (in Rs.) in 2000 | 12 | 20 | 20 | 25 | 110 |

(ii) For the year 2004, taking 1998 as base year

| Commodity | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Price in 1998 | 20 | 28 | 110 | 80 | 60 | 20 |
| Price in 2004 | 25 | 40 | 120 | 100 | 80 | 25 |

(iii) For the year 1996, 1997, 1998, 1999, Taking 1990 as base year

| Commodity |  | A | B | C | D |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Price in | 1990 | 5 | 8 | 10 | 22 |
|  | 1996 | 10 | 12 | 20 | 18 |
|  | 1997 | 12 | 15 | 20 | 16 |
|  | 1998 | 10 | 15 | 25 | 22 |
|  | 1999 | 15 | 20 | 28 | 22 |

2. Using Simple Average of Price Relative Method, find the price index for each of the following:
(i) For 2000, taking 1998 as base year

| Commodity | A | B | C | D | E |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Price (in Rs.) 1998 | 12 | 20 | 24 | 28 | 20 |
| Price (in Rs.) 2000 | 16 | 25 | 30 | 35 | 26 |

(ii) For the year 2004, taking 1999 as base year

| Commodity | A | B | C | D | E | F |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Price (in Rs.) in 1999 | 12 | 28 | 32 | 36 | 40 | 50 |
| Price (in Rs.) in 2004 | 16 | 35 | 40 | 45 | 50 | 60 |

(iii) For the years 2003 and 2004 Taking 1998 as base year

| Commodity | A | B | C | D |
| :--- | :---: | :---: | :---: | :---: |
| Price (in Rs.) in 1998 | 4 | 28 | 30 | 40 |
| Price (in Rs.) in 2003 | 5 | 35 | 36 | 50 |
| Price (in Rs.) in 2004 | 6 | 42 | 42 | 65 |

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## CHECK YOUR PROGRESS 38.1

3. (i) 124.78
(ii) $1999: 125 ; 2000: 140 ; 2001: 150 ; 2002: 175 ; 2003: 200$
(iii) 11.74 ; 128.26

## CHECK YOUR PROGRESS 38.2

(i) 128.33
(ii) 122.67

TERMINAL EXERCISE
1.
$\begin{array}{ll}\text { (i) } 115.43 & \text { (ii) } 122.64\end{array}$
(iii) 1996: 133.33 ; 1997: 140.00 ; $1998: 160.0$; 1999:188.88

2
(i) 127.67
(ii) 125.56
(iii) 2003:123.75; $2004: 150.625$

